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Giffen Behaviour and Asymmetric Substitutability*

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Abstract
Let a consumer consume two goods, and let good 1 be a Giffen good. Then a well-known necessary condition for such behaviour is that good 1 is an inferior good. This paper shows that an additional necessary for such behaviour is that good 1 is a gross substitute for good 2, and that good 2 is a gross complement to good 1 (*strong asymmetric gross substitutability*). It is argued that identifying asymmetric gross substitutability as an additional necessary condition gives better insight into Giffen behaviour, both on an analytical level and an intuitive level. In particular, the paper uses the concept of asymmetric gross substitutability to give a taxonomy of preferences, which includes preferences that are locally characterised by Giffen behaviour, and also uses this concept to introduce new decompositions of the effect of a change in own price on the demand for a good, different from those known in the literature.

Keywords: Giffen behaviour, asymmetric substitutability

JEL classification: D11.

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1. Introduction

Any paper that treats Giffen behaviour may be confronted with the criticism: to what extent is such behaviour relevant? While there is empirical evidence for Giffen behaviour (Battalio et al., 1991; Jensen and Miller, 2008), one may still argue that the law of demand is violated only infrequently. Yet, the point of Giffen behaviour is that it is a tool to get insight into what determines the price elasticity of demand. The fact that the income effect may exceed the substitution effect helps remind us that these two determining effects may not operate in the same direction. Also, even if one accepts Giffen goods do not occur very frequently, there still remains a class of goods that are very similar to Giffen goods, in that their price elasticity of demand is negative but very small, or is even zero. Getting insight into the extreme possibility of Giffen goods also gives us insight into these similar goods, which in this reasoning are important as they occur more frequently.

In Section 2, we derive a list of necessary (though not sufficient) conditions for Giffen behaviour. This includes well-known necessary conditions such as non-homothetic preferences, and the fact that the Giffen good must also be an inferior good while the other good consumed is a luxury (strong asymmetric income elasticity), which we refer to as strong asymmetric income elasticity. To these well-known conditions, we add the necessary conditions of weak asymmetric gross substitutability, where the cross-price effects on the two goods are unequal, and of strong asymmetric gross substitutability (De Jaegher, 2009), where good 1 is a gross substitute for good 2, while good 2 is a gross complement to good 1.

In Section 3, we show the relations between all these necessary conditions, and show that the set of preferences characterised by Giffen behaviour lies in the intersection of the set of preferences characterised by strong asymmetric income elasticity, and the set of preferences characterised by strong asymmetric gross substitutability. Moreover, it is shown that a strict focus on strong asymmetric income elasticity may cause us to include preferences that do not approach Giffen behaviour. While a necessary condition for Giffen behaviour on good 1 is that good 1 is an inferior good and good 2 a luxury, this is not a sufficient condition. Indeed, when good 1 is an inferior good and good 2 a luxury, it may be that good 1 is price elastic and good 2 is price inelastic, which is the opposite of what we need for Giffen behaviour.

Section 4 given an intuition for why the latter may occur. In particular, this may occur when low-quality goods are considered as candidate Giffen goods. We argue that such low-quality goods, while being inferior goods, may at the same time be price elastic, and more so than the other good consumed, making them bad examples of Giffen goods. Instead, it is argued that a good example of a Giffen good is a *high*-quality good, in the sense that we must have a good that is able to serve multiple functions if necessary.

Consider for instance a consumer who consumes only rice and meat. If the consumer's purchasing power is low, rice is able to perform both the function of providing the consumer with subsistence and with taste, and are in this sense a high-quality good. Potatoes are a gross substitute for meat, precisely because they can very well take over the role of subsistence from meat. Because meat has a substitute, it is price elastic. At the same time, meat is a gross complement to rice, because it complements rice in making food more tasty, but may not be able to perform well as an alternative source for subsistence. Because rice does not have a substitute, it is price inelastic. This intuition also applies when the other good consumed is a composite good consisting of all other goods rather than meat, where this composite commodity may be considered as giving flavour to life.

Section 5 treats alternative decompositions of the own-price effect to the well-known Hicksian (and Slutsky) decomposition. These well-known decompositions are founded in the necessary condition for Giffen behaviour of strong asymmetric income elasticity. Given the stress that we have put on strong asymmetric gross substitutability as an additional necessary condition, we treat alternative decompositions that are founded in this concept. We end with some conclusions in Section 6.

2. Derivation of necessary conditions

In this section, we treat several well-known necessary conditions for Giffen behaviour, and introduce some new necessary conditions. We focus on the case where two goods are consumed, and where good 1 is the potential Giffen good. The analysis is easily extended to multiple good when good 2 is thought of as a composite good.[1] In the context of this book, it is important to not that we will not always make all underlying assumptions explicit. E.g., when we apply the Slutsky equation, we simply assume that the conditions necessary for the application of this equation are valid.

The necessary conditions we derive all boil down to some form of asymmetry in the effect of income and price changes on the consumption of the two goods. The necessary conditions are ordered from weak necessary conditions to stronger ones. Section 2.1 shows that good 1 should be a necessity, and good 2 a luxury. Section 2.2 shows that good 1 should be a relatively better substitute for good 2 than good 2 is for good 1. Section 2.3 repeats the well-known necessary condition that good 1 should not only be a necessity but should also be an inferior good. Section 2.4 shows that good 1 should be a gross substitute for good 2, while good 2 should be a gross complement to good 1 (strong asymmetric gross substitutability). Section 2.5. finally shows that good 1 should be price inelastic, while good 2 should be price elastic (strong asymmetric price elasticity).

2.1. Unequal income elasticities: weak asymmetric income elasticity

A necessity (luxury) is defined as good *i* for which the income elasticity, denoted $\varepsilon_{x_i m}$, is smaller (larger) than one: $\varepsilon_{x_i m} < 1$ ($\varepsilon_{x_i m} > 1$). Define as weak asymmetric income elasticity the case where one good is a necessity, and the other good is a luxury.

Definition 1. Two goods 1 and 2 are locally characterized by *weak asymmetric income elasticity* if it is locally the case that $\varepsilon_{x_1 m} < 1$ and $\varepsilon_{x_2 m} > 1$.

This leads us to the following proposition.

Proposition 1. A necessary condition for good 1 to be locally a Giffen good (i.e. locally $(\partial x_1 / \partial p_1) > 0$) is that preferences are locally characterised by weak asymmetric income elasticity, where locally good 1 is a necessity and good 2 a luxury.
Proof:

As is well-known, the Hicksian decomposition of the own price-derivatives in a substitution effect and an income effect takes the form

[1] Specifically, by Hicks' Composite Commodity Theorem (for a proof, see Carter, 1995), all other goods than good 1 can be treated as a composite good as long as the relative prices of all these other goods do not change. A change in the price of good 2 is then interpreted as a price change that leaves the relative prices of all other goods than good 1 intact.

$$\frac{\partial x_1}{\partial p_1} = \frac{\partial h_1}{\partial p_1} - x_1 \frac{\partial x_1}{\partial m} \tag{1}$$

$$\frac{\partial x_2}{\partial p_2} = \frac{\partial h_2}{\partial p_2} - x_2 \frac{\partial x_2}{\partial m} \tag{2}$$

where h_i denotes the compensated Hicksian demand for good *i*. As $\partial h_1 / \partial p_1 < 0$, it follows that a necessary condition for good 1 to be a Giffen good is that $\partial x_1 / \partial m < 0$, meaning that a necessary condition is that $\varepsilon_{x_1 m} < 1$, or good 1 is a necessity.

Furthermore, by the Engel aggregation identity, we have

$$s_1 \varepsilon_{x_1 m} + s_2 \varepsilon_{x_2 m} = 1, \tag{3}$$

where $s_i = (p_i x_i)/m$ is the share of the budget that the consumer spends on good *i*. By budget balancedness, $s_1 + s_2 = 1$. It follows from this that if good 1 is a necessity ($\varepsilon_{x_1 m} < 1$), good 2 must be a luxury ($\varepsilon_{x_2 m} > 1$). QED

Another way to state Proposition 1 is that a necessary condition for Giffen behaviour is that the consumer has *non-homothetic preferences*. Concretely, homothetic utility functions have what we can refer to as *strong symmetric income elasticity* ($\varepsilon_{x_1 m} = \varepsilon_{x_2 m} = 1$), a property which by Proposition 1 should not apply under Giffen behaviour.

Corollary 1. A necessary condition for good 1 to be locally a Giffen good is that the consumer locally has non-homothetic preferences.
Proof:
We show that local homothetic preferences imply income elasticities that are locally all equal to 1. Take a bundle (x_1, x_2), and derive from this a bundle (tx_1, tx_2), for $t > 1$. By the fact that indifference curves are blown-up versions from one another, the marginal rate of substitution at bundles (x_1, x_2) and (tx_1, tx_2) is the same. This means that, when income is increased by a factor *t*, consumption of each good increases by a factor *t*. In other words, income elasticity is equal to 1 for both goods. The fact that local homothetic preferences implies unit income elasticities can be restates as saying that non-unit income elasticities (= weak asymmetric income elasticity) implies non-homothetic preferences. Since by Proposition 1, Giffen behaviour implies weak asymmetric income elasticity, and weak asymmetric income elasticity implies non-homothetic preferences, it follows that local non-homothetic preferences are a necessary condition for local Giffen behaviour. QED

As shown in Figure 1a, for any given indifference curve, we can find a corresponding homothetic indifference map by "blowing up" the given indifference curve u_0. We then have linear income expansion paths, denoted by the dashed lines. Non-homothetic preferences, where good 1 is a necessity (and possibly an inferior good) and good 2 is a luxury are now obtained by making the blown up versions of the given indifference curve flatter, as illustrated in Figure 1b. We then obtain convex income expansion paths. Graphically, a necessary condition for Giffen behaviour is that higher indifference curves tilt as in Figure 1b. In Figure 1b, good 1 is a necessity, but not an inferior good, as the income expansion paths do not have a negative slope.

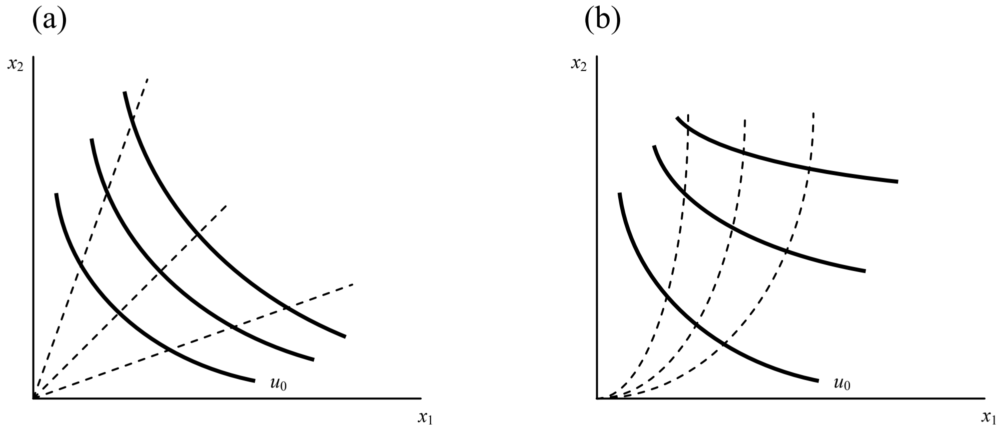


Figure 1 Homothetic and non-homothetic preferences

2.2. Inferior good/luxury: strong asymmetric income elasticity

By the analysis in Section 2.1, we can be more precise about the necessary condition referring to income elasticity. Define good *i* as an inferior good when $\varepsilon_{x_i m} < 0$. We can then additionally introduce the concept of strong asymmetric income elasticity.

Definition 2. Two goods 1 and 2 are locally characterized by *strong asymmetric income elasticity* if locally one good is an inferior good ($\varepsilon_{x_1 m} < 0$) and the other good a luxury ($\varepsilon_{x_2 m} > 1$).

A well-known result then allows us to formulate the necessary conditions for Giffen behaviour sharper:

Proposition 2. A necessary condition for good 1 to be locally a Giffen good is that preferences are locally characterised by strong asymmetric income elasticity, where locally good 1 is an inferior good and good 2 a luxury.
Proof: See the proof of Proposition 1, which not only shows the weaker statement of Proposition 1, but also the stronger statement of the current proposition.

2.3. Unequal gross cross-price effects: weak asymmetric gross substitutability

Section 2.1 shows that a necessary condition for Giffen behaviour is weak asymmetric income elasticity, meaning that we have non-homothetic preferences. We here show that this asymmetry can be expressed in another form, in the sense that a necessary condition for Giffen behaviour is that goods are *asymmetrically gross substitutable*. The expression $(\partial x_i / \partial p_j)$ (with $i \neq j$) may be seen as measuring the extent to which good *i* is a gross substitute for $(\partial x_i / \partial p_j > 0)$ or gross complement to $(\partial x_i / \partial p_j < 0)$ good *j*. (A)symmetric gross substitutability compares this measure across goods. We start by showing that with homothetic preferences, goods are what we call strongly symmetrically gross substitutable, we first define:

Definition 3. Two goods 1 and 2 are locally characterized by *strong symmetric gross substitutability* if $(\partial x_1 / \partial p_2) = (\partial x_2 / \partial p_1)$.

This leads us to the following proposition:

Proposition 3. The consumer locally has homothetic preferences over goods 1 and 2 ($\varepsilon_{x_1 m} = \varepsilon_{x_2 m} = 1$) if and only if it is locally true that $(\partial x_2 / \partial p_1) = (\partial x_1 / \partial p_2)$.
Proof: Consider the Hicksian decomposition of the cross-price effects:

$$\frac{\partial x_1}{\partial p_2} = \frac{\partial h_1}{\partial p_2} - x_2 \frac{\partial x_1}{\partial m} \tag{4}$$

$$\frac{\partial x_2}{\partial p_1} = \frac{\partial h_2}{\partial p_1} - x_1 \frac{\partial x_2}{\partial m} \tag{5}$$

As is well-known, given the symmetry of the Slutsky matrix, the Hicksian cross-price effects are equal, or $(\partial h_1 / \partial p_2) = (\partial h_2 / \partial p_1)$. Combining (4) and (5), we see that $(\partial x_2 / \partial p_1) = (\partial x_1 / \partial p_2) \Leftrightarrow \varepsilon_{x_1 m} = \varepsilon_{x_2 m}$. QED

The fact that local homothetic preferences are equivalent to strong symmetric gross substitutability is illustrated in Figure 2. In Figure 2(a), 2(b) and 2(c), homothetic preferences are each time obtained by constructing blown-up versions of the indifference curve in bold. For a bundle 1 on the bold indifference curve, we construct a line crossing the origin and bundle 1. On every bundle 2 on this line, consumption of each good has increased by the same factor. For an income increase (parallel shift in the budget line through 1) that makes bundle 2 affordable, bundle 2 should be chosen, so that the indifference curve through bundle 2 must have the same slope. In order to highlight the relation between homotheticity and symmetric substitutability, the income increase from bundle 1 to bundle 2 can be split up in a price decrease in good 1 (from bundle 1 to bundle 3), and an equi-proportional price decrease in good 2 (from bundle 3 to bundle 2). This is because two equi-proportional price decreases are equivalent to an income increase. In Figure 2a, each good is a gross complement to the other good, in Figure 2b, both goods are neither gross complements nor gross substitutes. In Figure 2c finally, each good is a gross substitute for the other good. The relation between homotheticity and symmetric substitutability is especially clear in Figure 2b. By the fact that the line connecting bundles 1 and 3 is horizontal, good 2 is neither a substitute for nor a complement to good 1. If the line connecting bundles 3 and 2 would not be vertical, so that it is not the case that good 1 is neither a substitute for nor a complement to good 1, bundle 2 would not lie on the line connecting bundle 1 to the origin, and we would not have homothetic preferences.

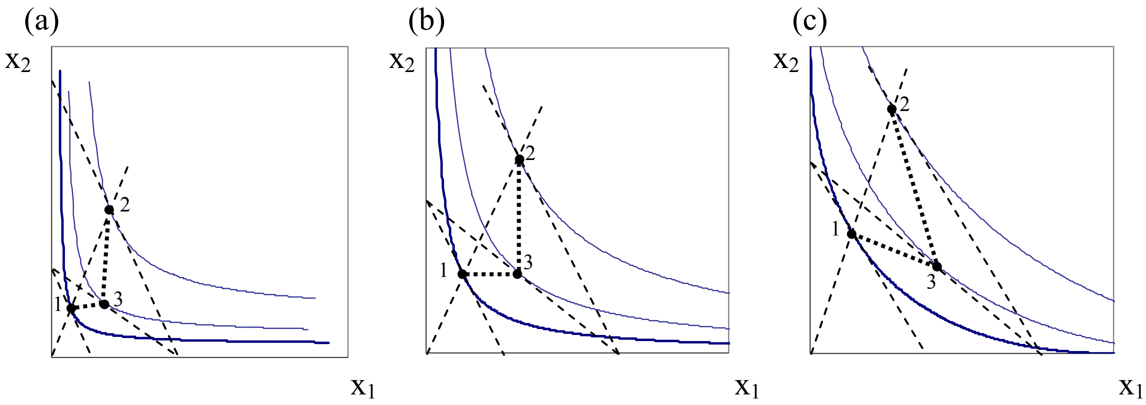


Figure 2. Homothetic preferences and strong symmetric gross substitutability

We now show that another way to state the necessary condition for Giffen behaviour that there is a necessity and a luxury is to say that the gross cross-price effects on each good are not equal. We refer to this property as *weak asymmetric gross substitutability*.

Definition 4. Two goods 1 and 2 are locally characterized by *weak asymmetric gross substitutability* if $(\partial x_1 / \partial p_2) \neq (\partial x_2 / \partial p_1)$.

This leads us to the following proposition:

Proposition 4. Good 1 is locally a necessity and good 2 is a luxury if and only if locally $(\partial x_1 / \partial p_2) \neq (\partial x_2 / \partial p_1)$ (= weak asymmetric substitutability). More precisely, good 1 is locally a necessity and good 2 is a luxury if and only if locally $(\partial x_2 / \partial p_1) < (\partial x_1 / \partial p_2)$.
Proof:
Given again that $(\partial h_1 / \partial p_2) = (\partial h_2 / \partial p_1)$, and combining (4) and (5), we see that $(\partial x_2 / \partial p_1) < (\partial x_1 / \partial p_2) \Leftrightarrow \varepsilon_{x_1 m} < \varepsilon_{x_2 m}$. By (3), $\varepsilon_{x_1 m} < \varepsilon_{x_2 m}$ implies that $\varepsilon_{x_1 m} < 1 < \varepsilon_{x_2 m}$. QED

Given that weak asymmetric income elasticity is a necessary condition for Giffen behaviour and is synonymous to weak asymmetric gross substitutability, the latter is also a necessary condition for Giffen behaviour. In general, a necessary condition for Giffen behaviour is asymmetry in the consumers' preferences. This asymmetry can be stated as weak asymmetric income elasticity, where good 1 is a necessity and good 2 is a luxury. As shown by Proposition 4, this turns out to be the same as saying that the gross cross-price effect of the price of good 2 on the consumption of good 1 is higher than the cross-price effect of the price of good 1 on the consumption of good 2. How to interpret this result intuitively? The case is made here for interpreting this as good 1 being a better gross substitute for good 2 than good 2 is for good 1. The reader may oppose that this is a contentious interpretation, as absolute cross-price effects are compared, rather than cross-price elasticities. Yet, the following counterarguments may be presented. *First*, while the individual cross-price effects are not unit-independent, the property of unequal absolute cross-price effects for non-homothetic preferences is unit independent. Thus, whatever it measures, the property $(\partial x_2 / \partial p_1) < (\partial x_1 / \partial p_2)$ is unit independent – where no claims need to be made that the extent to which $(\partial x_2 / \partial p_1)$ is smaller than $(\partial x_1 / \partial p_2)$ measures any *degree* of asymmetric gross substitutability.

Second, it should be noted that if one takes the difference between the cross-price elasticities as a measure of asymmetric substitutability, then it should be noted that the relation between $\varepsilon_{x_2 p_1}$ and $\varepsilon_{x_1 p_2}$ is affected by the budget shares spent on each good. Intuitively, if goods 1 and 2 are mutual gross substitutes, and if more is spent on good 2 than on good 1, then an increase in the price of good 2 has a larger impact than an increase in the price of good 1. In order to exclude the effects of budget shares, a case can be made for comparing the absolute cross-price effects rather than the cross-price elasticities. In this sense, in the case of homothetic preferences, due to the equal absolute cross-price effects, any differences in cross-price elasticities are purely due to differences in budget shares. Net of the effect of budget shares, goods may be considered as gross symmetrically substitutable, and as equally good gross substitutes for or gross complements to one another. In the same manner, with non-homothetic preferences, with good 1 a potential Giffen good, we interpret good 1 as being a better gross substitute for good 2 than good 2 is for good 1.

Intuitively, let good 1 be bread, and let good 2 be yachts. From the perspective of subsistence, bread is a better substitute for yachts than yachts are for bread. You can eat bread, but you cannot eat yachts. There is a one-to-one relationship of this fact with asymmetric income elasticity. Given that bread is a necessity, if the price of yachts increases, the income effect of this on bread is relatively small, so that the consumption of bread changes relatively much, as the impact of the substitution effect is only slightly limited by the income effect. Given that yachts are a luxury, if the price of bread increases, the income effect of this on the consumption of yachts is relatively large, so that the consumption of yachts decreases relatively little, as the impact of the substitution effect is considerably limited by the income effect.

The one-to-one relation between weak asymmetric income elasticity and weak asymmetric substitutability is shown graphically in Figure 3. Given that rational consumers are not subject to money illusion, the effect of an income increase from bundle 1 to bundle 2 can be split up in a decrease in the price of good 2 from bundle 1 to bundle 3, plus an equi-proportional decrease in the price of good 1 from bundle 3 to bundle 2 or bundle 2'. Bundle 2' is the bundle that is obtained if the consumer has homothetic preferences, and has income expansion paths that are straight lines through the origin. As shown in Proposition 1 and illustrated in Figure 2, the goods are then equally good gross substitutes. In Figure 3a, we make preferences non-homothetic by making good 2 a worse gross substitute for good 1, so that higher-up indifference curves become flatter; the income expansion path running through bundles 1 and 2 has a convex shape, so that good 1 is a necessity and good 2 a luxury. We thus move in the direction of good 1 being a Giffen good. In Figure 3b, we make preferences non-homothetic by making good 2 a better gross substitute for good 1, so that higher-up indifference curves become steeper; the income expansion path running through bundles 1 and 2 is concave so that good 1 is a luxury and good 2 a necessity. We thus move further away from good 1 being a Giffen good.

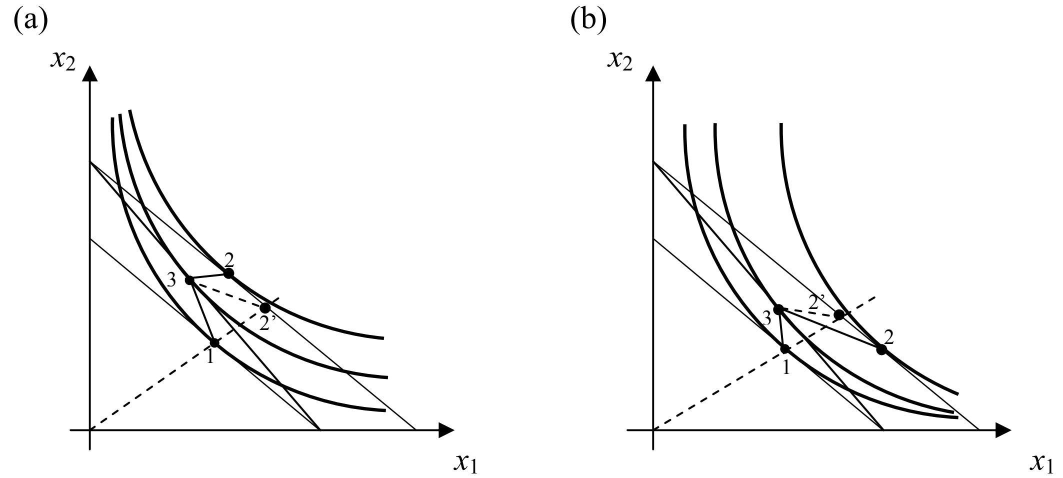


Figure 3 Non-homothetic preferences and weak asymmetric gross substitutability

2.4. Good 1 gross substitute for good 2, good 2 gross complement to good 1: strong asymmetric gross substitutability

We now show that a more detailed necessary condition for Giffen behaviour is that the gross cross-price effects on each good have opposite signs. Thus, in order to have Giffen behaviour, not only must preferences be asymmetric in the sense that one good 1 is a better gross substitute for good 2 than good 2 is for good 1, they must be asymmetric in the sense that good 1 is a gross substitute for good 2 while good 2 is a gross complement to good 1. We define such a phenomenon as *strong* asymmetric substitutability.

Definition 5. Two goods 1 and 2 are locally characterized by *strong asymmetric gross substitutability* if $sign(\partial x_1 / \partial p_2) \neq sign(\partial x_2 / \partial p_1)$.

This leads us to the following proposition:

Proposition 5. *A necessary condition for good 1 to be locally a Giffen good is that the preferences over goods 1 and 2 are locally characterized by strong asymmetric gross substitutability, where that good 1 is a gross substitute for good 2 (*$\partial x_1 / \partial p_2 > 0$*), while good 2 is a gross complement to good 1 (*$\partial x_2 / \partial p_1 < 0$*)*

Proof:

By budget balancedness, $p_1 x_1 + p_2 x_2 = m$. If good 1 is a Giffen good, and p_1 increases, then x_1 increases as well. It follows that $p_1 x_1$ increases, so that in order for the consumer to spend the entire budget, x_2 must fall. It follows that a necessary condition for good 1 to be a Giffen good is that $\partial x_2 / \partial p_1 < 0$, meaning that good 2 is a gross complement to good 1. We already know that any Giffen good is an inferior good, so that in equation (4), $\partial x_1 / \partial m < 0$. As the Hicksian cross-price effect is necessarily positive, it follows that when good 1 is a Giffen good, good 1 must be a gross substitute for good 2. QED

Strong asymmetric gross substitutability is not a familiar concept in the literature, and many economists seem to be unaware of its possibility, yet it is perfectly in line with rational preferences. Intuitively, let a family own a smaller car to drive locally in the city, and a larger car for larger trips. The larger car is a complement to the small car; it is not a substitute for the smaller car, because it is too big for parking in the city (imagine that the larger car is one of the bigger four-wheel drives on the market). But the smaller car may at the same time be a substitute for the larger car: if larger cars become too expensive, the family will only keep a smaller car for driving both small and larger distances (imagine that the smaller car is still a reasonable family car, so that larger distances are not that uncomfortable with the smaller car). For other potential examples, see De Jaegher (2009).

2.5. Strong asymmetric price elasticity

As we show in this section, a necessary condition for local Giffen behaviour is that locally, it is the case for good 1 that $\varepsilon_{x_1 p_1} > -1$ (the demand for good 1 either has a negative slope and is price inelastic, or has a positive slope), and that ($\varepsilon_{x_2 p_2} < -1$) (the demand for good 2 has a negative slope and is price elastic). We do this by showing that this necessary condition is fully equivalent to the necessary condition of strong asymmetric substitutability treated in Section 2.4.

We say that two goods are characterized by weak symmetric elasticity if both price elasticities are smaller than minus one, or both are larger than minus one.

Definition 6. Two goods 1 and 2 are locally characterised by *weak symmetric price elasticity* if locally either $\varepsilon_{x_1 p_1} < -1$ and $\varepsilon_{x_2 p_2} < -1$ (both goods have a downward-sloping demand and are price elastic), or $\varepsilon_{x_1 p_1} > -1$ and $\varepsilon_{x_2 p_2} > -1$ (each good has a downward-sloping demand and is price inelastic, or is upward sloping).

One could also define the concepts of strong symmetric price elasticity, where both goods have exactly the same elasticity, and weak asymmetric price elasticity, where one good is more elastic than the other, but it need not be the case that one good is elastic and the other inelastic. These concepts, however, tell little about preferences, because relative elasticity depends also on the relative budget shares of each good. This is why we only additionally define the concept of strong asymmetric elasticity.

Definition 7. Two goods 1 and 2 are locally characterised by *strong asymmetric price elasticity* if locally either $\varepsilon_{x_1 p_1} < -1$ (good 1 has a negatively sloped demand and is price elastic) and $\varepsilon_{x_2 p_2} > -1$ (good 2 has a negatively-sloped demand and is price inelastic, or it has a positively-sloped demand), or $\varepsilon_{x_1 p_1} > -1$ (good 1 has a negatively-sloped demand and is price inelastic, or it has a positively-sloped demand) and $\varepsilon_{x_2 p_2} < -1$ (good 2 has a negatively-sloped demand and is price elastic).

We can now state the following proposition, showing equivalence between strong asymmetric price elasticity and strong asymmetric gross substitutability, and between strong symmetric price .

Proposition 6. Strong asymmetric price elasticity is fully equivalent to strong asymmetric gross substitutability, and is a necessary condition for Giffen behaviour. Weak symmetric price elasticity is fully equivalent to weak symmetric gross substitutability, and if there is weak symmetric price elasticity, there cannot be Giffen behaviour.

Proof:

By budget balancedness, the consumer always uses up the complete budget, so that it must be the case that $x_i = m/p_i - (p_j/p_i)x_j$ for $i, j = 1,2$ and $i \neq j$. Taking the derivative of this expression with respect to p_i, and expressing in terms of elasticities, we obtain that

$$\varepsilon_{x_1 p_1} = -1 - (s_2/s_1)\varepsilon_{x_2 p_1} \quad (6)$$
$$\varepsilon_{x_2 p_2} = -1 - (s_1/s_2)\varepsilon_{x_1 p_2} \quad (7)$$

By equations (6) and (7), good 1 is a gross substitute for good 2 ($\varepsilon_{x_1 p_2} > 0$) and good 2 is a gross complement to good 1 ($\varepsilon_{x_2 p_1} < 0$) (i.e., strong asymmetric gross substitutability) if and only if $\varepsilon_{x_1 p_1} > -1$ (the demand for good 1 is downward sloping and inelastic, or is upward sloping), and $\varepsilon_{x_2 p_2} < -1$ (the demand for good 2 is downward-sloping and elastic) (i.e. strong asymmetric price elasticity). As by Proposition 5 strong asymmetric gross substitutability is a necessary condition for Giffen behaviour, and as we have just shown strong asymmetric gross substitutability is equivalent to strong asymmetric price elasticity, it follows that strong asymmetric price elasticity is a necessary condition for Giffen behaviour.

Further, by equations (6) and (7), both goods are gross complements to one another, meaning that, $\varepsilon_{x_1 p_2} < 0$ and $\varepsilon_{x_2 p_1} < 0$ (weak symmetric gross substitutability), if and only if $\varepsilon_{x_1 p_1} > -1$ and $\varepsilon_{x_2 p_2} > -1$. Given that strong gross asymmetric substitutability is a necessary condition for Giffen behaviour, it follows that both goods are gross complements to one another if and only if $0 > \varepsilon_{x_1 p_1} > -1$ and $0 > \varepsilon_{x_2 p_2} > -1$.

Similarly, both goods are gross substitutes to one another, meaning that $\varepsilon_{x_1 p_2} > 0$ and $\varepsilon_{x_2 p_1} > 0$ (weak symmetric gross substitutability), if and only if the demands for both goods are down-ward sloping and price elastic, meaning that $\varepsilon_{x_1 p_1} < -1$ and $\varepsilon_{x_2 p_2} < -1$. We have thus established that weak symmetric gross substitutability is fully equivalent to weak symmetric price elasticity. As strong asymmetric gross substitutability is a necessary condition for Giffen behaviour, it follows that if there is weak symmetric gross substitutability and with is weak symmetric price elasticity, there cannot be Giffen behaviour. QED

A corollary of Proposition 6 is that we can narrow down the case of weak symmetric price elasticity with $\varepsilon_{x_1 p_1} > -1$ and $\varepsilon_{x_2 p_2} > -1$ to the case where the demand for both goods are downward-sloping and price inelastic.

Corollary 2. Let preferences be locally characterised by weak symmetric price elasticity with $\varepsilon_{x_1 p_1} > -1$ and $\varepsilon_{x_2 p_2} > -1$. Then $0 > \varepsilon_{x_1 p_1} > -1$ and $0 > \varepsilon_{x_2 p_2} > -1$.

Proof: Given Proposition 6, if there is Giffen behaviour on good 1, it must be that the demand for good 2 is downward sloping. So it cannot be that $\varepsilon_{x_1 p_1} > 0$ and $\varepsilon_{x_2 p_2} > 0$. Further, if there is Giffen behaviour on good 1, then it must be that $\varepsilon_{x_2 p_2} < -1$, which is incompatible with the given case. It follows that there cannot be Giffen behaviour on any of the goods for the given case. QED

We conclude that necessary conditions for Giffen behaviour are found in several ways in which preferences are asymmetric. One good should be a necessity, and one good should be a luxury, and goods should not be equally-good gross substitutes for each other. The asymmetry should even be more extreme, in that good 1 should be inferior, and good 2 should be a luxury. Also, good 1 should be a gross substitute for good 2, and good 2 a gross complement to good 1, which is exactly equivalent to saying that good 1 should be price inelastic, and good 2 price elastic. The necessary conditions are summarised in Table 1. Note that $\varepsilon_{x_1 p_1} > -1$, $\varepsilon_{x_2 p_2} < -1$ is used as a necessary condition rather than the necessary (and sufficient) condition $\varepsilon_{x_1 p_1} > 0$, $\varepsilon_{x_2 p_2} < -1$ because we are interested in wider classes of local preferences that have some of the characteristics of Giffen behaviour.

	Good 1	Good 2
Weak asymmetric income elasticity	$\varepsilon_{x_1 m} < 1$	$\varepsilon_{x_2 m} > 1$
Weak asymmetric gross substitutability	$\partial x_1 / \partial p_2 > \partial x_2 / \partial p_1$	
Strong asymmetric income elasticity	$\varepsilon_{x_1 m} < 0$	$\varepsilon_{x_2 m} > 1$
Strong asymmetric gross substitutability	$\partial x_1 / \partial p_2 > 0$	$\partial x_2 / \partial p_1 < 0$
Strong asymmetric price elasticity	$\varepsilon_{x_1 p_1} > -1$	$\varepsilon_{x_2 p_2} < -1$

Table 1. Necessary conditions for Giffen behaviour

The reader will be well aware that a necessary condition for good 1 to be a Giffen good is that good 1 is an inferior good, and good 2 a luxury (*strong asymmetric income elasticity*), and may find the necessary condition that good 1 is price inelastic and good 2 price elastic (*strong asymmetric price elasticity*) self-evident. The additional necessary condition that good 1 is a gross substitute for good 2, and good 2 a gross complement to good 1 (*strong asymmetric gross substitutability*) is novel, and the reader may be surprised to find out that such a phenomenon is possible. But the reader may wonder about its relevance to the analysis of Giffen behaviour. If local preferences such that good 1 is an inferior good and good 2 is a luxury (strong asymmetric income elasticity) are a strict subset of local preferences such that good 1 is a substitute for good 2 and good 2 is a complement to good 1 (strong asymmetric substitutability), then the familiar strong asymmetric income elasticity remains the relevant concept to analyse Giffen behaviour, as it brings us closest to Giffen behaviour.

As shown in the next section, this is not the case. Preferences that are (locally) characterised by strong asymmetric income elasticity need not be characterised by strong asymmetric gross substitutability, and vice versa. Giffen behaviour is only possible if preferences have *both* these characteristics. This itself makes strong asymmetric gross substitutability a useful necessary condition that more precisely pins down the characteristics of Giffen behaviour. Further, preferences cannot be excluded such that good 1 is an inferior good and good 2 a luxury, but where good 1 is *price elastic* and good 2 is *price inelastic*, which is the opposite of what should be the case for Giffen behaviour. If the reader accepts that a case where good 1 is price inelastic and good 2 price elastic is qualitatively similar to Giffen behaviour, and constitutes what could be termed near-Giffen behaviour, then strong asymmetric gross substitutability can be argued to be a more relevant condition for identifying Giffen or near-Giffen behaviour. Given that strong asymmetric gross substitutability is fully equivalent to strong asymmetric price elasticity (see Proposition 6), the former guarantees that $\varepsilon_{x_1 p_1} > -1$ and that $\varepsilon_{x_2 p_2} < -1$, which leaves the possibility of Giffen behaviour on good 1, and even if good 1 has a downward sloping, price inelastic demand could in any case be said to approach Giffen behaviour. Further, given that strong asymmetric gross substitutability implies weak asymmetric gross substitutability, and given that the latter is fully equivalent to weak asymmetric income elasticity (Proposition 4), it also guarantees that good 1 is a necessity and good 2 is a luxury, again making such a case qualitatively similar to Giffen behaviour. All in all, an analysis of the relations between the necessary conditions listed in this section is called for, an exercise which is undertaken in Section 3.

3. Relation between necessary conditions

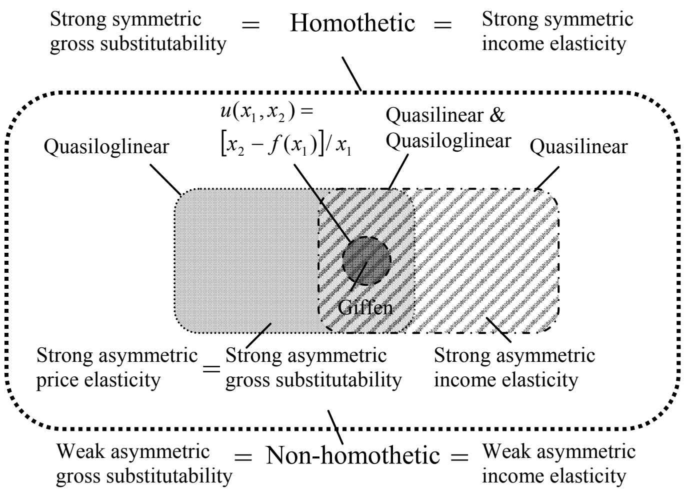


Figure 4 Relations between the sets of local preferences meeting the necessary conditions

In order to characterise the relation between the necessary conditions listed in Section 2, we represent the relations between sets of corresponding local preferences by means of Figure 4, where ovals represent these sets. The labels at the bottom refer to characteristics of local preferences *inside* the sets. The labels at the top of the figure each time refer to characteristics of the local preferences that lie exactly on the boundaries of one or more of the sets, i.e., local preferences which just fail to belong to one set, or local preferences which lie on the boundary separating one set from another. All the characteristics in Figure 4 refer to *local* characteristics of preferences: e.g. a consumer's preferences may only be locally homothetic.

The set of all preferences is represented by the area of the large dashed oval in Figure 4, including its dotted boundary. Exactly on the dotted boundary of this set lie the locally homothetic preferences, where locally all goods are neither luxuries nor necessities and have unit income elasticity (*strong symmetric income elasticity*), which is equivalent to having equal gross cross-price effects (*strong symmetric gross substitutability*). The open set contained *inside* the large dashed oval is the set of the locally non-homothetic preferences, where the two goods locally have different income elasticities (*weak asymmetric income elasticity*), which is fully equivalent to having different gross cross-price effects (*weak asymmetric gross substitutability*).

The open set consisting of the dashed area contained *within* the dashed-dotted oval is the set of local preferences where locally one good is an inferior good, and the other one good is a luxury (*strong asymmetric income elasticity*). Exactly on the dashed-dotted oval itself lie the local preferences where locally one good is neither an inferior good nor a normal good, and the other good is a luxury. These are the well-known *quasilinear preferences*. For all local preferences outside of the dashed-dotted oval, it is the case that both goods are normal goods (*weak symmetric income elasticity*).[2]

The open set consisting of the grey area contained *within* the dotted oval is the set of local preferences where locally one good is a gross substitute for the other good, and the other good is a gross complement to the first good (*strong asymmetric gross substitutability*), which is equivalent to the first good being price inelastic, and the second good being price elastic (*strong asymmetric price elasticity*). Exactly on the dotted oval itself lie the local preferences where locally one good is either gross substitute for or a gross complement to the other good, and the other good is neither a gross substitute for nor a gross complement to the first good. The first good is then has unit price elasticity, whereas the other good is either price elastic of price inelastic. As shown in De Jaegher (2008), preferences that are locally characterised by a *quasiloglinear utility function* have these characteristics, and take the functional form $u(x_1, x_2) = x_2 f(x_1)^{-1}$, where $x_2 = f(x_1)$ is a single downward sloping, strictly convex and continuous indifference curve from which this utility function is derived. For all local preferences outside of the dotted oval, it is locally the case that both goods are gross complements to one another, both goods are gross substitutes for one another, or both goods are neither gross substitutes for nor gross complements to one another[3] (*weak symmetric gross substitutability*). It is the case that both goods are ordinary goods and are both price inelastic, both price elastic, or both price unit elastic (*weak symmetric price elasticity*).

Local preferences that lie both exactly on the dotted oval and exactly on the dashed-dotted oval have the characteristics that good 1 is neither a normal nor an inferior good, is a gross substitute for good 2, and is unit elastic; good 2 is a neither a gross substitute for nor a gross complement to good 1, is price elastic, and is a luxury. As pointed out in De Jaegher (2008), preferences that are locally both quasilinear and quasiloglinear have these characteristics, and take the form $u(x_1, x_2) = x_1^{a} e^{x_2}$, where *a* is any positive real number.

Finally, the open set consisting of the dark grey area contained *within* the dashed circle is the set of local preferences where locally one good is a Giffen good. Exactly on the dashed circle lie the local preferences where locally one good is neither an ordinary good nor a Giffen good, and where the other good is an ordinary good. As shown in De Jaegher (2008), preferences that are locally characterised by the utility function $u(x_1, x_2) = [x_2 - f(x_1)]/x_1$, where $x_2 = f(x_1)$ is again a single downward sloping, strictly convex and continuous indifference curve from which this utility function is derived, have this property. Outside of the dashed circle lie all local preferences where both goods are ordinary goods.

In part, the rationale for the manner in which Figure 4 is drawn is immediately clear. Local preferences where locally one good is neither an inferior good nor a normal good cannot at the same time be homothetic, so that the set of local preferences characterised by strong asymmetric income elasticity cannot at the same time by homothetic, and must lie strictly within the set of non-homothetic preferences. Similarly, local preferences where locally, one good is neither a substitute for nor a complement to the other good, whereas the other good is a substitute for the first good, cannot at the same time be characterised by strong symmetric gross substitutability, and must therefore again lie strictly within the set of non-homothetic

[2] Homothetic utility functions are represented as a boundary case, because they lie on the boundary of the case where good 1 is a luxury and good 2 a necessity, and the case where good 1 is a necessity and good 2 a luxury. In a more extensive figure, one can represent these two cases as two separate sets each time with the same form as Figure 4, with homothetic utility functions on the boundary of these two sets.

[3] In the case that both goods are neither gross substitutes for nor gross complements to one another, we have the Cobb-Douglas utility functions, which are a special case of the homothetic utility functions.

preferences. Finally, local preferences where one good is neither a Giffen good nor an ordinary good by the analysis in Section 2 cannot have a good which is neither an inferior good nor a normal good, or a good which is neither a substitute for nor a complement to the other good. It follows that the set of local preferences characterised by Giffen behaviour must lie strictly within the set of local preferences characterised by strong asymmetric income elasticity and strong asymmetric gross substitutability. Finally, the set of local preferences characterised by Giffen behaviour is not empty: see Heijman and v. Mouche (2009) for references.

The reason why we have not drawn the set of local preferences characterised by strong asymmetric income elasticity as a subset of the local preferences characterised by strong asymmetric gross substitutability, or vice versa, needs further justification. Consider preferences locally characterised by a quasilinear utility function of the form $u(x_1, x_2) = x_2 + [1-(1/\varepsilon)]^{-1} x_1^{1-(1/\varepsilon)}$, with $\varepsilon > 1$. It can be checked that this utility function is well-behaved. The demand for good 1 equals $x_1 = (p_1 / p_2)^{-\varepsilon}$ (where ε is the absolute value of the fixed own-price elasticity of good 1), the demand for good 2 equals $x_2 = (m/p_2) - (p_1/p_2)^{1-\varepsilon}$. Good 1 is neither an inferior good nor a normal good, and good 2 is a luxury. It is the case that $\partial x_1 / \partial p_2 = -\varepsilon (p_1/p_2)^{-\varepsilon-1}(-(1/p_2^2))$ is larger than zero, and that $\partial x_2 / \partial p_1 = -(1-\varepsilon)(p_1/p_2)^{-\varepsilon}(1/p_2)$ is larger than zero if $\varepsilon > 1$. This shows that we can find local preferences on the boundary of the set of preferences characterised by strong asymmetric income elasticity, which are not characterised by strong asymmetric grods substitutability.

Next, consider preferences locally characterised by a quasiloglinear utility function of the form $u(x_1, x_2) = x_2 \left(1 - x_1^{[(\varepsilon-1)/\varepsilon]}\right)^{-[\varepsilon/(\varepsilon-1)]}$, where $0 < \varepsilon < 1$. It can again be checked that this utility function is well-behaved. The demand functions corresponding to this utility function are $x_1 = (m/p_1)^{\varepsilon}$ and $x_2 = (m/p_2) - (p_1/p_2)(m/p_1)^{\varepsilon}$. Good 1 is price inelastic, is a necessity but not an inferior good, and is neither a gross substitute for nor a gross complement to good 1. Good 2 is unit elastic, is a luxury, and is a gross complement to good 1. It follows that there are local preferences on the boundary of the preferences with strong asymmetric substitutability that are not characterised by strong asymmetric income elasticity.

To further highlight the importance of strong asymmetric gross substitutability for Giffen behaviour, it is now shown that strong asymmetric income elasticity, with a low income elasticity for good 1 and a high income elasticity for good 2 is no guarantee that good 1 has a downward sloping demand and is price inelastic or has an upward sloping demand, and that good 2 is relatively price elastic (as should be the case under Giffen behaviour or near-Giffen behaviour). Let a consumer's preferences locally be given by the quasilinear utility function of the form $u(x_1, x_2) = x_2 + (a/b)x_1 - 1/(2b)x_1^2$, where $a, b > 0$. The demand for good 1 locally equals $x_1 = a - b(p_1/p_2)$, the demand for good 2 locally equals $x_2 = (m/p_2) - (a/p_2) + b(p_1/p_2^2)$. It can be checked that for a = 10, b = 1, p_1 = 9, p_2 = 1, m = 100, while good 1 has zero income elasticity and good 2 is a luxury, good 1 is price elastic, while good 2 is price inelastic, which is completely the opposite of what we need for approaching Giffen behaviour on good 1. In this particular example, as the demand for good 1 is linear, as long as p_1 is high enough, the price elasticity of good 1 can take on any height desired.

In the next section, we give an intuition why a subclass of inferior goods may be price elastic rather than price inelastic, whereas the composite good is a luxury and is price inelastic. This is argued to be plausible in the case of the standard textbook example of an inferior good, namely a low-quality good, and where the composite good is then considered as a high-quality good.

4. Low-quality and high-quality goods

The previous analysis has shown that, in order to make it possible that good 1 is a Giffen good, not only does it need to be the case that good 1 is an inferior good and good 2 a luxury (strong asymmetric income elasticity), it also needs to be the case that good 1 is a gross substitute for good 2 while good 2 is a gross complement to good 1 (strong asymmetric gross substitutability). Further, it has been shown that the latter condition alone suffices to obtain a situation that bears resemblance to the case of Giffen behaviour, where good 1 is price inelastic and good 2 is price inelastic, whereas the former condition in isolation can lead to a situation that is quite different from the case of Giffen behaviour, namely one where good 1 is *price elastic* and good 2 is *price inelastic*. The current section attempts to support this result with intuition, and therefore mainly consists of verbal arguments – even though these are again partially backed up by analysis.

By Definition 2, strong asymmetric income elasticity means that one of the goods is an inferior good. The definition of inferior goods does not refer in any way to the quality of the good involved, yet the term *inferior* suggests an association with goods of inferior quality. Indeed, as argued by Varian (2003, p.96) "examples (of inferior goods) (…) include (…) any kind of low-quality good". In the context of a simple two-good dichotomy, an example is the case where a consumer consumes only a low-quality good such as hamburgers, and a high-quality good such as sirloin steak. Intuitively, when the consumer's income increases, the quantity of hamburgers consumed *relative* to the quantity of sirloin steaks decreases; put otherwise, the income expansion paths are convex. Thus the income elasticity of sirloin steaks is larger than the income elasticity of hamburgers. This need not imply that fewer hamburgers are consumed in absolute terms as income gets higher. People with a high income may continue to enjoy a hamburger every now and then, and a higher income may still lead to relatively small increases in the total consumption of hamburgers, so that we only have weak asymmetric income elasticity. But one can envisage also that the demand for hamburgers of a consumer would decrease with income. The fact that of two goods, one good has low and the other good has high quality is then equated to strong asymmetric income elasticity.

Analytically, let us take the difference between the left-hand sides of equations (1) and (2), and equate this to the difference between the right-hand sides, to obtain:

$$\left(\frac{\partial x_1}{\partial p_1}-\frac{\partial x_2}{\partial p_2}\right)=\left(\frac{\partial h_1}{\partial p_1}-\frac{\partial h_2}{\partial p_2}\right)-m\left(\varepsilon_{x_1 m}-\varepsilon_{x_2 m}\right) \tag{8}$$

Then, keeping the difference between the Hicksian own-price effects constant, this argument says that we decrease the quality of good 1 and increase the quality of good 2 by making the income elasticity of good 1 smaller and the income elasticity of good 2 larger. Graphically, this has the following effect on an indifference map. Start from a given indifference curve u_0, and assume that the shape of this particular indifference curve is not affected at all by changes in the relative quality of the goods. This assures that, as long as the consumer's starting point is a bundle on this fixed indifference curve, the size of the Hicksian own-price effects and with it the size of the substitution effect is unaffected by changes in the relative quality of the goods. This is true no matter how the change in relative quality affects the rest of the indifference curve map. In order to decrease the quality of good 1 with respect to the quality of good 2, by assumption, we can now make the income elasticity of good 1 smaller compared to the income elasticity of good 2. The simplest manner to do this is by taking quasilinear preferences, so that indifference curves are vertical shifts of u_0, as illustrated in Figure 5a. By making the income elasticity of good 1 smaller and smaller with respect to the income elasticity of good 2, we deviate further and further from homothetic preferences, so that we can eventually reach the case where the price elasticity of good 1 is zero, as is the case in Figure 5b. As the substitution effect remains fixed in size on u_0, it follows that the price elasticity of good 1 is decreased, and the price elasticity of good 2 is increased, as can be seen from equation (8). It thus seems that as we are making the quality of good 1 smaller compared to the quality of good 2, we are making good 1 less and less elastic, and are in this way moving towards the case of Giffen behaviour. In general, from this point of view, low-quality goods would seem price inelastic relative to high-quality goods. Indeed, it seems intuitive that e.g. a 10% discount on the name-brand (and presumably high-quality) variant of a commodity for sale in a supermarket chain would have a more important effect on demand than a 10% discount on the supermarket chain's own-brand (and presumably low-quality) variant of this commodity. The 10% discount means an increase in purchasing power, which by the fact that the name-brand version of the commodity is a luxury is mostly spent on the name-brand version itself.

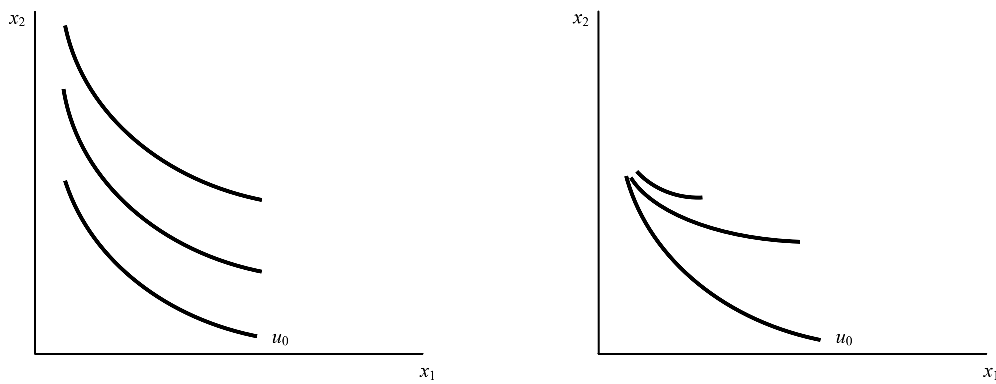


Figure 5 Quasilinear indifference curves and preferences with a zero own-price effect on good 1

Yet, the manner in which one indifference curve is positioned with respect to another is just one way in which the indifference map may be affected when the quality of good 1 is decreased with respect to the quality of good 2. The shape of each individual indifference curve could be affected as well. To fix ideas on how this shape may be affected, consider the extreme case where good 1 is of better quality than good 2 in the sense that good 1 is identical to good 2, with the exception that it gives more value per unit bought; for instance, good 1 is exactly the same commodity as good 2, but sold in a bigger package. The utility function then locally has the form $u(x_1, x_2) = ax_1 + bx_2$, which leads to the bold linear indifference curves represented in Figure 6. Additionally, it is assumed that preferences are homothetic, so that the tastes of consumers over low- and high-quality goods are not affected by purchasing power. In Figure 6a, $a > b$, so that good 1 has higher quality than good 2: it takes relatively many units of good 2 to substitute for good 1 along any indifference curve. An increase in the price of good 1 leads to a moderate change in the consumption of good 1, because the low-quality good 2 is not a good substitute for it. In Figure 6b, $b > a$, so that good 2 is now the higher-quality good. An increase in the price of good 1 leads to a relatively large decrease in the consumption of good 1, because good 2 is a good substitute for good 1. This suggests that high-quality goods are relatively price inelastic, and low-quality goods relatively price elastic. Intuitively, if ersatz coffee becomes more expensive, then the impact may be large, as a superior substitute is available in the form of real coffee. If ersatz coffee becomes expensive as well, why drink it at all? If real coffee becomes more expensive, the impact may be small, because only an inferior substitute is available in the form of ersatz coffee.

It should be noted that this argument is not specific to linear indifference curves, where it is always optimal for the consumer to consume either one good or the other. In reality, the consumer may value specific characteristics of low-quality goods, so that indifference curves have a convex shape. But this does not affect the argument that any given indifference curve becomes flatter as the quality of good 1 decreases with respect to the quality of good 2. Analytically, the argument that we are making here is that, when the quality of good 1 is decreased with respect to the quality of good 2, in equation (8), $\partial h_1 / \partial p_1$ is increased with respect to $\partial h_2 / \partial p_2$. If at the same time income elasticities remain constant because preferences over quality do not change with purchasing power, then by equation (8) this means that the low-quality good is relatively more elastic.

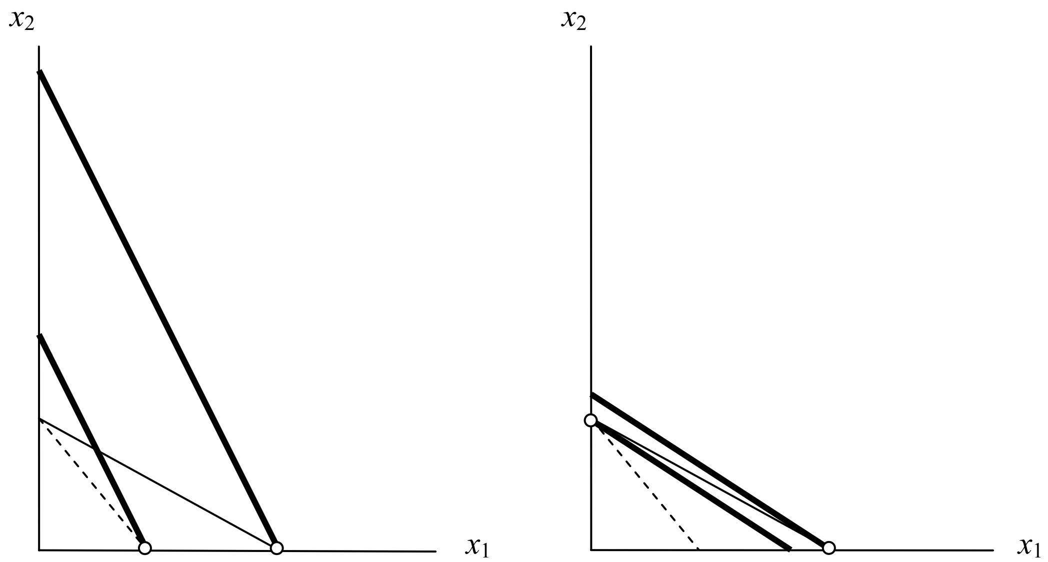


Figure 6 Price inelastic high-quality goods (left) and price elastic low-quality goods (right). Thick lines are indifference curves, thin lines are budget lines.

In this argument, there are two factors affecting the elasticity of goods as a function of their quality, and these two factors have opposite effects. From the perspective of the substitution effect, low-quality goods are relatively elastic, because they have good substitutes in the form of high-quality goods. From the perspective of the income effect, low-quality goods are inelastic: an increase in the price of low-quality goods decreases the purchasing power of the consumer, which from this point of view may lead him to buy more of the low-quality good.

If a decrease in the quality of good 1 relative to good 2 mainly affects the substitution effect, then it is the case that inferior goods are price elastic, and luxuries price inelastic, meaning that we are then further away from the case of Giffen behaviour, rather than closer to it. For instance, Figure 7 represents an example where, in the logic of the argument, good 1 is of high quality and good 2 is of low quality. The indifference curves are steep, because consumers need a lot of good 2 in exchange for one unit of good 1. Higher indifference curves are steeper, so that income increases lead to relatively more consumption of good 1, and relatively less consumption of good 2. The fact that good 2 is of low quality makes the substitution effect small, while the fact that good 1 is of high quality assures a positive income effect. In Figure 8, good 1 is of low quality and good 2 of high quality, as seen by the fact that the indifference curves are flat, and get flatter as utility increases. Nevertheless, compared to Figure 7, the price elasticity of demand is increased. This is because of the relatively large substitution effect, meaning that the effect of the argument that the demand for the low-quality good is elastic as there is a good substitute for it, is dominant. Comparing Figure 7 to Figure 8, decreasing the quality of good 1 relative to good 2 in this example mainly affects the substitution effect and only has limited effect on the income effect, so that a good becomes more price elastic as it quality decreases.

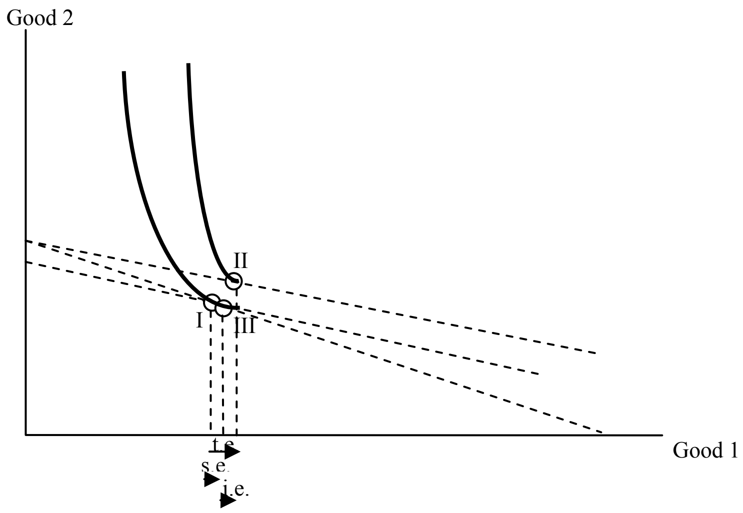


Figure 7. Good 1 = high-quality good, good 2 = low-quality good.

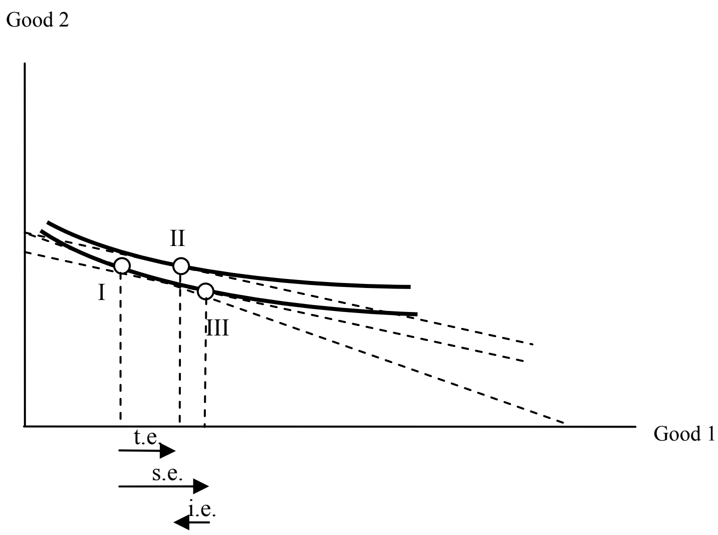


Figure 8. Good 1 = low-quality good, good 2 = high-quality good.

Given this intuition, we argue that in fact for good 1 to be close to a Giffen good, it must be a *high*-quality good. This is obvious from the fact that the consumer's current indifference curve is then relatively steep, so that the substitution effect for good 1 in equation (1) is also small. The income effect will now additionally be small if the consumer cares less about the quality of good 1 the higher is his purchasing power. This is possible in the following manner. Let us again take the example of the family that considers buying a smaller and/or a larger car. As we now want to argue, the smaller car is not in fact a low-quality good, but is instead a high-quality good, as it can both be used for driving and parking in the city, and (with moderate inconvenience) for larger trips (again, the smaller car is still a reasonably sized family car). As the larger car is not suitable for driving and parking in the city, it is from this perspective a low-quality good (again, it is one of the large four-wheel drives on the market). At the same time, the larger car is a luxury and the smaller car is a necessity. The fact that the larger car is of low quality and is at the same time a luxury is not a paradox: it is exactly when the family has high purchasing power that it can afford a good that can exclusively be put to the particular use of driving outside of the city. The fact that the smaller car both does not have good substitutes (small substitution effect) and is a necessity (small income effect) makes the small car a good that unambiguously has small price elasticity.

The relation of the smaller-car larger-car example with strong asymmetric gross substitutability is the following. If the price of the larger car increases, two things happen. First, the family's appreciation of the quality of the smaller car increases, and may increase heavily, because the family becomes willing to use the smaller car for the job normally done by the large car, namely making larger trips. Further, along the indifference curve, the family makes use of the high quality of smaller cars to substitute in favour of smaller cars. It follows that smaller cars are a gross substitute for large cars. If the price of the smaller car decreases, two things happen. First, because along one and the same indifference curve larger cars are not a very good substitute for smaller cars, the consumption of large cars does not change very much. But because the family has more purchasing power, it cares less about the ability of small cars to do both jobs of driving in the city and driving longer trips. For this reason, the family buys more larger cars. It follows that large cars are a gross complement to small cars.

It should be noted that our focus on Giffen behaviour as an instance of strong asymmetric substitutability can be linked to two approaches for analysing Giffen behaviour found in the literature (for a detailed analysis of the links to these two approaches, see De Jaegher, 2009). In the subsistence-constraint approach (Gilley and Karels, 1991), the consumer has standard preferences over two goods, but additionally has a subsistence constraint. As long as this constraint is slack, the consumer behaves normally. However, when hitting the subsistence constraint, the consumer's first priority is to at least stay on this constraint. If one of the goods is better at providing subsistence, then an increase in its price may cause larger consumption of it. The connection with strong asymmetric gross substitutability is that, for low purchasing power, the quality of subsistence becomes more important to the consumer. In the car example, subsistence means being able to drive both small and larger trips. Given that smaller cars are good at providing subsistence, more of them may get consumed as their price increases. A second approach to analysing Giffen behaviour is to apply Lancaster's (1966) characteristics approach (Jensen and Miller, 2008). In the car example, we could say that the consumer cares about two characteristics, namely being able to drive everywhere, and being able to drive comfortably (i.e., in a larger car). For low purchasing power, the characteristic of being able to drive everywhere is more important to the consumer, so that the smaller car is bought more often.

5. Alternative decompositions of the own price effect

A well-known characteristic of a Giffen good is that it is an inferior good. This justifies explaining Giffen behaviour in terms of Hicksian (or alternatively: Slutsky) decomposition into a substitution effect and an income effect, where for Giffen behaviour the positive income effect of a price increase dominates the negative substitution effect. As we have shown, a further characteristic of preferences over two goods that allow for a Giffen good is that the two goods are characterised by strong asymmetric gross substitutability, where the potential Giffen good is a gross substitute for the other good, and the other good is a gross complement to the Giffen good. We now develop a number of alternative decompositions of an own price change, that further highlight the role of asymmetric gross substitutability in determining the potential for Giffen behaviour. The analysis is mainly graphical, and serves to provide further intuition for the link between strong asymmetric gross substitutability and Giffen behaviour.

5.1. Fixed expenditure effect, and complementarity of the second good

We know that in order for good 1 to be a potential Giffen good, it must be the case that good 2 is a gross complement to good 1. Can we somehow decompose the own-price effect on good 1 in an effect that reflects the degree to which good 2 is a gross complement to or gross substitute for good 1, and another effect? This can simply be done in the following way. *First*, consider what happens if good 2 is neither a gross substitute for nor a gross complement to good 1, meaning that the consumption of good 2 does not change after a decrease in the price of good 1. In the two-good case, in terms of the consumption of good 1, this simply means that the expenditure on good 1 remains fixed. This is why we call this the *fixed expenditure effect*. Graphically, in Figure 9, the fixed expenditure effect of a decrease in the price of good 1 is simply obtained by finding the bundle on the new budget line with the same level of consumption of good 2. This effect is unambiguous: a consumer who keeps his expenditure on good 1 fixed will consume more of good 1 when its price decreases. *Second*, consider the effect of good 2 being either a gross complement to or a gross substitute for good 1. If good 2 is a gross complement to good 1, then the consumption of good 2 is increased compared to its fixed level in the fixed expenditure effect, meaning that the consumption of good 1 is necessarily decreased. Intuitively, if good 2 is a gross complement to good 1 (Figure 9a), then the consumer will not only want to spend the budget increase caused by the price decrease of good 1 on good 1 itself, but will want some of good 2 along with this. This reduces the price effect on good 1. Put otherwise, the consumer is not satisfied with keeping his expenditure on good 1 fixed, but instead chooses to spend less on good 1 after the price decrease. We call this the *deviation effect*. If good 2 is instead a gross substitute for good 1, then the consumption of good 2 is decreased compared to its fixed level in the fixed expenditure effect, meaning that the consumption of good 1 is necessarily further increased. In this case, the deviation effect says that the consumer decides that rather than keeping his expenditure on good 1 fixed after the price decrease, he will instead spend more on good 1. As good 2 is a gross substitute for good 1, it can in part serve the same function as good 1; but when good 1 becomes cheaper, the consumer will let this function be served mainly by good 1. It is clear from this that when good 2 is a gross substitute for good 1, the price elasticity of demand is relatively large, whereas it is relatively small when good 2 is a gross complement to good 1.

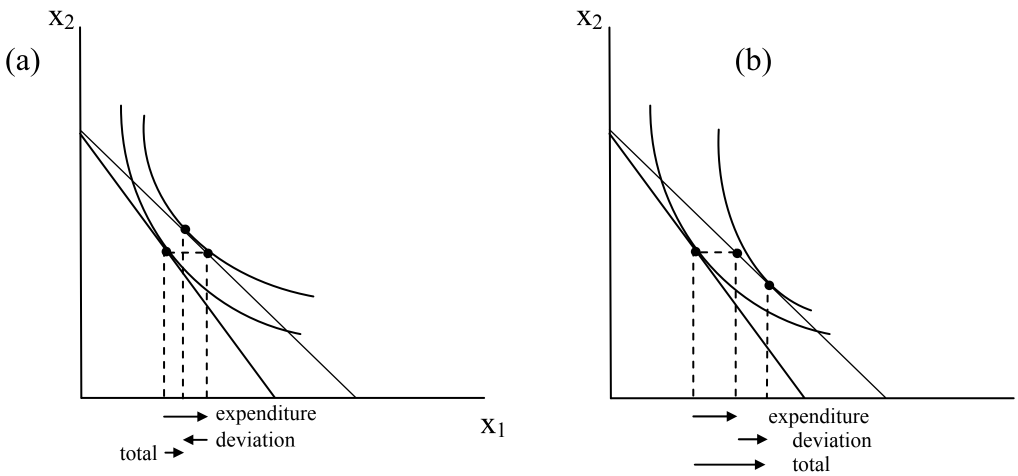


Figure 9 : Alternative decomposition : fixed expenditure effect, and deviation effect

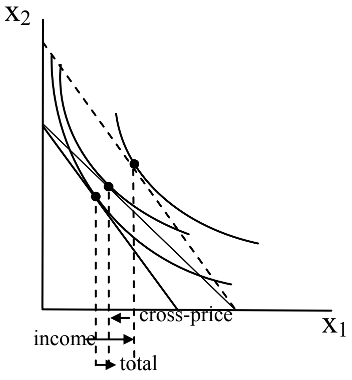
This alternative decomposition is more than a theoretical construct, and makes sense as a decision rule for a boundedly rational consumer who uses simple rules of thumb to come to his demand decisions. Such a simple rule of thumb is to always keep one's expenditure on good 1 fixed, no matter what the price of the good – which at the same time means that the consumption of all goods other than good 1 remains fixed. The specified alternative decomposition may have a psychological counterpart, in which the consumer as a first step follows such a decision rule (fixed expenditure effect), and then in second step considers how much the newly obtained consumption bundle is desirable, and possibly decides to deviate from the choice suggested by the fixed expenditure effect, and e.g. to give up some consumption of good 2 in favour of additional consumption of good 1.

Formally, this decomposition is given in elasticity form by equation (6), namely $\varepsilon_{x_1 p_1} = -1 - (s_2 / s_1)\varepsilon_{x_2 p_1}$. The fixed expenditure effect simply says that the own-price elasticity is -1, the deviation effect shows the deviation from this. Using equation (7), a similar decomposition can be used to show the effect of good 1 being a gross substitute for good 2 on the price elasticity of good 2.

5.2. Inflation (deflation) effect, and substitutability of the first good

The previous section explains the effect of good 2 being a gross complement to or gross substitute for good 1. Let us now look at the effect of good 1 being a gross complement to or a gross substitute for good 2. An alternative decomposition in which such an effect is isolated, is obtained in the following way. Consider a decrease in the price of good 1. In a *first* effect, the *deflation effect*, the consumer acts as if all prices would have deflated by the same proportion as the price of good 1. Thus, the deflation effect is nothing but another version of the income effect. The second effect is the cross-price effect. Having considered the deflation effect, the consumer notices in fact that it is only the price of good 1 that has decreased. Therefore, in order to get to the final situation, the consumer needs to consider his choice after an *increase* in the price of good 2. If, as in Figure 10a, good 1 is a gross complement to good 2, the price increase of good 2 will decrease the consumption of good 1, so that the price elasticity of good 1 is reduced. If, as in Figure 10b, good 2 is a gross substitute for good 1, the price increase of good 2 will increase the consumption of good 1, so that the price elasticity of good 1 is made even larger.

(a)



(b)

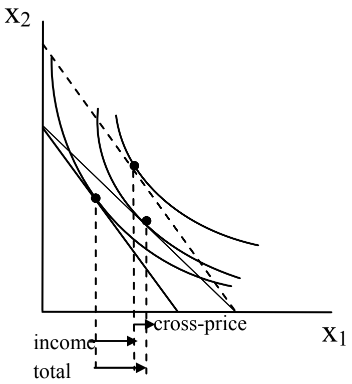


Figure 10 Alternative decomposition: deflation effect and cross-price effect

For a formalisation of this decomposition, given the assumption that consumers are not subject to money illusion, the demand for good 1 should be homogeneous of degree zero. By Euler's formula, in the case of *n* goods, it follows that

$$\sum_k \frac{\partial x_i}{\partial p_k} p_k + \frac{\partial x_i}{\partial m} m = 0 \tag{9}$$

In elasticity form, this becomes: $\sum_k \varepsilon_{x_i p_k} + \varepsilon_{x_i m} = 0$, or $\varepsilon_{x_i p_i} = -\sum_{k \neq i} \varepsilon_{x_i p_k} - \varepsilon_{x_i m}$. It follows that, for two goods:

$$\varepsilon_{x_1 p_1} = -\varepsilon_{x_1 m} - \varepsilon_{x_1 p_2}\,. \tag{10}$$

This simply says that an increase in the price of good 1 is equivalent to the price of good 1 staying fixed, and income and the price of good 2 decreasing by the same percentage. We have previously derived that in order for good 1 to be a Giffen good, good 1 must be an inferior good. When good 1 is an inferior good, it is a gross substitute for good 2: both the substitution effect and the income effect go in the same direction (see equation (4)). Equation (10) shows that good 1 should not be too strong a gross substitute for good 2, because otherwise own-price elasticity is still negative. Thus, in the smaller car – larger car example, in order for the smaller car to be a Giffen good, the smaller car should be a gross substitute for the large car, but not too strong a gross substitute.

The decomposition in equation (10) may again have a psychological counterpart. As a first step, the consumer chooses his consumption of good 1 as if the *x*% increase in the price of good 1 were equivalent to an overall loss in purchasing power of *x*%. As a second step, the consumer considers the fact that in reality, relative to such an overall loss in purchasing power, good 2 has become cheaper, and adapts his consumption of good 1 accordingly.

5.3. Strong symmetric substitutability, and deviation

The decompositions in Sections 5.1. and 5.2 separately consider the fact that for Giffen behaviour, good 1 must be a gross substitute for good 2 and that good 2 must be a gross complement to good 1. We now provide a decomposition which considers these two facts simultaneously, by looking for the demand bundle obtained when the two goods are strongly symmetrically substitutable, and by considering how the consumer deviates from this. Graphically, Figure 11 provides the standard Hicksian decomposition of total effect of a decrease in price of good 1 (from bundle I to bundle II) into a substitution effect (bundle I to bundle III) and an income effect (bundle III to bundle II). The income effect is here decomposed into two further effects. *First*, a new bundle IV is introduced which the consumer consumes if he considers the two goods as symmetric gross substitutes – meaning by Proposition 3 that the consumer has homothetic preferences. The bundle IV can simply be found by drawing a linear income expansion path that crosses both the origin and bundle III. We call the move from bundle III to bundle IV the symmetric gross substitutability effect. By the analysis in Section 4, bundle IV can be considered as the bundle that the consumer consumes if his preferences over low- and high-quality goods do not change as his purchasing power changes. *Second*, the change in consumption from bundle IV to bundle I reflects the effect of asymmetric gross substitutability, in this case the effect of good 1 being a better gross substitute for good 2 than good 2 is for good 1. By the analysis in Section 4, this effect at the same time may be considered as measuring the extent to which the consumer's preferences over quality change as purchasing power is increased, where in the case of Figure 11, if good 1 is considered as the high-quality good, the preferences for high quality decrease. As shown in (11), a formal representation of this decomposition follows straightforwardly from the Hicksian decomposition in elasticity form, which can be derived from equation (1), where $\varepsilon_{h_1 p_1}$ is compensated own price elasticity, and s_1 is the share of the budget spent on good 1. In elasticity terms, the symmetric gross substitutability effect equals $-s_1 * 1$, and the asymmetric gross substitutability effect equals $s_1(1-\varepsilon_{x_1 m})$:

$$\varepsilon_{x_1 p_1} = \varepsilon_{h_1 p_1} - s_1\left[1-(1-\varepsilon_{x_1 m})\right] \tag{11}$$

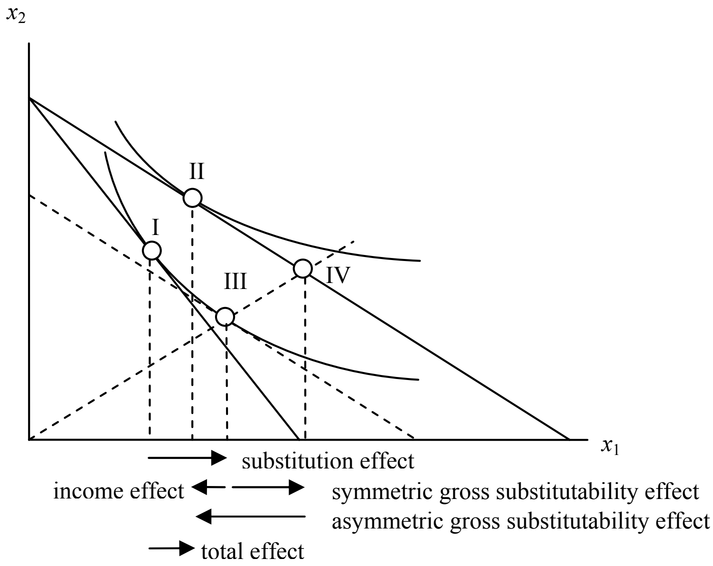


Figure 11 Decomposition into symmetric and asymmetric substitutability

6. Conclusion and summary

Whether or not Giffen behaviour is actually observed empirically, insight into the determinants of Giffen behaviour is useful to gain insight into the behaviour of rational consumers. This paper has shown that, on top of the traditional condition of inferiority of the Giffen good, the Giffen good should additionally be asymmetrically substitutable for the other good, in that the Giffen good is a gross substitute for the other good, but the other good is a gross complement to the Giffen good. While these two conditions are necessary conditions, it was shown that, when the necessary condition of inferiority is met, where one good is an inferior good an the other good a luxury, it may be that the inferior good is price elastic and the luxury is price inelastic. This is the opposite of what we need for approaching Giffen behaviour on the inferior good. As we argue, intuitively this may occur when the inferior good is a low-quality good and the luxury is a high-quality good. The demand for the low-quality good is then price elastic, because a good substitute is available for it in the form of the luxury.

With asymmetric substitutability, however, it is certain that good 1 is price inelastic and good 2 price elastic, and we do have a situation that always resembles Giffen behaviour. In this sense, the necessary condition that one good is a substitute for the other good, whereas the other good is a complement to the first good, better characterises Giffen or near-Giffen behaviour. Intuitively, rather than being a low-quality good, it can be argued that the Giffen good is a high-quality good, in the sense of being effective in letting the consumer subsist. At the same time, the luxury is a low-quality good, in that it in isolation it is ineffective in letting the consumer subsist. Put otherwise, the inferior good is a better substitute for the luxury than the other way around. This effect already makes the inferior good have a low price elasticity compared to the luxury. Further, when the consumer's purchasing power increases, he is less concerned with subsistence, and if we equate subsistence with quality thus cares less about quality. This further decreases the size of the price elasticity of the inferior good, which for this reason may become a Giffen good.

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