Tjalling C. Koopmans Research Institute

Discussion Paper Series nr: 05-18

The Market for Mergers and the Boundaries of the Firm

Matthew Rhodes-Kropf
David T. Robinson

Universiteit Utrecht
Utrecht School of Economics

Tjalling C. Koopmans Research Institute
Utrecht School of Economics
Utrecht University

Vredenburg 138
3511 BG Utrecht
The Netherlands
telephone +31 30 253 9800
fax +31 30 253 7373
website www.koopmansinstitute.uu.nl

The Tjalling C. Koopmans Institute is the research institute and research school of Utrecht School of Economics.
It was founded in 2003, and named after Professor Tjalling C. Koopmans, Dutch-born Nobel Prize laureate in economics of 1975.

In the discussion papers series the Koopmans Institute publishes results of ongoing research for early dissemination of research results, and to enhance discussion with colleagues.

Please send any comments and suggestions on the Koopmans institute, or this series to M.Damhuis@econ.uu.nl

ontwerp voorblad: WRIK Utrecht

How to reach the authors

Please direct all correspondence to the first author.

Matthew Rhodes-Kropf
Columbia Business School
3022 Braodway, Uris Hall 803
New York, NY 10027
U.S.A.
E-mail: mr554@columbia.edu

David T. Robinson
Fuqua School of Business
Duke University
Box 90120
Durham, NC
27708-0120
U.S.A.
E-mail: DavidR@duke.edu

Utrecht School of Economics
Tjalling C. Koopmans Research Institute
Discussion Paper Series 05-18

The Market for Mergers and the Boundaries of the Firm

Matthew Rhodes-Kropf[a]
David T. Robinson[b]

[a] Graduate School of Business
Columbia University
[b] Fuqua School of Business
Duke University

14 October 2004

Abstract
Mergers are the mechanisms that redraw the boundaries of the firm. In this paper, we relate incomplete contracts, upon which much of our understanding of firm boundaries is based, to empirical regularities in the market for mergers and acquisitions. We begin by empirically challenging conventional wisdom about mergers and acquisitions: high M/B acquirers typically do not purchase low M/B targets. Instead, mergers typically pair together firms with similar M/B ratios. To show why this occurs, we build a continuous time model of investment and merger activity that combines search, relative scarcity, and asset complementarity. Our model shows that the 'like buys like' empirical finding is a natural consequence of a prediction from the property rights theory of the firm; namely, that complementary assets should be placed under common control. A number of new empirical predictions emerge from our analysis. First, if asset complementarity is important, then we should see small differences in the M/B of targets and acquirers. It also predicts that the difference in M/B ratios should increase when discount rates are high and valuations are low. In additional tests, we show that both of these predictions are borne out by the data. Our findings suggest that the incomplete contracts theory of the firm is central to understanding the empirical regularities of the market for mergers and acquisitions.

Acknowledgements
We would like to thank Boyan Jovanavic, Charles Jones, Gordon Philips, Stefano Rossi, Tano Santos and S. Viswanathan for useful discussions, and workshop participants at Columbia, Duke, the 2004 EFA meetings, the National Bureau of Economic Research Summer Institute, the Stockholm School of Economics, SUNY Bu_alo, SUNY Binghamton and Texas Christian for helpful suggestions on earlier versions of this paper. Any errors are our own.

The boundaries of the firm are constantly being drawn and redrawn in the market for corporate control. In 2003 alone, over $500 billion in US merger activity occurred in over 8,200 transactions.[1] This merger activity should reflect the importance of the placement of the boundaries of the firm. The goal of this paper is to bridge the gap between the theory of the firm and empirical evidence on mergers, and demonstrate the significance of the theory of the firm for understanding empirical regularities in the market for mergers and acquisitions.

One of the most well-established stylized facts about M&A activity is that the typical merger involves an acquirer with high asset valuations purchasing a target with low asset valuations. This is commonly interpreted as evidence in favor of a 'Q-theory of mergers,' in which mergers involve redeploying the assets of underperforming targets towards more profitable uses under the better management of the high-performing acquirer.[2]

Section I of this paper challenges this conventional wisdom with a fresh look at who buys whom in mergers and acquisitions. Our analysis shows that mergers bring together firms with similar M/B ratios. While the average M/B for an acquirer is slightly higher than the average for a target, the spread is typically less than one-sixth of a standard deviation in industry M/B. Bidders from high M/B deciles acquire other high decile targets; low decile bidders acquire other low decile targets. This pattern holds even after making industry adjustments. Thus, instead of 'high buys low' describing most mergers, we show that 'like buys like' is the central stylized fact surrounding merger activity.

Figures 1 and 2 illustrate the striking nature of this pattern. Figure 1 plots the distribution of the difference in market-to-book ratios between acquirers and targets and shows that a large fraction of transactions cannot be described as high buying low. In Figure 2 we explore this further with a bivariate distribution of acquirer and target market-to-book ratios. With acquirers on the left horizontal axis and targets on the right axis, the height of the graph represents the merger activity between each M/B decile. The distinct ridge running along the diagonal of the graph shows just how much of the merger activity occurs between firms with similar M/B ratios.

This 'like buys like' result is surprising when viewed through the lens of existing theory. No current theory suggests any reason for the M/B of targets and acquirers to be so similar. The theories of agency (Jensen, 1986), hubris (Roll, 1986), transaction costs (Coase, 1937), or even simple beliefs in the potential for synergies, suggest limited patterns in M/B. They do not imply

[1] Source: Mergerstat Review. Data include US and Cross-border transactions.

[2] This idea has its foundations in Manne (1965) who argues that low value, badly managed firms will be bought by better managed firms. It is also an extension of Tobin's Q theory of investment: if mergers are simply another form of investment then high M/B firms should invest by buying the assets of those firms that have low opportunities, and hence low M/B ratios. Recent work by Jovanovic and Rousseau (2002) puts this idea into a dynamic context to generate waves of activity. They show empirically that when M/B dispersion is high (implying a lot of room for the high M/B firms to improve the low M/B firms) then merger activity increases. This also suggests the Q-theory is at work.

convergence of bidder and target market/book ratios.[3] Theories of misvaluation (Rhodes-Kropf and Viswanathan (2004) and Shleifer and Vishny (2003)) suggest target and acquirer M/Bs are high on average, but the theories do not predict bidder/target similarity in M/B ratios. And the q-theory of mergers (Jovanovic and Rousseau, 2002) actually suggests the opposite result: the highest M/B firms should acquire the lowest. It is reasonable to assume that hubris, agency or q-theory are partial or even complete motivations in some mergers. However, since they cannot explain the pattern in M/B ratios that we observe in the data, these theories do not seem to capture the overriding motivation for mergers.

Thus, the second step in our analysis is to provide a new theory of mergers. Our theory combines search, scarcity, and the matching of assets to show that M/B ratios of merging firms naturally tend to equate when the merging firms have potentially complementary assets. Our paper demonstrates the small target/acquirer differential in M/B is a natural consequence of one of the key predictions from the property rights theory of the firm of Hart and Moore (1990) and Hart (1995), namely, that complementary assets should be bound together under common ownership. Thus, the data suggest that mergers are the process through which complementary assets are joined into a single firm.

At the core of the model is a continuous time version of a Diamond-Mortensen-Pissarides search model joined with a basic neoclassical investment model. There are three key ingredients in our analysis.

The first two ingredients are search and scarcity. In our model, firms initiate standard investment projects. At the same time, firms also search for Pareto-improving asset combinations with other firms. In our model, firms cannot contract on the creation or distribution of the surplus generated by the asset combination: placing assets under common control is the only way to realize the synergies from asset combinations.

When firms find an acceptable partner, they then bargain over the available surplus from merger. Whether a firm accepts or rejects a particular merger negotiation depends on whether it prefers the terms of the current offer to the expected net gains from waiting, which are in turn determined by the likelihood of future merger opportunities, as well as the expected surplus from future transactions.

Thus, the M/B ratio essentially contains two parts, one from the net present value of future investment projects and one from the net present value of future merger activity. The NPV from future investment arises from the skill or quality of the characteristics inherent to a firm. However, the NPV related to merger arises from the relative bargaining power of the merging firms, not from any inherent benefit they bring to the merger. This is because unlike investment,

[3] There exist many theories of mergers, for example Gorton, Kahl, and Rosen (2000) suggest they are a defensive mechanism, but none that we know of suggests a matching in M/B.

a merger must be negotiated. Therefore, the benefit each party receives depends on their negotiating position, which in turn depends on each firm's ability to locate another merger partner. Since both firms are necessary for the merger, the firm with the relatively more scarce assets will more easily locate another merger partner and therefore garner more of the merger gains. Thus, greater relative scarcity leads to higher ex ante M/B ratios.

The third ingredient is complementarity. We assume that gains from the merger are related to the firms' compatibility or complementarity with one another. That is, mergers will create greater surplus if the partners are a 'better match' along one or more dimensions. For example the best pharmaceutical company can do more with the best new drug than the second best pharmaceutical can do. Thus, the second best pharmaceutical firm could buy the best new drug firm but would have to give up a larger fraction of the synergy. The compatibilities that we are suggesting could arise along any dimension. For example, better production, better technology, better culture, etc. This element of our analysis builds on work by Becker (1981), Kremer (1993), Burdett and Coles (1997), and is related to Shimer and Smith (2000).

The fact that firms look for complementary partners does not directly imply that firms will have similar M/B ratios. We show that each firm trades off their desire to merge with a better partner with the endogenously reduced bargaining power they will have in such a merger. In equilibrium, if complementarity is important, successful mergers will exhibit a high degree of matching. The best targets and the best acquirers have the best outside opportunities and create the most synergies. They endogenously choose to search for each other and therefore, in equilibrium firms who have the highest M/B ratios in their respective industries will choose to merge. Our point is that relative scarcity determines who gets more surplus from the merger while complementarities cause the best to merge with the best.

The assumption that merging firms have complementary assets arises from Hart and Moore (1990), Hart (1995) and the incomplete contracting literature. When there are significant complementarities between assets then simultaneous ownership by one firm reduces the hold-up problems and under-investment that results from the incomplete contracting. If this idea is important then a shock that increases the complementarities across firms should lead to merger activity.

Two key predictions emerge from our analysis. First, our model predicts small differences in the relative M/B rank of targets and acquirers. As Figure 2 shows, this prediction is borne out by the data. Second, our model predicts that the difference in M/B ratios between merging firms should increase when discount rates are high and valuation levels are low. As Section V shows, this too is borne out by the data. Thus, the theory offers a guide: when thinking about the motivation behind mergers, we must focus on the traits that produce complementarities between partners. The data seem to reflect the idea that firms search for complementary partners and the market expects for firms to a large extent to share the gains from merging and have similar outside investment opportunities. Overall, our work suggests that incomplete contracts play an important role in determining the boundaries of the firm and therefore, who should merge with whom.

The balance of the paper is organized as follows. First, we explore the stylized facts surrounding merger activity. This is presented in Section I. Next, we build a model of merger activity based on search and scarcity. This is presented in Sections II and III. In Section IV, we analyze the role that asset complementarity plays in search. Then, in Section V we re-examine the empirical evidence to test the predictions of the model. Section VI discuses extensions, and Section VII concludes.

I Do High Value Acquirers Buy Low Value Targets?

This section revisits the conventional wisdom that the typical merger involves an acquirer with a high asset valuation purchasing a target with a low asset valuation. This basic finding is discussed in a great deal of prior empirical work, including Servaes (1991), Rau and Vermaelen (1998), Martin (1996), Loughran and Vijh (1997), Lang, Stulz and Walkling (1989), Andrade, Mitchell and Stafford (2002) and others. In their survey paper, Holmström and Kaplan (2001) argue that corporate governance issues led to the merger waves of the 80s and 90s. Their work implicitly squares with the 'high buys low' idea inasmuch as firms with poor corporate governance have low market values and are taken over by higher valued bidders. Typically this result is couched as evidence for favorable asset redeployment; i.e., that high quality managers are overtaking poorly run firms.

We revisit the relative market-to-book question by reexamining the empirical evidence on mergers and acquisitions. Our data include 3,400 merger transactions that were announced after 1980 between publicly listed bidders and targets in the United States. The merger data come from Securities Data Corporation's Mergers&Acquisitions database; these data are merged with CRSP and Compustat to calculate log market-to-book ratios.[4]

Figure 1 depicts the density of the difference in market-to-book valuations for bidders and targets. Positive values correspond to 'high buys low' transactions; these occur roughly 60% of the time. The plot shows that the mean value of the difference is positive, which indicates that on average, high market-to-book acquirers purchase lower market-to-book targets. However, the region to the left of the origin on the x-axis corresponds to the roughly 40% of the time

[4] We impose a three-month time lag between the date of the accounting and market information and the announcement of the merger to make sure that our measurements of the market-to-book ratio are not polluted by the announcement. For the exact details of our calculation, see Rhodes-Kropf, Robinson, and Viswanathan (2004).

in which the market-to-book of the acquirer is below that of the target.[5] This 'low buys high' result echoes findings in Rhodes-Kropf, Robinson, and Viswanathan (2004).

To explore this issue more deeply, we examine the joint distribution of acquirer and target Market-to-book ratios. Rather than simply examining the difference, examining the bivariate distribution allows us to see which types of firms are most often involved in mergers, which speaks directly to the question of who buys whom.

In Table 1, we group the population of bidders and targets into bins according to annual breakpoints of the distribution of market-to-book for all NYSE traded firms.[6] The i, j^{th} cell in Table 1 reports the frequency of mergers occurring between targets in decile i and bidders in decile j. The breakpoints are recomputed annually to reflect changes in the distribution of market-to-book ratios for all NYSE firms. Thus, any clustering that appears in the table is not a result of time-series clustering of merger activity.

The table illustrates a high degree of correlation between bidder and target m/b ratios; Pearson's χ^2 test for independence of bidder and target m/b ratios has a value of 854.91, with an associated p-value of 0.00. In fact, the mean difference is $\frac{8}{10}$ of a decile, meaning that the average transaction couples bidders and targets that are no more than one decile apart in the distribution of m/b ratios.

The table illustrates several interesting features of merger activity. First, merger activity seems to cluster down the main diagonal. This is the 'like buys like' diagonal. This means that in many cases, bidders and targets come from nearby points of the market-to-book distribution. The prevailing wisdom that high buys low is borne out by the fact that most mergers lie below the main diagonal; these correspond to mergers in which the acquirer is in a higher M/B decile than the target. This indicates that, if anything, bidders have slightly higher market-to-book ratios than targets, but that there asset valuations are generally quite similar. This table shows why most research has focused on the 'high buys low' result as that type of transaction is dominate. However, this table also suggests that something is driving firms with similar market-to-book ratios to merger.

Thus, Table 1 motivated this paper as it indicates that mergers may exhibit assortative matching. Instead of seeing firms with disparate valuations merging, we find that firms with similar valuations merge with one another. This holds even though mergers occur at times when the average industry Q-dispersion is high (Rhodes-Kropf, Robinson, and Viswanathan, 2002).

To explore this idea further, Table 2 reports the average bidder/target M/B spread by transaction type. The column labelled 'Mean Scaled M/B Difference' reports the bidder/target difference as a fraction of the within-industry standard deviation of M/B for the year of the

[5]These figures match Andrade, Mitchell, Stafford (2001), who report that roughly 2/3 of transactions involve an acquirer with a higher q than its target.

[6]The breakpoints were obtained from Ken French's website.

transaction. So, for example, if mergers occurred between an acquirer that was one standard deviation above the mean acquiring a firm that was one standard deviation below the mean (a prototypical 'high buys low' transaction) the scaled difference would equal two (200%).

In fact, the scaled difference is a paltry 14% when averaged across all transactions. When the acquirer's M/B exceeds that of the target, the scaled difference is only 77% of a standard deviation on average. This occurs in roughly 62% of the sample. The remaining 38% of the sample has a scaled difference of negative 89% of a standard deviation.

The third bank of numbers in Table 2 splits the data according to whether 'high buys high,' 'low buys low,' etc., where we gauge whether bidder and/or target were above or below their industry median values in a given year. When both are below the industry median, the scaled M/B spread is quite small, only 2% of an industry standard deviation. When high buys high, the difference is again quite small, although larger: it rises to 11.46% of a standard deviation. These low values support the idea of assortative matching, but naturally control for the fact that mergers may be occurring within or across industries at a point in time. Only when we examine transactions that involve bidders and targets on opposite sides of the median line do we find scaled M/B differences greater than 1. And it is important to note that these transactions are relatively uncommon: the most common transaction type is for both bidder and target to be above industry median (high buys high). The second-most common type is for both firms to be below the industry median.

We hasten to add that the results that we show here are not isolated to a particular point in time. As table 3 demonstrates, there is no obvious clustering of the low buys high result in a given year. In particular, our results are not driven by what happened in the late 1990s, or by the merger characteristics of the 1980s.

The analysis of the entire distribution of market-to-book ratios shows that the empirical phenomenon is less clear than conventional wisdom would suggest. While it is true that a comparison of means indicates that the M/B for acquirers is higher than targets, the fact is that most of the time both firms have high M/B. Modest, but consistent, average differences in M/B between bidder and target mask the fact that most of the time 'like buys like.' This pairing of firms with similar valuations implies that mergers exhibit assortative matching.

In light of this new evidence on who buys whom in mergers and acquisitions, our next task is to develop a model that squares with these facts and which generates new predictions. The model is discussed in the next three sections. In section V, we return to the data to empirically explore the new predictions of the theory.

II The Model

The model focuses on two types of firms, A and B, with two different types of assets, K_A and K_B. Both firms produce output y according to the production technology

$$y_i = z_i K_i^{\alpha}. \tag{1}$$

The parameter z_i represents the firm's production skill and can be interpreted as management quality or ability. This parameter is constant across states and over time. For simplicity, neither firm's capital stock depreciates. The production parameter $\alpha \in (0,1)$ implies decreasing returns to scale, which in turn guarantees that the capital stock is determinant. The particular choice of production function allows simple closed form solutions but does not drive any results.

Firms are also expected to find some positive net present value projects in the future. We assume that the probability of the arrival of an opportunity is the same for any future time interval of length Δ.[7] This ensures that at any moment in time the net present value of expected future opportunities is the same. Let us call the net present value of the future opportunities O_i.

The price of output is exogenously determined and normalized to 1. The appropriate discount rate is $r > 0$; therefore, the quantity $Y_i \equiv \frac{y_i}{r} + O_i$ represents the capitalized value of the infinite stream of future output and opportunities.

We will allow continuous and frictionless adjustment of the capital stock. Thus, the capital stock is always set optimally. The cost of one unit of capital is normalized to one dollar.

If firms are joined then the management ability parameter becomes z_M, where M denotes the merged firm. After the merger is consummated, the joint production function becomes

$$y_{AB} = z_M (K_A^{\alpha} + K_B^{\alpha}). \tag{2}$$

For example, if mergers are occurring for Q-theoretic reasons then good management should apply its expertise to the bad firm and $z_M = \max[z_A, z_B]$, but any z_M function is possible. The simple assumption that a merger alters only the z parameter will not drive any result. We will assume that the outside opportunities are not improved by the merger, but this is not important as our results would only be magnified if outside opportunities also improved.

There are gains from a merger if

$$y_A + y_B < z_M (K_A^{*\alpha} + K_B^{*\alpha}) - r\left(K_A^* + K_B^* - K_A - K_B\right), \tag{3}$$

[7] At each instant, the probability of an opportunity arriving over the next time interval Δ is $e^{-\Delta\lambda^O}$, where λ^O is the arrival intensity.

where K_i^* is the optimal capital stock chosen if the merger occurs. Thus, gains from merger are more likely if z_M is larger.

There are two states of nature, the No Merger state (NM) and the Mergers are Possible state (MP), $\Sigma \in \{\Sigma^{NM}, \Sigma^{MP}\}$, with associated state intensities λ^{NM} and λ^{MP}. Since time evolves continuously, this means that at each instant, the probability of remaining in state Σ over the next time interval Δ is $e^{-\Delta\lambda^{\Sigma}}$. The model begins in the state Σ^{NM}, which means that no profitable merger opportunities are available. Specifically, management ability is assumed to be such that $z_i > z_M$ $\forall i$.

If the economy is in the Σ^{NM} state, there is probability $1-e^{-\Delta\lambda^{NM}}$ that a positive shock will occur to z_M. If a shock occurs, the state switches to Σ^{MP}, and profitable merger opportunities are available. This shock captures the idea from Gort (1969) that exogenous factors, such as the discovery of a new technology or production process, create periods of organizational flux. These periods give rise to opportunities for new organizational structures that will improve production in the economy.

We assume that firms do not merge before the shock in preparation for the benefit because they are unaware of which type of assets they will need when the shock occurs. It seems reasonable that firms did not try to locate an internet partner in the 80's before the internet was invented. The nature of a shock is such that we do not know what form it will take until it occurs.

This shock allows a possible benefit from joining the assets of firm A and firm B. After the shock we will assume that, z_M is large enough to ensure condition (3) holds since we are interested in merger activity. We assume that the state of nature, the value of z_M and whether the shock has occurred are common knowledge to both firms.

It is important to note that the increase in firm output only occurs if a firm A and a firm B merge. Firms cannot contract on the creation or distribution of the surplus generated by the asset combination: placing assets under common control is the only way to realize the synergies from asset combinations.[8] Furthermore, if a firm remains a stand alone entity and simply invests in more assets, then z_i remains the same as before the shock. The idea is that firms develop different kinds of assets. The synergies in a merger occur because of the complementary nature of the assets of different firms. For example, an inventor may have a new product and another firm may have the assets that market and distribute. While we assume the frictionless adjustment of capital of type A for any firm A, we assume that type A firms cannot create the assets of type B. After the shock it is precisely because type A firms cannot develop any assets they desire that we consider the assets of type B to be potentially scarce. Therefore, a merger has a potential benefit. We could think that a firm A may have tried and failed to develop

[8] See section IV for a discussion of this assumption.

assets like those of B, or not. The point is that at a particular point in time B has had an asset realization and A has had an asset realization, and it is precisely because attempting to invest to achieve the other firm's assets is too costly relative to buying a type B firm (or impossible) that the A and B are scarce and may choose to merge.

Let π_i^{NM} represent the present value of firm i in the No Merger state. This value includes an expectation over all possible future states of the world. As noted above it is possible that the state jumps to a state where Mergers are Possible (MP). When mergers are possible we assume that firms must search for a suitable partner. The delay in finding a partner represents the actual time firms must spend looking for a partner as well as the significant time spent in due diligence to ensure the match is viable. Let π_i^{MP} represent the present value of firm i in the Mergers are Possible state before they have located a potential partner. If a firm locates a potential partner then they must negotiate the terms of the deal. If the deal is consummated then on the announcement of a completed deal a firm's value changes to π_i^M, where π_i^M represents the expected value of a merger to firm i. If a deal cannot be reached then firms continue to search for another potential partner. Thus, their value remains at π_i^{MP}. At any time during this search process the state may return to the No Mergers state and then the value of firm i would return to π_i^{NM}. We will develop this search and negotiation more fully below.

Figure 3 depicts a game tree for the model. It shows how the economy can move from the no merger state to the merger state. Once in the merger state, mergers can either occur or not occur, depending on whether merger partners are found and can agree on a deal.

A. Investment When No Mergers Are Possible

The world begins in the No Mergers state (NM). As long as this state persists, each firm chooses its investment, I, to solve

$$\max_{I_i} \left[(K_i + I_i + \Delta z_i (K_i + I_i)^{\alpha}) e^{-r\Delta} - I_i \right], \tag{4}$$

where the subscript i represents either A or B. This expression describes the value of the firm as the value of assets in place, plus the value of the production that will occur over the interval Δ, minus the required investment. Since investment is frictionless there is no need to invest more or less than what is instantaneously needed. The FOC is

$$\frac{\Delta \alpha z_i (K_i + I_i)^{\alpha - 1} e^{-r\Delta}}{1 - e^{-r\Delta}} = 1. \tag{5}$$

By letting $\Delta \to 0$ we have

$$\frac{\alpha z_i (K_i + I_i)^{\alpha - 1}}{r} = 1. \tag{6}$$

Therefore, each firm invests until the marginal invested dollar will increase the value of the firm by exactly one dollar.

$$I_i = \sqrt[\alpha-1]{\frac{r}{z_i\alpha}} - K_i \tag{7}$$

Since investment is costless and frictionless that capital stock is always

$$K_i^{NM*} = \sqrt[\alpha-1]{\frac{r}{z_i\alpha}}, \tag{8}$$

where the $NM*$ signifies that this is the optimal capital choice in the No Mergers state. It is clear that larger α and larger z_i firms will be larger and generate more profits.

During any small moment of time, Δ, each firm earns $\Delta z_i \left(K_i^{NM*}\right)^\alpha$. There is also a chance that a shock occurs or that a shock does not occur. Let π_i^{NM} represent the expected value of the No Merger state, and let π_i^{MP} represent the expected value if the state is such that Mergers are Possible. The expected value of the No Mergers state can be written as

$$\pi_i^{NM} = e^{-\Delta\lambda^{NM}}\pi_i^{NM}e^{-r\Delta} + (1 - e^{-\Delta\lambda^{NM}})\pi_i^{MP}e^{-r\Delta} + \Delta z_i \left(K_i^{NM*}\right)^\alpha e^{-r\Delta}. \tag{9}$$

That is, the expected value in the No Mergers state equals the discounted expected value of the No Mergers state after Δ time has passed ($\pi_i^{NM}e^{-r\Delta}$) times the probability that the No Mergers state persists ($e^{-\Delta\lambda^{NM}}$), plus, the discounted expected value of the Mergers are Possible state ($\pi_i^{MP}e^{-r\Delta}$) times the probability that the state jumps ($1 - e^{-\Delta\lambda^{NM}}$), plus the discounted value of the output produced during that time period ($\Delta z_i \left(K_i^{NM*}\right)^\alpha e^{-r\Delta}$).

To solve for the expected value in the No Merger state (NM) we must find the expected value in the Mergers are Possible state (MP). This requires us to examine the search a negotiation process. However, to understand negotiations we must first examine the benefits that would arise from a merger.

B. The Gains from Merger

After the merger, the merged firm chooses its capital stock to solve

$$\max_{K_A,K_B} \left[\Delta z_M((K_A + I_A)^\alpha + (K_B + I_B)^\alpha)e^{-r\Delta} - I_A - I_B + (K_A + K_B + I_A + I_B)\, e^{-r\Delta}\right]. \tag{10}$$

The FOCs are

$$\frac{\Delta\alpha z_M(K_i + I_i)^{\alpha-1}e^{-r\Delta}}{1 - e^{-r\Delta}} = 1 \tag{11}$$

Again, in the continuous time limit we have

$$I_i = \sqrt[\alpha-1]{\frac{r}{z_i\alpha}} - K_i. \tag{12}$$

The optimal amount of each type of asset now differs because of the change in z.

$$K_i^{M*} = \sqrt[\alpha-1]{\frac{r}{z_M \alpha}}, \tag{13}$$

where the $M*$ signifies that this is the optimal investment in assets of type i post Merger. We assume nothing changes in a firm in the periods after the merger, therefore the capital stock will remain the same for all future periods. Thus, the value of the merged firm is

$$\frac{z_M((K_A^{M*})^\alpha + (K_B^{M*})^\alpha)}{r}. \tag{14}$$

The optimal capital stock in the state where Mergers are Possible but before the merger, is the same as in the No Merger state. That is, $K_i^{NM*} = K_i^{MP*}$. For now, we assume that firms cannot commit to a non-optimal level of capital post merger negotiations (whether or not the merger occurs) in an attempt to change the piece of the pie they receive. We will show in equilibrium (see proof of Proposition 1) that even if firms could commit to a non-optimal level of capital they have no incentive to do so. Thus, the value of the merged firm is the value of the output at the optimal investment level minus the cost of the increased investment.

$$s \equiv \frac{z_M((K_A^{M*})^\alpha + (K_B^{M*})^\alpha)}{r} - (K_A^{M*} + K_B^{M*} - K_A^{MP*} - K_B^{MP*}) + O_A + O_B. \tag{15}$$

The set of possible agreements is $\Pi = \{(\pi_A^M, \pi_B^M) : 0 \leq \pi_A^M \leq s \text{ and } \pi_B^M = s - \pi_A^M\}$, where π_i^M is the share of the merged firm to player i ($i \in \{A, B\}$). For each $\pi_i^M \in [0, s]$, π_i^M is the expected profit player i obtains if an agreement is reached and a merger occurs.

In equilibrium, if a firm finds a partner it is possible to strike a deal as long as the utility from a deal is greater than the outside opportunity for both firms. If a firm rejects a merger partner then they remain in the Mergers are Possible state with expected value π_i^{MP}. Thus, π_i^{MP} represents the utility from continuing to search and also the disagreement utility.

To determine how firms share the surplus generated by the merger we must decide on a model for the negotiations. While many different choices for the model of negotiations will work for our purposes, the simplest is the Nash bargaining solution, which in this case is just the solution to

$$\max_{(\pi_A^M, \pi_B^M) \in \Pi} (\pi_A^M - \pi_A^{MP})(\pi_B^M - \pi_B^{MP}). \tag{16}$$

The well known solution to the bargaining problem is presented in the following Lemma.[9]

[9] The generalized Nash bargaining solution is a simple extension but adds no insight and is omitted.

Lemma 1 *In equilibrium the resulting merger shares for firms A and B are*

$$\pi_A^M = \frac{1}{2}(s - \pi_B^{MP} + \pi_A^{MP}) \text{ and } \pi_B^M = \frac{1}{2}(s - \pi_A^{MP} + \pi_B^{MP}), \tag{17}$$

where the π_i^{MP} are the disagreement expected values and s is defined by equation (15).

With this understanding of the negotiation and merger we can now look at the state when mergers are possible and examine the search process.

C. The Search for a Merger Partner

If mergers create value then they will occur, but only if firm A and B can find each other and agree on a merger price. To explore this process we combine the Nash bargaining model with the classic search model of Diamond-Mortensen-Pissarides (for examples see Diamond (1993), Mortensen and Pissarides (1994), and for a review see Petrongolo and Pissarides (2001)).[10] This allows for a negotiated outcome that depends on each party's ability to locate another suitable merger partner.

If firms must search for a partner, we must define the probability that they locate one. We assume that the measure of firms with the assets of firm A is M_A and the measure of firms with the assets of firm B is M_B. As is standard in search models, we define $\theta \equiv M_A/M_B$. This ratio is important because the relative availability of each type of firm will determine the arrival rate of merger opportunities and therefore influence each firms bargaining ability. This fraction represents the *relative scarcity* of each type of asset. If θ is high there are many more firms with type A assets than type B, and vice versa.

Given the availability of each type of firm, the number of negotiations per unit of time is given by the matching function $\psi(M_A, M_B)$. This function is assumed to be increasing in both arguments, concave, and homogenous of degree one. This last assumption ensures that the arrival rate of merger opportunities depends only on the relative scarcity of the assets, θ, which in turn means that the overall size of the market does not impact each firm in a different manner. Each individual firm experiences the same flow probability of a match. Thus, the arrival rate of a merger opportunity is a Poisson process. The arrival rate of a merger is

$$\psi(M_A, M_B)/M_A = \psi(1, \frac{M_B}{M_A}) \equiv q_A(\theta), \tag{18}$$

for firm A. By the properties of the matching function, $q_A'(\theta) \leq 0$, the elasticity of $q_A(\theta)$ is between zero and unity, and q_A satisfies standard Inada conditions. Thus, firm A is more likely to meet an available firm B if the ratio of type A to type B firms is low. From firm B's point

[10]For a complete development of the model see Pissarides (1990). Our exposition follows this work. This combination was recently used by Inderst and Müller (2002) in examining venture investing.

of view the arrival rate of mergers is $\theta q_A(\theta) \equiv q_B(\theta)$. This differs from the viewpoint of firm A because of the difference in relative scarcity of their assets. $q'_B(\theta) \geq 0$, thus Firm B is more likely to meet an available firm A if the ratio of type A to type B firms is high.

During any short period of time, Δ, there is a probability that firm A finds a merger partner, $\Delta q_A(\theta)$ and a probability that firm A does not find a partner, $(1 - \Delta q_A(\theta))$, and must continue the search. During this search period there is also the probability that the Mergers are Possible state ends. This happens because of another technological shift that eliminates the usefulness of any more combinations of assets A and B. As noted above, the intensity of the Mergers are Possible state is λ^{MP}, so the probability that mergers are still viable after a search of time Δ is $e^{-\Delta\lambda^{MP}}$. If the Merger are Possible state ends then each firm receives the expected value that they originally achieved at the beginning of the model in the No Merger state, π_i^{NM}.

We assume that the measure of each type of firm is unchanging and therefore π_i^{MP} is the same at any point in time. This stationarity requires the simultaneous creation of more firms with assets like those of firm A and B to replace those that merge. We can think of these new firms as coming from spin-offs of other combinations of assets that are no longer synergistic or from fundamental firm creation. We know that new firm creation and spin-off activity are highly correlated with merger activity. Let m_i denote the rate of creation of new type i firms. Stationarity requires the inflows to equal the outflows. Therefore, $m_i = q_i(\theta)M_i$.[11]

Given the discount rate r, the disagreement utility of firm A is defined by

$$\begin{aligned}\pi_A^{MP} = {} & \Delta q_A(\theta)e^{-\Delta\lambda^{MP}}\pi_A^M e^{-r\Delta} + (1 - \Delta q_A(\theta))e^{-\Delta\lambda^{MP}}\pi_A^{MP}e^{-r\Delta} \\ & + (1 - e^{-\Delta\lambda^{MP}})\pi_A^{NM}e^{-r\Delta} + \Delta z_A \left(K_A^{NM*}\right)^\alpha e^{-r\Delta}.\end{aligned} \tag{19}$$

The four summands can be interpreted as follows. The expected value when mergers are possible (π_A^{MP}) equals the probability of finding a partner ($\Delta q_A(\theta)$) times the probability that an agreement is still valuable ($e^{-\Delta\lambda^{MP}}$) times the discounted value of finding a merger partner and agreeing with them ($\pi_A^M e^{-r\Delta}$). The second term is the probability a partner cannot be found $(1 - \Delta q_A(\theta))$ times the probability the state persists ($e^{-\Delta\lambda^{MP}}$) times the discounted value continuing to search (π_A^{MP}). The third term is the probability that the state jumps to the No Merger state $(1 - e^{-\Delta\lambda^{MP}})$ times the discounted expected value of the No Mergers state ($\pi_A^{NM}e^{-r\Delta}$). The fourth term is the discounted value of the output produced during that time period ($\Delta z_A \left(K_A^{NM*}\right)^\alpha e^{-r\Delta}$).

The model is now completely defined and we have enough information to solve for both the

[11] In standard labor search models the inflow are from employees who leave their job. This is because labor models are focused on the rate of unemployment. There is no analog in mergers as we are not interested in the 'rate' that firms stay unmerged. None of our results depend on this simplification as we could have inflows arise only from spin-offs.

market values and the market-to-book ratios.

III Relative Scarcity and Equilibrium M/B

This section will demonstrate the basic solution to the model and the endogenous nature of the M/B ratio. A firm's M/B reflects their stand alone opportunities plus the expected gains from the merger even before the merger occurs. But a firm's M/B depends not on the total gain from the merger, but rather on the share that firm will receive. We show how the total merger gain is split depends on the relative scarcity of the assets each firm brings to the merger. This establishes the notion of relative bargaining power that will be important when we think about what kind of partner a firm should try to find.

The first proposition provides the complete solution to the model.

Proposition 1 *Assuming condition (3) holds, expected profit is given by:*

$$\pi_i^{NM} = \frac{1}{\lambda^{NM} + r} \left(\lambda^{NM} \pi_i^{MP} + rY_i\right), \tag{20}$$

$$\pi_i^{MP} = \frac{(2D + q_j(\theta)) Y_i + q_i(\theta) (s - Y_j)}{2D + q_i(\theta) + q_j(\theta)}, \tag{21}$$

$$\pi_i^{M} = \frac{(D + q_i(\theta)) (s - Y_j) + (D + q_j(\theta)) Y_i}{2D + q_i(\theta) + q_j(\theta)}, \tag{22}$$

where $D = r\frac{\lambda^{MP} + \lambda^{NM} + r}{\lambda^{NM} + r}$, $i \neq j$, $\forall i, j \in \{A, B\}$.

Proof. See appendix. ■

These solutions are actually rather intuitive as they are simply weighted averages of the possible future outcomes. The profit in the No Mergers state is just a weighted average of the expected profit from entering the state where Mergers are Possible and staying as a stand-alone entity. If, for example, the No Mergers state persists indefinitely ($\lambda^{NM} \to 0$) then the firms expected profit would just be Y_i, the present value of future output as a stand-alone entity. The expected value after a merger has occurred, or the expected value in the Mergers are Possible state is just a weighted average of the surplus remaining after the other firm receives his stand alone value, $s - Y_j$, and the firms own stand alone value, Y_i. The weights of course depend on the firm's relative bargaining powers. The bargaining powers depend on the ability to find another partner and the probability that it will still be viable to merge with them (the Mergers are Possible state persists) and on the opportunity cost of waiting, r.

A couple of particular cases help provide intuition. If each type of firm is equally likely, $q_i(\theta) = q_j(\theta)$, then the bargaining powers are equal. Therefore, the profit from a merger becomes $\frac{1}{2}(s - Y_j + Y_i)$. That is, firms split the surplus from the merger after each takes the present value of their own firm as a stand alone entity. This also occurs, in the extreme case, as the instantaneous probability that the state switches to the No Mergers state increases to one ($\lambda^{MP} \to \infty$), the relative bargaining powers from continuing to search equate because neither firm will be able to find another partner in time. Thus, once again, the firms split the surplus after each receives his stand alone value.

Knowledge of the expected profit from each possible outcome leads directly to the next proposition that tells us how each firms share in the merger is affected by their scarcity.

Proposition 2 *The negotiated share of the combined firm that each firm receives is larger the more scarce their assets.*

Proof. See appendix. ■

This proposition shows that the more scarce a firm's assets the greater its bargaining power because it can more easily locate another merger partner. Thus, its share of the merged entity is larger.

A. Endogenous Market-to-Book Ratios

We now understand how the synergies from the merger will be shared and how this will effect the expected value in all states of the world. Combining this understanding with our calculations of capital we can examine the Market-to-Book ratio.

The Market-to-Book ratio at any point in time is the present value divided by the chosen capital stock. It is easy to see that the market values and thus the market to book ratios pre merger increase if the probability of a merger increases.[12] The central point we wish to make in this section is that firms M/B ratios before a merger are affected not by the value they bring to the future merger, but by their relative bargaining power. This bargaining power depends on their ability to locate another partner, i.e., the firm's relative scarcity.

Proposition 3 *Increasing the scarcity of a firm's assets increases the probability that firm is in a merger, increases the share of the synergy they receive and increases their M/B ratios in all states (assuming the synergy is positive).*

Proof. See Appendix. ■

This proposition shows us that the reflection of future merger activity in M/B ratios depends on the relative scarcity of the firm's assets. This is because a merger must be negotiated. Therefore, the benefit each party receives depends on their negotiating ability, which in turn depends on each firm's ability to locate another merger partner. Since both firms are necessary

[12] The probability of a merger increases if the persistence of the No Mergers state decreases (λ^{NM} increases) or the persistence of the Mergers are Possible state increases (λ^{MP} decreases)

for the merger, the firm with the relatively more scarce assets will more easily locate another merger partner and therefore garner more of the merger gains. Thus, greater relative scarcity leads to higher ex ante M/B ratios.

This proposition tells us that the relationship between the M/B ratios of acquirers and targets could be anything: high buys low, low buys high, or like buys like, as it depends on the relative scarcity of the merging firms. Thus, we should think about mergers differently from neoclassical investment. Standard Q-theory logic is that a firm has a high M/B because it has great investment opportunities. Since, the logic goes, mergers are just another form of investment high M/B firms should buy low M/B firms. We argue that investment and mergers are fundamentally different because in a merger the acquirer has to split the surplus with the target. Whereas with an investment the investor creates and thus earns all of the surplus. Thus, firms merge for reasons of organizational structure i.e., to create synergies. The firm with the higher M/B is the firm with the relatively more scarce resources not necessarily the firm who brings the better management skill.

Our theory actually subsumes standard Q theory logic. In standard thinking the firm with the great investment opportunities has the higher M/B because the net present value of these future investment are impounded into their current price. If mergers are just another form of investment then high M/B firms should buy low M/B firms. But what if it were the case that many firms had great management and great ideas, but there were only a few firms with the assets needed to implement these ideas. The good managers would then compete to buy the relatively more scarce assets. Thus the firm with the assets and no ideas would acquire most of the surplus from the merger. Ex ante the market would realize this and award the firm with no investment ideas the higher M/B. We are not suggesting that this is a likely world. Our point is that there is an unspecified assumption that is a part of the standard thinking: management skill is relatively scarce and furthermore, is relatively more scarce than any asset held by the target.

We are suggesting that there are essentially two parts to M/B. Part of the M/B or Q is from the stand alone value of the firm. However, there is also a merger Q in our model. We have shown the M/B ratios reflect the gains from the merger before the merger occurs. This is the merger Q. The merger Q, however, must be split between two firms. Thus, to understand Q in the context of mergers we must understand how firms will split the gains from merging. Economic logic tells us that the relatively more scarce firm will have the greater bargaining power and therefore appropriate the greater share of the surplus. The market anticipates a firm's relative scarcity and alters its market value before the merger accordingly. Thus, a firm that is relatively more scarce may have a high M/B even if it has no investment opportunities on its own!

It is natural, of course, to think that skill is relatively more scarce than poorly performing assets so the Q-theory suggestion that high M/B buys low requires only the additional assumption that managerial skill is rare and assets that need to be redeployed are not. Thus, if mergers are about the redeployment of assets to better uses then there should be a large differential between the target and acquirer M/B as one is relatively more scarce. However, as noted this does not seem consistent with the data. Proposition 3 shows us that the only way to make current merger theories such as Q-theory, agency or hubris consistent with a small acquirer M/B - target M/B differential is to assume that they are equally scarce. As there is no obvious economic justification in any current theory for such an assumption we propose a new way to think about mergers.

IV Incomplete Contracts and Complementary Assets

One of the central predictions of Hart and Moore (1990) and Hart (1995) is that in a world of incomplete contracts, highly complementary assets should be contained in a single firm. This suggests that firms with complementary assets should be more willing to pay the costs of merging to gain the benefits associated with concentrated ownership. This section models the process through which this occurs.

As we stated in the introduction and the model set up, incomplete contracts enter our model in the following way. We assume that when two parties are paired, they cannot contract on the provision and division of the surplus created by combining their assets. The only way to capture the synergies is to place the assets under common control.

One way to arrive at this reduced-form representation of mergers is to assume, in the spirit of Williamson (1985), Grossman and Hart (1986) or Hart and Moore (1990), that the realization of synergies involves relationship-specific investments. Firms that chose to contract with each other instead of merging would later face hold-up problems as either firm could threaten to search for another partner. This would lead to underinvestment in relationship specific objectives, such as quality, along a particular dimension on which firms could match.

Following Hart and Moore (1990), these incomplete contracting problems are most severe between assets that are highly complementary. This implies that firms with the highest degree of complementarity have the strongest incentive to merge, since the opportunity cost of underinvestment is highest between them.[13]

To incorporate this logic into our model we introduce the potential for firms to match along a number of distinct dimensions. For example, firms might differ according to culture, quality, location, etc. Potential partners can choose to match on only a few dimensions, and generate

[13] See Tirole (1999) for an interesting summary of the incomplete contracting literature.

relatively low surplus, or they can choose to find a partner who matches on a large number of dimensions, and generate a great deal of surplus.

Firms trade off searching for increased synergies with the diminished bargaining power they may possess if they face a high quality partner. In equilibrium we show that firms continue to search for a partner until they find a match on many dimensions. Any firm would like a 'better' partner, ceteris parabis, and any match may have the potential to generate some surplus, but better partners have more bargaining power. Thus, lower quality firms choose to search for lower quality partners to get a larger fraction of smaller synergies. Because high quality firms can generate more surplus by pairing with other high quality firms, lower quality firms prefer to settle for lower quality partners than to wait for a higher quality partner but face a disadvantaged bargaining position. In equilibrium, this is reflected in their M/B ratios: the best type A firms (who endogenously have the highest M/B) merge with the best type B firms, and targets and acquirers have M/B ratios that come from similar deciles within their industries.

The complementarity between firms could arise from any number of dimensions. For example, consider the best new product firm. Should they look to merge with the best marketing and distribution firm? It seems natural that this would be the best match. Who has the higher M/B ratio or 'better' management? Neither obviously stands out, but there is a clear potential for the sum to be greater than the parts. When Cisco buys a technology startup, are they redeploying assets to a better use? No; they are looking for a firm who has the best new technology that will most complement their own. When Cisco finds the potential partner that in equilibrium chooses to merge with them, who has the greater bargaining power? Whomever is more scarce. This might be Cisco because they have the needed market power, or the startup because they have the best technology. Our point is that relative scarcity determines who gets more surplus from the merger while complementarities cause the best to merge with the best.

A. Search with Asset Complementarity

To implement these ideas we extend the model developed in the previous sections to incorporate some of the ideas in Burdett and Coles (1997). Within the set of firms with type A or type B assets there exists a subset of types. Thus, we partition the measure of type A and B firms into subcategories. For simplicity we assume each of the n subcategories are equally likely. (This does not affect our results but simplifies the analysis.) A firm from any subcategory will be denoted by a subscript k or h on A or B, where k and h are elements from the set $N = \{1, ..., n\}$ which represents the set of n subcategories. Thus, when a type A firm locates a potential type B merger partner there will be a probability $\rho \equiv 1/n$ that the type B firm is of any particular subtype, B_h. We will assume that this is the same as from firm B's point of view. That is, when B finds an A partner there is a ρ chance that firm A is any particular subtype, A_k.

The number of subcategories of firms is determined by the number of dimensions on which firms can differ. For a given number of dimensions d, the number of subcategories n is $n = 2^{d-1}$. For example, if firms differ on only 1 dimension (i.e., they are either type A or B), then there is only one subcategory. If firms can differ along a second dimension, then $n = 2^{2-1} = 2$. If firms can have a third feature, then are are four subtypes of A firms, $n = 2^{3-1} = 4$, and so forth.

We assume that firms with the same subtype have the same ability to generate synergies and the same outside opportunities, O_{A_k}. Our naming convention will give the lowest number to the highest quality firm. Thus, the highest quality type A firms are subtype A_1, then next highest quality are A_2. Higher quality means the ability to generate more synergies in a merger and have greater outside opportunities. For simplicity we assume all type A firms have the same stand alone value (same z_A).

Let $y_{A_kB_h}$ represent the output of the different types of mergers. Output differs because $z_M \equiv z_{A_kB_h}$ differs. The possibility that firms endogenously sort on type, called assortative matching, requires that the firm's production function exhibits strict supermodularity (see Becker (1981)). The assumption of supermodularity in our context means that if type 1 firms match with type 2 firms then output would rise if all firms rematched with their 'own' type.

$$\sum_{k\in N} y_{A_kB_k} \geq \sum_{k\in N, h\in N} y_{A_kB_h}. \tag{23}$$

This would be true, for example, if $z_M \equiv z_{A_kB_h} = z_{A_k}z_{B_h}$and $z_i > 1\ \forall i$.

Note that this does not assume that type 1 firms will match with type 1 firms, only that it would be pareto-improving for them to do so. Shimer and Smith (2000) have shown that in general assortative matching in a search model with our assumptions requires low enough search costs. We will use this idea here to show how the possibility of assortative matching shows up endogenously in M/B ratios before the merger.

These new assumptions change very little about the model. Investment after the merger will now depend on $z_M \equiv z_{A_kB_h}$ so,

$$K_{A_k}^{M*} = \sqrt[\alpha-1]{\frac{r}{z_{A_kB_h}\alpha}}. \tag{24}$$

Therefore, the synergy will now depend on the types of the merging firms

$$s_{A_kB_h} \equiv \frac{z_{A_kB_h}\left(\left(K_{A_k}^{M*}\right)^{\alpha} + \left(K_{B_h}^{M*}\right)^{\alpha}\right)}{r} - \left(K_{A_k}^{M*} + K_{B_h}^{M*} - K_{A_k}^{MP*} - K_{B_h}^{MP*}\right) + O_{A_k} + O_{B_h}. \tag{25}$$

The expression for the profit in the No Mergers state also changes little. The No Merger profit can be expressed just as it is in Equation (9) with the addition of a subscript k or h representing the subtypes. The equilibrium to the bargaining in a Merger can also be expressed similarly to Lemma 1, Equation (17); it is necessary only to change notation to recognize that profits and synergies now depend on the subtype of the merging firms. π_i^M is changed to $\pi_{i_k}^M(j_h)$ to recognize that the profits of a merger depend on the subtype of firm i and the subtype of firm j, where $i \neq j,\ i,j \in \{A, B\}$. Also s is changed to $s_{A_k B_h}$ and π_i^{MP} to $\pi_{i_k}^{MP}$. The difference between this extension and the original problem arises in the profit from the Mergers are Possible state which is also a firm's disagreement utility. Equation (19) the disagreement utility of a type A firm now becomes

$$\pi_{A_k}^{MP} = \Delta q_A(\theta) e^{-\Delta\lambda^{MP}} \left[\sum_{h \in N} \left[\rho \max\left(\pi_{A_k}^M(B_h), \pi_{A_k}^{MP} \right) \right] \right] e^{-r\Delta} + (1 - \Delta q_A(\theta)) e^{-\Delta\lambda^{MP}} \pi_{A_k}^{MP} e^{-r\Delta} \tag{26}$$

$$+ (1 - e^{-\Delta\lambda^{MP}}) \pi_{A_k}^{NM} e^{-r\Delta} + \Delta z_{A_k} \left(K_{A_k}^{NM*} \right)^{\alpha} e^{-r\Delta}.$$

The difference in this and equation (19) is in the first term. The first term is the probability of finding a partner ($\Delta q_A(\theta)$) times the probability that an agreement is still valuable ($e^{-\Delta\lambda^{MP}}$) times the discount rate ($e^{-r\Delta}$). This is then multiplied by the expected payoff from each partner. This is the sum of the profits when each subtype of partner is located times the probability of that subtype, where the max determines if a merger is consummated or the firms continue to search. This summation and max function are the key additions to the model due to the multiplicity of types. The expected profit from searching for a partner now depends on all the different types of partners a firm may find.[14]

The first step in demonstrating the effect of greater matching on M/B ratios is the following lemma, which describes the conditions under which assortative matching will hold.

Lemma 2 *If*

$$2D \left[s_{A_k B_h} - Y_j - Y_i \right] + \rho q_j(\theta) \left[s_{A_k B_h} - s_{A_h B_h} \right] < \rho q_i(\theta) \left[s_{A_k B_k} - s_{A_k B_h} \right], \tag{27}$$

$\forall k \neq h,\ k, h \in N,\ \forall i \neq j,\ i, j \in \{A, B\}$, *then assortative matching will occur.*

Proof. See appendix. ■

If condition (27) holds for a given N, then we say that firms are in the assortative matching regime. If so, firms will merge only if they have the same 'quality', i.e., their d dimensions match and they belong to the same subcategory. This result is similar to Burdett and Coles (1997) who focus on marriage and find that in equilibrium only men and women from the same 'class' marry.[15] If Condition (27) holds for a finer partition N (i.e., for a larger list of characteristics d), then we say that the degree of assortative matching has increased.

Assortative matching in our context requires the supermodularity of synergies to be large

[14] The equation that represents the merger value from a type B firm's point of view is parallel and omitted.
[15] Eeckhout (1999) looks at a very general set up and shows existence and uniqueness.

relative to the opportunity costs of searching. Thus, for example, $s_{A_1B_1} + s_{A_2B_2}$ must be significantly greater than $s_{A_1B_2} + s_{A_2B_1}$. The assumption of supermodularity in the merger production function, assumption (23) made above ensures that $s_{A_1B_2} - s_{A_2B_2} < s_{A_1B_1} - s_{A_2B_1}$. Therefore, if there were no costs to continuing to search and the types were equally scarce then we would be guaranteed assortative matching a la Becker (1973). However, there are two effects which alter the willingness of differing types to consummate deals.

The first effect relates to $2D\left[s_{A_kB_h} - Y_j - Y_i\right]$. The expression in brackets is the gain from a merger between A_k and B_h, which is positive as long as the synergies are positive. Thus, if type k and type h firms do generate synergies it pushes them toward accepting a deal, and more synergies lead to a greater chance of a deal. This effect becomes more important the more costly it is to continue to search (larger $D = r\frac{\lambda^{MP}+\lambda^{NM}+r}{\lambda^{NM}+r}$). With high search costs the current deal looks better than going back to searching. Thus, if λ^{MP} increases, λ^{NM} decrease, or r increases then a deal between different types of firms is more likely to get done.

The second effect relates to the scarcity of the type A and type B firms. The more scarce the type A firm is the greater $q_A(\theta)$ is. The greater $q_A(\theta)$ the more bargaining power the firm has and the more the firm benefits from waiting for another firm of the same subtype because the firm will get all the extra production. And ρ is the probability a firm is of a particular subtype. The greater this probability the more important the relative scarcity effect becomes.

Lemma 2 essentially says that it must be worthwhile for a firm to continue to search for a partner who matches on d dimensions or they will simply accept a merger with the first partner they find who matches on $d-1$ dimensions. Lemma 2 shows that as long as condition (27) holds firms will match. Increasing the number of dimensions over which firms match increases $s_{A_kB_k}$. So, as long as condition (27) holds, firms will increase the number of dimension on which they match. If firms are not matching on a dimension that is worth the search costs, then all firms could be made better off by searching on that dimension. Thus, firms endogenously decide the number of dimension over which they choose to search. If there are more dimensions that may be marginally beneficial but are not worth the cost of searching, then they occur in mergers only by happenstance.

This brings us to the central proposition of the paper.

Proposition 4 *Under assortative matching, firms' M/B ratios will exhibit* rank-ordering*: type A firms with the largest M/B will merge with the type B firms with the largest M/B, while type A firms with the second largest M/B will merge with the type B firms with the second largest M/B, and so forth.*

Proof. See appendix. ■

The best type A firms create both more synergies and have more outside opportunities than any other type A firm. Therefore, the best type A firms have the largest market-to-book ratios of all type A firms. The best type A firms merge with the best type B firms so both have the largest synergy gain and outside opportunities relative to other type A or B firms. Thus, endogenously, firms who choose to merge have M/B ratios that have a similar rank relative to other firms of their own type.

The endogenous outcome arises because firms trade-off their desire to merge with a better partner with the reduced bargaining power they would have in such a merger. In equilibrium mergers that complete will match on multiple characteristics and this will endogenously result in similar M/B ranks. Thus, the idea of complementarities in merger partners impact bargaining power and the choice of partner and through that mechanism endogenously pushes the M/B rank differential of completed deals toward zero.

It is interesting to note that if we had instead used the Q-theory assumption that $z_M \equiv z_{A_k B_h} = \max[z_{A_k}, z_{B_h}]$ we would have found the exact opposite result. Firms would choose to merge with the type most unlike them and the M/B of merging firms would be far apart.

The following corollary gives a second important prediction.

Corollary 1 *If the discount rate is low then assortative matching is more likely to hold and the difference in the rank of market-to-book ratios of merging firms will be smaller.*

Proof. See appendix. ■

This prediction is striking precisely because both intuition and q-theory suggest the opposite. In low discount rate environments the M/B differential across firms is greater (as shown by Jovanovic and Rousseau, 2002) therefore, there is more opportunity for a high M/B firm to buy a low M/B firm. However, in Section V we provide empirical evidence that supports this prediction. Even though the M/B dispersion is lower in high discount rate times, the firms that merge have greater M/B differentials. This is because as search becomes more costly matching breaks down.

Harford (2003) finds that mergers that occur during waves create more value than non-wave mergers. Since Rhodes-Kropf, Robinson and Viswanathan (2002) show that waves occur when valuations are high there is more likely to be matching during waves. Thus mergers during waves should create more value because of the greater matching.

The effect of complementarity leads to one more prediction. As noted above, if firms choose to match on more dimensions, then the subset of firms with whom they can match is smaller. That is, if firms match on many criteria then ρ becomes smaller as it becomes less likely that firms can find a partner with all of the correct dimensions.[16] This affects the relative bargaining

[16]Note that matching on more dimensions does not change the relative scarcity of the firms who merge. To provide intuition, imagine that there were 60 type A firms and 40 type B firms, then matching on a second characteristic would mean that there were 30 type A1 firms and 20 type B1 firms. Thus, a higher degree of matching does not

strengths of the merging firms.

Corollary 2 *Increasing the degree of assortative matching causes firms to split the gains from merger more equally.*

Proof. See appendix ■

Adding another dimension to consider in a merger match has two effects. First, if the new dimension is important for the creation of synergies then the matched mergers will increase in value. That is, $s_{A_k B_k}$ will be greater. Second, every dimension divides the subset of potential partners of each firm in half. This in turn reduces the probability of finding such a partner. Firms are endogenously making a trade-off. The naturally more scarce firm is giving up bargaining power conveyed by his relative scarcity in order to gain from increased profits. Simultaneously the less scarce firm is willing to keep searching precisely because the more scarce firm has reduced their relative bargaining power. If the more scarce firms bargaining power did not endogenously decrease then the less scarce firm would take a merger with another partner! Both firms must simultaneously choose to wait for the other in order to achieve assortative matching. Thus targets and acquirers naturally tend to share the gains from merger as firms choose to match along more dimensions.

The intuition is simple. If a match is rare then when it occurs neither side is willing to walk away. The relative scarcity becomes less important as the absolute scarcity of both firms dominates. Firms become 'equally scarce' because neither firm is likely to find as perfect a match. Thus, the firms have equal bargaining power.

Overall, our theory is a "Birds of a Feather" theory of mergers that builds on the desire of better firms to get together to endogenously deliver the remarkable tightness of M/B ratio ranks that we see in the data. Our theory further predicts how this tightness should vary through different discount rate environments and in mergers of equals vs unequals. We are suggesting that the desire to match on different dimensions is an important driver in mergers and acquisitions. The next section explores the empirical relevance of these predictions.

V Re-examining the Empirical Evidence

In this section we return to the data and explore the empirical predictions of assortative matching for merger activity. The first empirical prediction we explore is that bidders and targets should be drawn from similar points in the distribution of M/B ratios. Put differently, this prediction states that M/B ratios should be equal in a relative sense, rather than an absolute sense. The second prediction is that the equality of market-to-book ratios should be highest in high

change firms *relative* scarcity (60/40 =30/20).

valuation regimes, since in these periods the returns to searching for better matches increase.

A. Exploring the Relative Ranking Prediction

To explore the first prediction, we plot the bivariate distribution of M/B rankings for bidders and targets. This allows us to explore the empirical validity of Theorem 4, which predicts that the best firm from industry A should merge with the best firm from industry B even if the M/B across the two industries is very different. Since bidders and targets often come from different industries the theory tells us to rescale the market/book ratio before ranking them so that outliers from disparate industries are not mistakenly ranked close together (for example, a low valuation acquirer in a high-average-value industry could purchase a high valuation target in a low-average-valuation industry). To do this, we express the M/B ratio as a deviation from each firm's industry median.

This bivariate distribution is presented in Figure 2. The two horizontal axes in Figure 2 are the acquirer (left) and target (right) market-to-book ratios, grouped into deciles. The vertical axis counts the number of transactions that took place between bidders and targets with that market-to-book pairing. Thus, 'high buys low' transactions cluster at the left side of the graph, and 'low buys high' transactions cluster at the right side of the graph.

Under the null hypothesis that there is no relation between bidder and target market-to-book ratio, we should expect to see a more or less uniform grid of values. On the other hand, an extreme form of the high buys low prediction would yield a graph that was peaked on the left edge, where bidder valuation is highest and target valuation is lowest.

According to our theory, the bulk of merger activity should lie along the main diagonal. And indeed it does. The bulk of activity involves either 'high buying high' or 'low buying low': this can be seen by noting that the graph is shaped like a saddle, with peaks at the 'high, high' decile and the 'low,low' decile. Relatively little activity occurs in the 'high buys low' or 'low buys high' tails of the distribution. Instead, the fact that the bulk of merger activity lies on the main diagonal indicates that 'like buys like' when mergers occur.

Thus, the 'like buys like' effect presented in Table 1, which motivated our investigation, is actually an artifact of a much stronger phenomenon: firms are merging with other firms with a very similar relative rank. The fact that the absolute difference in M/B rank between targets and acquirer (see table 1) is small occurs because the M/B distribution across the industries of merging firms is similar. This tells us that even when one industry is redeploying the assets of another firm in a Q theory sense (so absolute M/B differential is large), the best firm in industry A does not buy the worst firm in industry B, but rather the best firm A buys the best B and the worst firm A buys the worst B (so relative ranks are similar).

B. Exploring the Discount Rate Prediction

The second prediction is that the tendency for bidder and target M/B ratios to converge should be the strongest in high valuation periods, i.e., when discount rates are low. To explore this prediction, we break the data into high valuation and low valuation regimes and examine bidder/target M/B spreads in these different regimes. This is presented in Table 4.

Panel A of Table 4 groups the data according to whether the merger occurred in a period of high or low valuation for the target firm. We determine 'high' or 'low' valuation by whether the industry median M/B ratio was above or below its long-run average value. In column (1), it reports the overall M/B dispersion, which is slightly higher in high valuation periods (.89 vs. .82). Yet in these high valuation periods, when dispersions are higher, the spread in bidder/target M/B ratio is 1/10th of what it is in low valuation periods. This evidence supports the idea that assortative matching is higher in high valuation periods than in low periods.

Next we break the sample into high/low valuation periods for both bidder and target. The central test comes from comparing market/book differentials in (low, low) valuation periods with those in (high, high) periods. In (high, high) periods, the difference in bidder and target M/B ratios is statistically insignificant from zero. On the other hand, in the (low, low) period, bidders have statistically lower market/book ratios than targets.

Thus, in the high valuation, high M/B dispersion times the M/B differential between merging firms is smaller, as predicted by the model.

VI Extensions and Discussion

In the last section we would like to consider three important issues. The first subsection considers the question of who should buy whom. The second subsection considers the time series implication of our model. And the last subsection considers the expected market reaction to merger announcements predicted by our model.

A. Who buys whom?

The discussion above talks about how the merger synergies will be split between the two firms but does not imply that one firm should buy the other. In fact, throughout the model we have required the firms to merge in order to achieve the synergy but it has not mattered who bought whom. However, it is more likely that since different firms are affected by any shock in different ways one firm's management may be better able to manage the joint firm. For simplicity lets assume that firm A is better able to manage the joint firm. Casually, this suggests that firm A should be the acquirer, the firm who is better able to manage the assets in the long run.

However, in theory it should be possible for firm B to buy firm A and then install firm A's management. In fact this possibility would turn standard Q-theory of mergers on its head, as low Q firms could buy high Q firms and enlist the target's management to run the firm. However, if this is even slightly more costly than the reverse, then it will not occur. For example, if there exist private benefits of control with non monetary payoffs then once B is in charge he may not turn over the firm to the management of A even if this is optimal. This is non-contractible pre-merger because firms cannot contract with themselves as the law only enforces contracts between parties with separate legal standing. Therefore, any ex ante contract becomes unenforceable once the firms merge. It is reasonable to assume that costs and incomplete contracts ensure that the acquirer is the firm that will manage the assets in the long run. Thus, the acquirer is the firm whose managers are best able to manage the joint firm.[17]

Presumably, the ability to manage the joint firm is scarce. In fact this is essentially the point in the Q theory of mergers. In that theory one firm has good ideas/management (presumably a scarce resource) and the other firm only has assets (presumably not very scarce). The following corollary to proposition 4 shows that in our theory this would cause the acquirer to have a larger M/B than the target on average, but this would not mean that the high M/B firm should always buy the low M/B firm.

Corollary 3 *If increased management talent increases the scarcity of the firm because management talent is rare, then on average acquirers will have larger M/B ratio than targets.*

Proof. If increased management talent increases θ then proposition 3 shows that everything else equal firms with management talent with have larger M/B. If the firm with the greater management talent must be the acquirer then the acquirer with have the greater M/B. ∎

Note that if for some strange reason management talent and ideas became plentiful and the assets of firm B became scarce, then firm A would still buy firm B but firm B would have the larger M/B. This seems unlikely in reality. Thus, it is not a firm's M/B that causes them to be the acquirer, but rather that fact that they need to be the acquirer tends to make their assets (i.e., management ability) more scarce and leads on average to slightly higher M/B for acquirers than for targets. Our point is that we must also account of the scarcity of the target's assets. If they are relatively more scarce than management ability then the target will have the larger M/B.

The idea that the management of firm A can better utilize the assets of firm B, as the Q theory argues, is likely a part of any merger. However, we argue that there are many other aspects of the assets of the firms that adjust their relative scarcity causing the a split of the surplus of the merger and ensuring that the M/B of the target is sometime higher than, and

[17] Technically we can assume that z_M is only large enough to create synergies if firm A manages the merged firm.

often close to, the acquirer's.[18] This is precisely what we see in the data: the difference between the M/B of the acquirer and target is small relative to the range of M/B ratios at any point in time. Furthermore, we find that in 40% of mergers the low M/B firm buys the high M/B firm, and also, it is not the case that high M/B buys low M/B but rather high M/B buys a little less high M/B and low M/B buys a little lower M/B. Furthermore, Maksimovic and Phillips (2001) empirically show that in a substantial fraction of mergers the target firms' plants are more efficient then those of the acquirer. They show that in those cases the productivity of the acuirer's plants subsequently increases. Also, the findings of Rhodes-Kropf, Robinson and Viswanathan (2002) suggest that firms with low long run growth prospects buy firms with higher than average long run growth prospects. These findings make much more sense in a world where firms are splitting synergies and assortatively matching. For example, it is perfectly logical that managers in shrinking industries take their management talent, corporate culture, etc and merge with firms that need to grow rapidly. Even when one industry is going to buy the assets from another the best acquirer will buy the best assets. Thus each firm will have the highest M/B within its own industry.

In general, firms with better talent will help a better firm grow more. Either firm may have the larger M/B ex ante depending on who needs who more. However, on average it is true that high M/B firms buy slightly lower M/B firms. Why is the M/B of acquires slightly larger than targets? Because the Q theory of mergers is not wrong, it is just part of a larger story. Scarce management talent does pushes up the M/B of the acquirer ensuring that the acquirer has a slightly higher M/B on average. However, since management talent is only one of many potentially scarce resources in the merger the difference in M/Bs of the target and acquirer is not large on average and often the target has the higher M/B.

B. Time series of Market-to-Book

We can now use our understanding of the M/B ratios to examine how they should change across time. The central idea of this section is that M/B ratios should rise and then fall around mergers. The rise leading up to the merger comes from the markets increasing expectation that the merger will occur. Then after the merger, if the merger occurred, the M/B will fall because the firm finally implements the expected investment. Or, if the merger does not occur and the Mergers are Possible state ends then the M/B ratio reduces to the level when No Mergers are viable. This prediction, particularly the drop in M/B and increase in investment after the merger, is novel to our model and if empirically true will provide greater confidence in our

[18] A minor extension of this idea is that the ability to manage a large firm is a resource usually held by the larger firm and usually needed after a merger. If the ability to manage a larger firm is an important part of a merger then we would tend to find large firms buying smaller firms.

theory. However, the difficulties in an empirical implementation prevent us from testing this here.

The next proposition demonstrates the path of the M/B around mergers.

Proposition 5 $\pi_i^{NM}/K_i^{NM*} < \pi_i^{MP}/K_i^{MP*} < \pi_i^{M}/K_i^{MP*}$ *and* $\left(\pi_i^M + \pi_j^M\right) / \left(K_i^{MP*} + K_j^{MP*}\right) > z_M(\left(K_i^{M*}\right)^{\alpha} + \left(K_j^{M*}\right)^{\alpha})/\left(r\left(K_i^{M*} + K_j^{M*}\right)\right)$, *as long as the synergies are positive.*

Proof. See Appendix ■

This proposition suggests three types of test. The first is an event study around mergers. We should find an increase in M/B before the merger and a decrease after. Of course, one would have to be careful in how they measured book value after the mergers as they would not want the merger accounting to confound the results. The second test requires a corollary to Proposition 5.

Corollary 4 $K_i^{M*} > K_i^{MP*}$ *and* $\left(z_M(\left(K_i^{M*}\right)^{\alpha} + \left(K_j^{M*}\right)^{\alpha}\right)/r > \pi_i^M + \pi_j^M$

Proof. The first inequality is true because $z_M > z_i \forall i$ and the second is true as long as $\left(z_M(\left(K_i^{M*}\right)^{\alpha} + \left(K_j^{M*}\right)^{\alpha}\right)/r > s$ which is shown in the proof of proposition 5 to be true. ■

This corollary tells us that the decrease in the M/B after the merger of two firms is due to an anticipated increase in investment rather than a decrease in market value. Thus, the second test is to see if investment increases after a merger. The standard Q theory would suggest that the merger itself was an investment. Thus, either the need to invest is the same as before the merger or, more likely, the need to invest has been partially satiated by the merger. Thus, investment should stay the same or decrease. Great care should be employed in looking for this prediction as investment is intertwined with financial constraints and other factors that may be correlated with merger activity. That said, it is interesting to note that Jovanovic and Rousseau (2002) find that firms that acquired asset through acquisitions also made a lot of investments.

The third test combines Corollary 4 with Proposition 5 with the idea that the M/B of all firms should rise as the probability of future mergers increases. Together these suggest an event study around a deregulating industry. This is essentially an increase in the probability that the state jumps to Mergers are Possible. Then a further increase for those firms who announce mergers. Then after 'enough' mergers have occurred and the Mergers are Possible state ends, a decrease in the M/B of those firms that did not merge due to a decrease in their market price, and a decrease in the M/B of firms that merged due to and increase in investment.

Thus our scarcity and assortative theory of mergers predicts novel patterns in the time series of M/Bs around mergers.

C. Market Reaction

Market reactions are about the difference between the expected value of a potential merger and the realized expected value of an actual merger. Much of the value of the merger may already be in the M/B ratio before the merger is announced. An ex ante M/B is about the ability to create synergies, the probability of creating them, and the ability to negotiate for the surplus. Thus, market reaction is not about the true gain from the merger but only about the difference between the realized merger and the expected outcome. Thus, in order to look at market reactions we need to endogenize the expectations of the market. Our model allows us to do this.

There are two important empirical papers that examine market reactions in the context of the Q theory of mergers, Servaes (1991) and Lang, Stultz and Walkling (1989). These papers argue that mergers where a high M/B acquirer buys a low M/B target should have the greatest market reaction because these are the 'good' mergers. They find some support for this idea but they also find many results that seem inconsistent with this idea. For example, Servaes finds that the acquirer's Q has no affect on the target's reaction. Lang et al find that the Q estimates from the year before the merger do not explain the bidders gain, but when low Q buys low Q then the target gains are higher. Furthermore, many of the bidder and target reactions are shown to be unrelated to differences in M/B. We would argue that this is because there are many different types of mergers and no reason that the best mergers are the ones where the target is relatively less scarce. Of course, some mergers are redeploying assets to a better use and therefore some mergers where high buys low will have high synergies. Thus, we would not be surprised to find that some high buys low mergers have positive announcement effects. However, we will show below that in general the correlation between M/B difference and announcement effect should be weak.

The market reaction is simply the difference in the price before and after the merger divided by the price before the merger, $\left(\pi_i^M - \pi_i^{MP}\right)/\pi_i^{MP}$. The following proposition demonstrates how the reaction is affected by the synergies, the probability of creating them, and the ability to negotiate for the surplus.

Proposition 6 *The market reaction to the announcement of a merger is greater if mergers are less likely (Mergers are Possible state has higher persistence or No Mergers state has lower persistence). Increasing the synergy increases the market reaction. The relative scarcity of the two firms has an ambiguous affect of the market reaction.*

Proof. See Appendix. ■

The first two effects are intuitive. The ambiguity relating to the scarcity arises from the offsetting effects. If a firm is relatively more scarce they will capture more of the surplus from the merger, but the market expects the firm to be more likely to find a partner. Thus, the reaction of the market may be larger or smaller depending on which affect dominates. If a firm is relatively less scarce the market expects they are unlikely to find a partner so the news of a merger would be surprising. However, a relatively less scarce firm will have weak negotiating powers and therefore also not extract much of the benefit.

However, even our model is not general enough to truly explain what part of a merger is not expected by the market. For example bidders may differ from targets in that they announce that they are looking for a target, there in changing the market expectations. Furthermore, in our model average reactions will be positive. We believe that the negative reaction that has been documented for acquirer stock prices arises from the possibility that the acquirer is overvalued. This idea is not a part of this model but is developed fully in Rhodes-Kropf and Viswanathan (2004) and Rhodes-Kropf, Robinson and Viswanathan (2002). Jovanovic and Braguinsky (2004) and McCardle and Viswanathan (1994) argue that the market reaction is due to learning about the investment opportunities of the target and acquirer. Market reaction is really about the benefits of this merger relative to the other possible potential options. What is the probability of a merger? What is the value from a merger? What do we expect each firm to extract from the merger? Overall market reactions are an interesting but difficult place to look to understand why mergers are occurring or the value they are creating.

VII Conclusion

One of the key results of the property rights theory of the firm is that complementary assets should be under common control. This means that in the market for corporate control we should observe firms with complementary assets or technologies joining together, redrawing the boundaries of the firm in such a way that complementary assets are placed under the command of a single firm.

This paper develops a search model with matching and asset complementarity that ties the property rights theory of the firm to new facts about who buys whom in merger transactions. While conventional wisdom suggests that high asset value firms buy low asset value firms, we show that a more appropriate interpretation is that firms with similar asset valuations purchase one another. That is, mergers exhibit assortative matching: instead of 'high buys low,' we see that 'like buys like.' We argue that this assortative matching is a direct result of asset complementarity and costly search.

In our theory the decision to merge balances the expected benefits of pairing with the current potential partner against the expected benefits of waiting and finding a more suitable partner. In the model, the identity of the bidder or target is determined by the fact that incomplete contracts require one party to oversee the joint assets of the newly merged firm. This party is one whose managerial talent is best suited to the merged resources of the new entity. On the other hand, the market-to-book ratios of the bidder and target are determined by the relative bargaining power of each party during the merger negotiation. A firm with relatively more scarce assets will, in general, command a larger fraction of the surplus from merger, since it will be able to effectively threaten to break off merger negotiations and find an outside offer of equal or greater value. Markets impound the expected value of this added surplus into market prices ex ante, which means that firms with relatively more scarce assets will have a higher market-to-book ratio, even if their investment opportunities are no different than their counterparty in the merger transaction.

When we add asset complementarity to the model, we find that firms with similar market-to-book ratios end up merging with one another. The assumption that merging firms have complementary assets arises from Hart and Moore (1990), Hart (1995) and the incomplete contracting literature. In our model, each firm trades off its desire to merge with a better partner with the endogenously reduced bargaining power they will have in such a merger. In equilibrium, if complementarity is important, successful mergers will exhibit a high degree of matching. The best targets and the best acquirers have the best outside opportunities and create the most synergies. They endogenously choose to search for each other. Therefore, in equilibrium firms who have the highest M/B ratios in their respective industries will choose to merge. Our point is that relative scarcity determines who gets more surplus from the merger while complementarities cause the best to merge with the best.

The model predicts that the difference in M/B ratios should increase when discount rates are high and valuation levels are low. It also predicts that the relative rank of bidder and target M/B ratios is driven towards equality as a result of matching. When we revisit the data, we find that both these predictions hold.

There are a number of fruitful avenues for future work. Our model allows us to bridge the gap between the property rights theory of the firm and the empirical evidence on merger and acquisition activity. Our findings indicate that mergers reflect conscious efforts to redraw the boundaries of the firm in a manner that best allows complementary assets to be placed under common control. Developing new empirical tests of these predictions may shed better light on the motives for merger activity by more fully articulating the ways in which complementarities arise. For example, future work could explore the pattern of assortative matching over the business cycle or through merger waves.

Our work also has implications for how firms are sold. Boone and Mulherin (2004) document the fact that some firms solicit many merger offers while others negotiate with a single partner. This reflects some equilibrium in the search market in which perceived scarcity of assets and the potential desire to find optimal asset complementarities drive firms to solicit an optimal number of merger offers. Our model offers one way to think about the process by which firms go about searching for and identifying partners.

Furthermore, our empirical findings suggest that merger activity may be a rich area for examining the general theories and predictions from assortative matching models. The advantage of merger markets over more traditional applications of search theory, such as marriage markets, is that in merger activity the market values of the firms in question allow us to identify more cleanly matching parameters that may be difficult to observe in other settings.

Our analysis also complements theories of misvaluation, such as Rhodes-Kropf and Viswanathan (2004). In Rhodes-Kropf and Viswanathan, mergers occur for un-modelled fundamental reasons that are confounded or magnified by the possibility of misvaluation. Instead of focusing on why firms merge, their work focusses on when they occur and what transaction medium they use. This paper instead focus on the fundamental reason for mergers. Combining these ideas could better explain why mergers cluster in time at the industry level (Mitchell and Mulherin, 1996; Harford, 2003; Rhodes-Kropf, Robinson, Viswanathan, 2002).

In addition, our main empirical finding—that like buys like—casts a new light on many questions in corporate finance. Do diversifying mergers exhibit stronger or weaker assortative matching? Do the long-run valuation consequences of mergers vary according to the degree of assortative matching that occurs? Are like-buys-like mergers more likely to shed unrelated assets around the time of the merger? Are like-buys-like transactions more likely to be initiated by better governed firms? Can these findings explain why some conglomerate firms trade at a discount relative to a portfolio of stand-alone firms? We leave each of these tasks for future research.

References

[1] Andrade, G., M. Mitchell, and E. Stafford, 2002, New Evidence and Perspectives on Mergers, *Journal of Economic Perspectives*, 15, 103-120.

[2] Becker, G., 1973, A Theory of Marriage: Part I, *The Journal of Political Economy*, 81, 813-846.

[3] Becker, G., Treatise on the Family (Cambridge: Harvard University Press, 1981)

[4] Boone, and Mulherin, 2004, How Are Firms Sold?, unpublished Working Paper, Claremont-McKenna College and The College of William & Mary.

[5] Burdett, K., and M. G. Coles, 1997, Marriage and Class, *The Quarterly Journal of Economics*, 112, 141-168.

[6] Coase, R., 1937, The Nature of the Firm, *Economica*, n.s., 4, 386-405. Reprinted in Readings in Price Theory, ed. by g. Stigler and K. Boulding, Homewook, Ill.: Irwin, 1952.

[7] Diamond, P. A., 1993, Search, Sticky Prices, and Inflation, *The Review of Economic Studies*, 60, 53-68.

[8] Eeckhout, J., 1999, Bilateral Search and Vertical Heterogeneity, *International Economic Review*, 40, 869-887.

[9] Gort, M., 1969, An Economic Disturbance Theory of Mergers, *Quarterly Journal of Economics*, 83, 624-642.

[10] Gorton, G, M. Kahl, and R. Rosen, 2000, Eat Or Be Eaten: A Theory Of Mergers and Merger Waves, Unpublished working paper, Wharton.

[11] Grossman, Sanford J., and O. Hart, 1986, The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration, *The Journal of Political Economy*, 94, 691-719.

[12] Harford, Jarrad, 2003, Efficient and Distortional Components to Industry Merger Waves, Forthcoming *Journal of Financial Economics*.

[13] Hart, Oliver, 1995, Firms Contracts and Financial Structure (Oxford University Press, Oxford, UK).

[14] Hart, O., and J. Moore, 1990, Property Rights and the Nature of the Firm, *The Journal of Political Economy*, 98, 1119-1158.

[15] Holmström, B. and S. N. Kaplan, 2001, Corporate Governance and Merger Activity in the United States: Making Sense of the 1980s and 1990s, *Journal of Economic Perspectives* 15, 121-144.

[16] Inderst, R. and H. M. Müller, 2002, The Effect of Capital Market Characteristics on the Value of Start-up Firms, Forthcoming *Journal of Financial Economics*

[17] Jensen, M., 1986, Agency Cost of Free Cash Flow, Corporate Finance and Takeovers, *American Economic Review* 76, 323-329

[18] Jovanovic, B., and S. Braguinsky, 2004, Bidder Discounts and Target Premia in Takeovers, *The American Economic Review*, 94, 46-56.

[19] Jovanovic, B., and P. Rousseau, 2002, The Q-Theory of Mergers, *The American Economic Review, (Papers and Proceedings)*, 92(2), 198-204

[20] Kremer, Michael, 1993, The O-Ring Theory of Economic Development, *The Quarterly Journal of Economics* 103, 551-575

[21] Lang, L., R. Stulz, and R. Walkling, 1989, Managerial performance, tobin's Q, and the gains from successful tender offers. *Journal of Financial Economics*, 24, 137-154.

[22] Loughran, T. Vijh, A. 1997, Do long-term shareholders benefit from corporate acquisitions? *Journal of Finance*, 52, 1765-1790.

[23] Maksimovic, Vojislav, and Gordon Phillips, 2001, The Market for Corporate Assets: Who Engages in Mergers and Asset Sales and Are There Efficiency Gains?, *Journal of Finance*, 56, 2019–2065.

[24] Manne, H. 1965, Mergers and the Market for Corporate Control, *Journal of Political Economy*, 73, 110–120

[25] Martin, K., 1996, The method of payment in corporate acquisitions, investment opportunities, and managerial ownership, *Journal of Finance* 51, 1227-1246.

[26] McCardle, K.F. and S. Viswanathan, 1994, The Direct Entry Versus Takeover Decision and Stock Price Performance Around Takeovers, *Journal of Business*, 67, 1-43.

[27] Mitchell, M.L., and J.H. Mulherin, 1996, The impact of industry shocks on takeover and restructuring activity, *Journal of Financial Economics* 41, 193-229.

[28] Mortensen, D. T. and C. A. Pissarides, 1994, Job Creation and Job Destruction in the Theory of Unemployment, *The Review of Economic Studies*, 61, 397-415.

[29] Petrongolo, B. and C.A. Pissarides, 2001, Looking into the black box: a survey of the matching function. *Journal of Economic Literature* 38, 390-431.

[30] Pissarides, C.A., 1990, Equilibrium unemployment theory. Basil Blackwell, Cambridge, MA.

[31] Rau, P., Vermaelen, T. 1998, Glamour, value and the post-acquisition performance of acquiring firms. *Journal of Financial Economics* 49, 223-253.

[32] Rhodes-Kropf, M., D. Robinson and S. Viswanathan., 2004, Valuation Waves and Merger Activity: the Empirical Evidence, Forthcoming *Journal of Financial Economics*.

[33] Rhodes-Kropf, M. and S. Viswanathan., 2004, Market Valuation and Merger Waves, *Journal of Finance* 59.

[34] Roll, R., 1986, The hubris hypothesis of corporate takeovers, *Journal of Business*, 59, 197-216.

[35] Servaes, H., 1991, Tobin's q and the gains from takeovers. *Journal of Finance* 46, 409-419.

[36] Shimer, Robert and Lones Smith, 2000, Assortative Matching and Search, *Econometrica* 68, 343-369.

[37] Shleifer, A. and R. Vishny, 2003, Stock Market Driven Acquisitions, Forthcoming, *Journal of Financial Economics*

[38] Tirole, J., 1999, Incomplete Contracts: Where Do We Stand?, *Econometrica*, 67, 741-781.

[39] Williamson, O. E., 1985, The Economic Institutions of Capitalism: Firms, Markets, Relational Contracting. New York: Free Press.

Proof of Proposition 1: We begin with Equations (9) and (19). These can be rearranged and written as

$$\pi_i^{MP} = c_{1i}(\Delta)\pi_i^{M} + c_{2i}(\Delta)\pi_i^{NM} + c_{3i}(\Delta), \tag{A1}$$

where

$$\begin{aligned} c_{1i}(\Delta) &= \frac{\Delta q_i(\theta)\exp(-\Delta\lambda^{MP})\exp(-r\Delta)}{1-(1-\Delta q_i(\theta))\exp(-\Delta\lambda^{MP})\exp(-r\Delta)}, \\ c_{2i}(\Delta) &= \frac{(1-\exp(-\Delta\lambda^{MP}))\exp(-r\Delta)}{1-(1-\Delta q_i(\theta))\exp(-\Delta\lambda^{MP})\exp(-r\Delta)}, \\ c_{3i}(\Delta) &= \frac{\Delta z_i\left(K_i^{NM*}\right)^{\alpha}\exp(-r\Delta)}{1-(1-\Delta q_i(\theta))\exp(-\Delta\lambda^{MP})\exp(-r\Delta)}. \end{aligned} \tag{A2}$$

And

$$\pi_i^{NM} = c_{4i}(\Delta)\pi_i^{MP} + c_{5i}(\Delta), \tag{A3}$$

where

$$\begin{aligned} c_{4i}(\Delta) &= \frac{(1-\exp(-\Delta\lambda^{NM}))\exp(-r\Delta)}{1-\exp(-\Delta\lambda^{NM})\exp(-r\Delta)}, \\ c_{5i}(\Delta) &= \frac{\Delta z_i\left(K_i^{NM*}\right)^{\alpha}\exp(-r\Delta)}{1-\exp(-\Delta\lambda^{NM})\exp(-r\Delta)}. \end{aligned} \tag{A4}$$

Solving for π_i^{MP} we find

$$\pi_i^{MP} = \frac{c_{1i}(\Delta)\pi_i^{M} + c_{2i}(\Delta)c_{5i}(\Delta) + c_{3i}(\Delta)}{1-c_{2i}(\Delta)c_{4i}(\Delta)}. \tag{A5}$$

Using Lemma 1 we know

$$\pi_i^{M} = \frac{1}{2}(s-\pi_j^{MP}+\pi_i^{MP}). \tag{A6}$$

Solving we find

$$\pi_i^{MP} = \frac{c_{1i}(\Delta)\frac{1}{2}s - c_{1i}(\Delta)\frac{1}{2}\pi_j^{MP} + c_{2i}(\Delta)c_{5i}(\Delta) + c_{3i}(\Delta)}{1-c_{2i}(\Delta)c_{4i}(\Delta)-\frac{1}{2}c_{1i}(\Delta)}. \tag{A7}$$

Since this equation is true for both firm A and B we have two equations and two unknowns (throughout this paper $i \neq j,\ i,j \in \{A,B\}$). Solving we find

$$\pi_i^{MP} = \frac{c_{1i}(\Delta)\frac{1}{2}s - \frac{c_{1i}(\Delta)\frac{1}{2}c_{1j}(\Delta)\frac{1}{2}s + c_{1i}(\Delta)\frac{1}{2}c_{2j}(\Delta)c_{5j}(\Delta) + c_{1i}(\Delta)\frac{1}{2}c_{3j}(\Delta)}{1-c_{2j}(\Delta)c_{4j}(\Delta)-\frac{1}{2}c_{1j}(\Delta)} + c_{2i}(\Delta)c_{5i}(\Delta) + c_{3i}(\Delta)}{1-c_{2i}(\Delta)c_{4i}(\Delta)-\frac{1}{2}c_{1i}(\Delta) - \frac{c_{1i}(\Delta)\frac{1}{2}c_{1j}(\Delta)\frac{1}{2}}{1-c_{2j}(\Delta)c_{4j}(\Delta)-\frac{1}{2}c_{1j}(\Delta)}}. \tag{A8}$$

Taking the limit as $\Delta \to 0$ and using L'Hôpital's rule we find

$$\lim_{\Delta\to 0} c_{1i}(\Delta) = \lim_{\Delta\to 0} \frac{\Delta q_i(\theta)\exp(-\Delta\lambda^{MP})\exp(-r\Delta)}{1-(1-\Delta q_i(\theta))\exp(-\Delta\lambda^{MP})\exp(-r\Delta)} = \frac{q_i(\theta)}{\lambda^{MP}+r+q_i(\theta)} = c_{1i}(0),$$
$$\lim_{\Delta\to 0} c_{2i}(\Delta) = \lim_{\Delta\to 0} \frac{(1-\exp(-\Delta\lambda^{MP}))\exp(-r\Delta)}{1-(1-\Delta q_i(\theta))\exp(-\Delta\lambda^{MP})\exp(-r\Delta)} = \frac{\lambda^{MP}}{\lambda^{MP}+r+q_i(\theta)} = c_{2i}(0),$$
$$\lim_{\Delta\to 0} c_{3i}(\Delta) = \lim_{\Delta\to 0} \frac{\Delta z_i\left(K_i^{NM*}\right)^{\alpha}\exp(-r\Delta)}{1-(1-\Delta q_i(\theta))\exp(-\Delta\lambda^{MP})\exp(-r\Delta)} = \frac{z_i\left(K_i^{NM*}\right)^{\alpha}}{\lambda^{MP}+r+q_i(\theta)} = c_{3i}(0),$$
$$\lim_{\Delta\to 0} c_{4i}(\Delta) = \lim_{\Delta\to 0} \frac{(1-\exp(-\Delta\lambda^{NM}))\exp(-r\Delta)}{1-\exp(-\Delta\lambda^{NM})\exp(-r\Delta)} = \frac{\lambda^{NM}}{\lambda^{NM}+r} = c_{4i}(0),$$
$$\lim_{\Delta\to 0} c_{5i}(\Delta) = \lim_{\Delta\to 0} \frac{\Delta z_i\left(K_i^{NM*}\right)^{\alpha}\exp(-r\Delta)}{1-\exp(-\Delta\lambda^{NM})\exp(-r\Delta)} = \frac{z_i\left(K_i^{NM*}\right)^{\alpha}}{\lambda^{NM}+r} = c_{5i}(0).$$

Therefore, using the fact that if $x_n \to x$ and $y_n \to y$ then $x_n y_n \to xy$ we find that

$$\pi_i^{MP} = \frac{q_i(\theta)\frac{1}{2}s + \left(\frac{\lambda^{MP}}{\lambda^{NM}+r}+1\right) z_i\left(K_i^{NM*}\right)^{\alpha} - q_i(\theta)\frac{1}{2}\frac{q_j(\theta)\frac{1}{2}s+(\frac{\lambda^{MP}}{\lambda^{NM}+r}+1)z_j\left(K_j^{NM*}\right)^{\alpha}}{\lambda^{MP}+r+\frac{1}{2}q_j(\theta)-\lambda^{MP}\frac{\lambda^{NM}}{\lambda^{NM}+r}}}{\lambda^{MP}+r+\frac{1}{2}q_i(\theta)-\lambda^{MP}\frac{\lambda^{NM}}{\lambda^{NM}+r} - q_i(\theta)\frac{1}{2}\frac{q_j(\theta)\frac{1}{2}}{\lambda^{MP}+r+\frac{1}{2}q_j(\theta)-\lambda^{MP}\frac{\lambda^{NM}}{\lambda^{NM}+r}}}. \tag{A9}$$

Let $D = \lambda^{MP} - \lambda^{MP}\frac{\lambda^{NM}}{\lambda^{NM}+r} + r = r\frac{\lambda^{MP}+\lambda^{NM}+r}{\lambda^{NM}+r}$ and remember that output $y_i = z_i\left(K_i^{NM*}\right)^{\alpha}$. Therefore,

$$\pi_i^{MP} = \frac{\left[q_i(\theta)\frac{1}{2}s + DY_i\right] - q_i(\theta)\frac{1}{2}\frac{q_j(\theta)\frac{1}{2}s+DY_j}{D+\frac{1}{2}q_j(\theta)}}{D+\frac{1}{2}q_i(\theta) - q_i(\theta)\frac{1}{2}\frac{q_j(\theta)\frac{1}{2}}{D+\frac{1}{2}q_j(\theta)}}, \tag{A10}$$

which reduces to

$$\pi_i^{MP} = \frac{(2D+q_j(\theta))\,Y_i + q_i(\theta)\,(s-Y_j)}{2D+q_i(\theta)+q_j(\theta)}. \tag{A11}$$

Using lemma 1 we can substitute into the negotiation equilibrium, Equation (19), and solve for the merger profit

$$\pi_i^{M} = \frac{(D+q_i(\theta))\,(s-Y_j) + (D+q_j(\theta))\,Y_i}{2D+q_i(\theta)+q_j(\theta)}. \tag{A12}$$

Finally we can substitute into Equation (9)

$$\pi_i^{NM} = \frac{\lambda^{NM}}{\lambda^{NM}+r}\frac{(2D+q_j(\theta))\,Y_i + q_i(\theta)\,(s-Y_j)}{2D+q_i(\theta)+q_j(\theta)} + \frac{y_i}{\lambda^{NM}+r}. \tag{A13}$$

We must also check that $\pi_i^{MP} < \pi_i^{M}$ because we assumed in the writing of Equation (19) that if a suitable partner is found a merger would occur. This reduces to checking that

$$0 < s - Y_j - Y_i. \tag{A14}$$

Which is true as long as the merger is value enhance which we assumed to start the problem.

We must also check that firms have no incentive to commit to a non-optimal level of capital as we assumed to solve the problem. Specifically $\frac{d}{dK_i}(\pi_i^M - K_i) \leq 0$. The derivative is

$$\frac{D+q_i(\theta)+(D+q_j(\theta))\frac{\alpha z_i (K_i)^{\alpha-1}}{r}}{2D+q_i(\theta)+q_j(\theta)} - 1 \leq 0. \tag{A15}$$

This reduces to

$$K_i \leq \sqrt[\alpha-1]{\frac{r}{\alpha z_i}}. \tag{A16}$$

We know that $K_i^{NM} = \sqrt[\alpha-1]{\frac{r}{\alpha z_i}}$ therefore there is no benefit to firm i to changing the level of investment just before a merger in an attempt to get more of the merger surplus.

Proof of Proposition 2: The more scarce that firm A is, the smaller θ is. The more scarce that firm B is, the larger θ is. The derivative of each firm's share with respect to θ is

$$\frac{\partial \pi_A^M}{\partial \theta} = \frac{D\left(s - Y_B - Y_A\right)\left(q_A'(\theta) - q_B'(\theta)\right)}{\left(2D + q_A(\theta) + q_B(\theta)\right)^2}, \qquad \text{(A17)}$$
$$\frac{\partial \pi_B^M}{\partial \theta} = \frac{D\left(s - Y_A - Y_B\right)\left(q_B'(\theta) - q_A'(\theta)\right)}{\left(2D + q_A(\theta) + q_B(\theta)\right)^2}.$$

We know $q_A'(\theta) < 0$ and $q_B'(\theta) > 0$, therefore, $\frac{\partial \pi_A^M}{\partial \theta} < 0$ and $\frac{\partial \pi_B^M}{\partial \theta} > 0$.

Proof of Proposition 3: Remember that firm A's scarcity is increases if θ decreases and firm B's scarcity increases if θ increases. The first part of the proposition is true because $q_A'(\theta) \leq 0$ and $q_B'(\theta) \geq 0$. The second part is shown in Proposition 2. And the last part is true if $\frac{\partial \pi_A^{NM}}{\partial \theta} < 0, \frac{\partial \pi_A^{MP}}{\partial \theta} < 0, \frac{\partial \pi_A^M}{\partial \theta} < 0, \frac{\partial \pi_B^{NM}}{\partial \theta} > 0, \frac{\partial \pi_B^{MP}}{\partial \theta} > 0 \frac{\partial \pi_A^M}{\partial \theta} > 0$, since capital in every state is unaffected by θ.

$$\frac{\partial \pi_A^{NM}}{\partial \theta} = \frac{\lambda^{NM}}{\lambda^{NM} + r} \frac{\left[2Dq_A'(\theta) + q_B(\theta)q_A'(\theta) - q_A(\theta)q_B'(\theta)\right]\left(s - Y_B - Y_A\right)}{\left(2D + q_A(\theta) + q_B(\theta)\right)^2}, \qquad \text{(A18)}$$

$$\frac{\partial \pi_A^{MP}}{\partial \theta} = \frac{\left[2Dq_A'(\theta) + q_B(\theta)q_A'(\theta) - q_A(\theta)q_B'(\theta)\right]\left(s - Y_B - Y_A\right)}{\left(2D + q_A(\theta) + q_B(\theta)\right)^2}, \qquad \text{(A19)}$$

$$\frac{\partial \pi_A^M}{\partial \theta} = \frac{\left[Dq_A'(\theta) + q_B(\theta)q_A'(\theta) - Dq_B'(\theta) - q_A(\theta)q_B'(\theta)\right]\left(s - Y_B - Y_A\right)}{\left(2D + q_A(\theta) + q_B(\theta)\right)^2}, \qquad \text{(A20)}$$

where $D = r\frac{\lambda^{MP} + \lambda^{NM} + r}{\lambda^{NM} + r}$ and output $y_A = z_A\left(K_A^{NM*}\right)^\alpha$. $\frac{\partial \pi_A^{NM}}{\partial \theta} < 0$ and $\frac{\partial \pi_A^{MP}}{\partial \theta} < 0$ if

$$\left[2Dq_A'(\theta) + q_B(\theta)q_A'(\theta) - q_A(\theta)q_B'(\theta)\right] < 0, \qquad \text{(A21)}$$

and $\frac{\partial \pi_A^M}{\partial \theta} < 0$ if

$$\left[Dq_A'(\theta) + q_B(\theta)q_A'(\theta) - Dq_B'(\theta) - q_A(\theta)q_B'(\theta)\right] < 0. \qquad \text{(A22)}$$

Both are true since $q_A'(\theta) \leq 0$ and $q_B'(\theta) \geq 0$. The proofs for firm B are parallel and are omitted.

Proof of Lemma 2: Let $i \neq j$, $i, j \in \{A, B\}$ and h, k subscripts represent subtypes, $h, k \in N$. We will conjecture that $\pi_{i_k}^{MP} < \pi_{i_k}^M(j_k)$, which we will confirm in equilibrium. And, we will conjecture that $\pi_{i_k}^M(j_h) < \pi_{i_k}^{MP}$ $\forall$ $k \neq h$ and then we will look at what conditions will make this true in equilibrium. In this case, the profit when Mergers are Possible reduces to

$$\pi_{A_k}^{MP} = \rho\Delta q_A(\theta)e^{-\Delta\lambda^{MP}}\pi_{A_k}^M(B_k)e^{-r\Delta} + (1 - \rho\Delta q_A(\theta))e^{-\Delta\lambda^{MP}}\pi_{A_k}^{MP}e^{-r\Delta} \qquad \text{(A23)}$$
$$+ (1 - e^{-\Delta\lambda^{MP}})\pi_{A_k}^{NM}e^{-r\Delta} + \Delta z_{A_k}\left(K_{A_k}^{NM*}\right)^\alpha e^{-r\Delta}.$$

Following steps virtually identical to the proof of Proposition 1 results in the solution:

$$\pi_{i_k}^{NM} = \frac{\lambda^{NM}}{\lambda^{NM} + r}\left(\pi_{i_k}^{MP} + rY_i\right), \qquad \text{(A24)}$$

$$\pi_{i_k}^{MP} = \frac{\left(2D + \rho q_j(\theta)\right)Y_i + \rho q_i(\theta)\left(s_{A_k B_k} - Y_j\right)}{2D + \rho q_i(\theta) + \rho q_j(\theta)}, \qquad \text{(A25)}$$

$$\pi_{i_k}^M(j_k) = \frac{\left(D + \rho q_i(\theta)\right)\left(s_{A_k B_k} - Y_j\right) + \left(D + \rho q_j(\theta)\right)Y_i}{2D + \rho q_i(\theta) + \rho q_j(\theta)}, \qquad \text{(A26)}$$

where $D = r\frac{\lambda^{MP} + \lambda^{NM} + r}{\lambda^{NM} + r}$ and output is $y_i = z_i\left(K_{i_k}^{NM*}\right)^\alpha$. It is now clear that $\pi_{i_k}^{MP} < \pi_{i_k}^M(j_k)$

as conjectured.

A subtype k firm has a disagreement utility of $\pi_{i_k}^{MP}$ and a type h firm has a disagreement utility of $\pi_{i_h}^{MP}$. The synergy between a subtype k and a subtype h firm would be $s_{A_kB_h}$. For two firms subtype $k \neq h$ to be unwilling to consummate a deal it must be that

$$\pi_{i_k}^{M}(j_h) = \frac{1}{2}(s_{A_kB_h} - \pi_{j_h}^{MP} + \pi_{i_k}^{MP}) < \pi_{i_k}^{MP}. \tag{A27}$$

This reduces to

$$s_{A_kB_h} - \pi_{j_h}^{MP} < \pi_{i_k}^{MP}. \tag{A28}$$

Using the equilibrium disagreement utilities and the definition of the synergy we find different types will not merge with each other as long as

$$s_{A_kB_h} - \frac{(2D+\rho q_i(\theta))\,Y_j + \rho q_j(\theta)\,(s_{A_hB_h} - Y_i)}{2D+\rho q_i(\theta)+\rho q_j(\theta)} < \frac{(2D+\rho q_j(\theta))\,Y_i + \rho q_i(\theta)\,(s_{A_kB_k} - Y_j)}{2D+\rho q_i(\theta)+\rho q_j(\theta)} \forall k \neq h. \tag{A29}$$

$$\begin{aligned} & s_{A_kB_h}\left[2D+\rho q_i(\theta)+\rho q_j(\theta)\right] - \left[(2D+\rho q_i(\theta))\,Y_j + \rho q_j(\theta)\,(s_{A_hB_h} - Y_i)\right] \\ & < (2D+\rho q_j(\theta))\,Y_i + \rho q_i(\theta)\,(s_{A_kB_k} - Y_j)\,\forall k \neq h. \end{aligned} \tag{A30}$$

$$2D\left[s_{A_kB_h} - Y_j - Y_i\right] + \rho q_j(\theta)\left[s_{A_kB_h} - s_{A_hB_h}\right] < \rho q_i(\theta)\left[s_{A_kB_k} - s_{A_kB_h}\right] \forall k \neq h. \tag{A31}$$

Thus, if this condition holds then the only mergers will be between the same types. It might seem that a high number type firm that found a 'better' lower number type partner would always like to merge so that $\pi_{A_2}^{M}(B_1) > \pi_{A_2}^{MP}$ for example. This is not the case. Certainly a type 2 firm who finds a type 1 firm generates more production than if they merge with a type 2 firm, but the type 1 firm has more bargaining power! Thus, both firms may find it mutually beneficial to continue searching.[19]

Simultaneously, for two firms subtype $k = h$ to be willing to consummate a deal it must be that

$$\pi_{i_k}^{M}(j_k) = \frac{1}{2}(s_{A_kB_k} - \pi_{j_k}^{MP} + \pi_{i_k}^{MP}) > \pi_{i_k}^{MP}. \tag{A32}$$

This reduces to

$$s_{A_kB_k} - \pi_{j_k}^{MP} > \pi_{i_k}^{MP}. \tag{A33}$$

Using the equilibrium disagreement utilities and the definition of the synergy we find that the same subtypes will merge with each other as long as

$$s_{A_kB_k} - \frac{(2D+\rho q_i(\theta))\,Y_j + \rho q_j(\theta)\,(s_{A_kB_k} - Y_i)}{2D+\rho q_i(\theta)+\rho q_j(\theta)} > \frac{(2D+\rho q_j(\theta))\,Y_i + \rho q_i(\theta)\,(s_{A_kB_k} - Y_j)}{2D+\rho q_i(\theta)+\rho q_j(\theta)} \forall k. \tag{A34}$$

$$\begin{aligned} & s_{A_kB_k}\left[2D+\rho q_i(\theta)+\rho q_j(\theta)\right] - \left[(2D+\rho q_i(\theta))\,Y_j + \rho q_j(\theta)\,(s_{A_kB_k} - Y_i)\right] \\ & > (2D+\rho q_j(\theta))\,Y_i + \rho q_i(\theta)\,(s_{A_kB_k} - Y_j)\,\forall k. \end{aligned} \tag{A35}$$

$$2D\left[s_{A_kB_k} - Y_j - Y_i\right] > 0. \tag{A36}$$

Which is true by the assumption that synergies are positive.

Proof of Proposition 4: The M/B ratio has two components: part that is due to the stand alone firm value and part that is from the potential gain from the merger. Remembering that $K_i^{NM*} = K_i^{MP*}$, we can write the portion of M/B due to potential merger gains as $\left(\pi_{i_k}^{MP} - Y_{i_k}\right)/K_i^{NM*}$. If firms could not merge their M/B ratio would be $Y_i/K_i^{NM*} =$

[19] It may also seem that we must be concerned that even though $\pi_{A_2}^{M}(B_1) < \pi_{A_1}^{MP}$ that for the merger partner $\pi_{B_1}^{M}(A_2) > \pi_{B_1}^{MP}$. However, any solution to the Nash bargaining solution must give both firms more than their reservation value so $\pi_{A_2}^{M}(B_1) < \pi_{A_1}^{MP}$ implies $\pi_{B_1}^{M}(A_2) < \pi_{B_1}^{MP}$.

$\frac{z_i\left(K_i^{NM*}\right)^{\alpha-1}}{r} + \frac{O_{i_k}}{K_i^{NM*}} = \frac{1}{\alpha} + \frac{O_{i_k}}{K_i^{NM*}}$. Thus, the total M/B ratio pre merger is

$$\frac{\pi_{i_k}^{MP}}{K_i^{NM*}} = \frac{1}{\alpha} + \frac{O_{i_k}}{K_i^{NM*}} + \frac{\pi_{i_k}^{MP} - Y_{i_k}}{K_i^{NM*}}, \tag{A37}$$

in the Mergers are Possible state. And

$$\frac{\pi_{i_k}^{NM}}{K_i^{NM*}} = \frac{\lambda^{NM}}{\lambda^{NM}+r}\left(\frac{\pi_{i_k}^{MP} + rY_i}{K_i^{NM*}}\right), \tag{A38}$$

in the No Mergers state.

Lemma 2 ensures that If condition (27) holds then firms will assortatively match. So firms with the same subtype will merge, i.e., A_k will merge with B_k. Given that condition (27) holds, Lemma 2 provides the equilibrium solution. Therefore, the M/B ratios can be rewritten

$$\frac{\pi_{i_k}^{MP}}{K_i^{NM*}} = \frac{1}{\alpha} + \frac{O_{i_k}}{K_i^{NM*}} + \frac{\rho q_i(\theta)\left(s_{A_kB_k} - Y_{j_k} - Y_{i_k}\right)}{\left[2D + \rho q_i(\theta) + \rho q_j(\theta)\right] K_i^{NM*}}, \tag{A39}$$

$$\frac{\pi_{i_k}^{NM}}{K_i^{NM*}} = \frac{\lambda^{NM}}{\lambda^{NM}+r}\left((1+r)\left(\frac{1}{\alpha} + \frac{O_{i_k}}{K_i^{NM*}}\right) + \frac{\rho q_i(\theta)\left(s_{A_kB_k} - Y_{j_k} - Y_{i_k}\right)}{\left[2D + \rho q_i(\theta) + \rho q_j(\theta)\right] K_i^{NM*}}\right). \tag{A40}$$

Our naming condition is such that firms with the lowest k have the highest outside opportunities, O_{i_k}, and the most gain from the merger, $\left(s_{A_kB_k} - Y_{j_k} - Y_{i_k}\right)$. Furthermore, K_i^{NM*}, $q_i(\theta)$ and $1/\alpha$ are not affected by the subtype of the firm. Therefore, firms in the lowest ordinal subcategory have the highest M/B ratios pre merger. Therefore, the search for complementarities results in mergers in which type A firms with the largest M/B will merge with the type B firms with the largest M/B, while type A firms with the second largest M/B will merge with the type B firms with the second largest M/B, etc. Q.E.D.

Proof of Corollary 1: Condition (27) is

$$2D\left[s_{A_kB_h} - Y_j - Y_i\right] + \rho q_j(\theta)\left[s_{A_kB_h} - s_{A_hB_h}\right] < \rho q_i(\theta)\left[s_{A_kB_k} - s_{A_kB_h}\right] \forall k \neq h. \tag{A41}$$

When this condition holds we need to show that raising the discount rate makes it less likely to hold. This condition can be rewritten for a particular k and h as

$$s_{A_kB_h} - Y_i - Y_j < \frac{\rho q_i(\theta)\left(s_{A_kB_k} - Y_i - Y_j\right)}{2D + \rho q_i(\theta) + \rho q_j(\theta)} + \frac{\rho q_j(\theta)\left(s_{A_hB_h} - Y_i - Y_j\right)}{2D + \rho q_i(\theta) + \rho q_j(\theta)}. \tag{A42}$$

Therefore we need to show that

$$\frac{d}{dr}\frac{\rho q_i(\theta)\left(s_{A_kB_k} - Y_i - Y_j\right)}{2D + \rho q_i(\theta) + \rho q_j(\theta)} + \frac{d}{dr}\frac{\rho q_j(\theta)\left(s_{A_hB_h} - Y_i - Y_j\right)}{2D + \rho q_i(\theta) + \rho q_j(\theta)} - \frac{d}{dr}\left(s_{A_kB_h} - Y_i - Y_j\right) < 0. \tag{A43}$$

Since since $\frac{\partial}{\partial r}D > 0,$[20] it will be sufficient to show

$$\frac{\rho q_i(\theta)\frac{d}{dr}\left(s_{A_kB_k} - Y_i - Y_j\right) + \rho q_j(\theta)\frac{d}{dr}\left(s_{A_hB_h} - Y_i - Y_j\right)}{2D + \rho q_i(\theta) + \rho q_j(\theta)} - \frac{d}{dr}\left(s_{A_kB_h} - Y_i - Y_j\right) < 0, \tag{A44}$$

when

$$s_{A_kB_h} - Y_i - Y_j < \frac{\rho q_i(\theta)\left(s_{A_kB_k} - Y_i - Y_j\right)}{2D + \rho q_i(\theta) + \rho q_j(\theta)} + \frac{\rho q_j(\theta)\left(s_{A_hB_h} - Y_i - Y_j\right)}{2D + \rho q_i(\theta) + \rho q_j(\theta)}. \tag{A45}$$

The term in the derivative that we drop is negative, therefore, if condition A44 holds when matching is occurring then raising the discount will make condition (27) less likely to hold.

[20] $\frac{\partial}{\partial r}D = \frac{\lambda^{MP}\lambda^{NM}}{\left(\lambda^{NM}+r\right)^2} + 1$

First note that

$$s_{A_kB_h} - Y_{j_k} - Y_{i_k} = \frac{z_{A_kB_h}\left(\left(K_{A_k}^{M*}\right)^{\alpha} + \left(K_{B_h}^{M*}\right)^{\alpha}\right)}{r} - \left(K_{A_k}^{M*} + K_{B_h}^{M*} - K_{A_k}^{NM*} - K_{B_h}^{NM*}\right) - \frac{z_A\left(K_A^{NM*}\right)^{\alpha}}{r} - \frac{z_B\left(K_B^{NM*}\right)^{\alpha}}{r}. \tag{A46}$$

Furthermore, all optimal capital stocks are determined by the equation

$$K^* = \sqrt[\alpha-1]{\frac{r}{z\alpha}}. \tag{A47}$$

Therefore,

$$\frac{z\left(K^*\right)^{\alpha}}{r} - K^* = \frac{z^{\frac{1}{1-\alpha}}\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)}{r^{\frac{1}{1-\alpha}}}, \tag{A48}$$

and

$$\frac{\partial}{\partial r}\frac{z^{\frac{1}{1-\alpha}}\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)}{r^{\frac{1}{1-\alpha}}} = -\frac{1}{1-\alpha}\frac{z^{\frac{1}{1-\alpha}}\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)}{r^{\frac{\alpha}{1-\alpha}}}, \tag{A49}$$

and

$$\frac{d}{dr}\left(s_{A_kB_h} - Y_{j_k} - Y_{i_k}\right) = -\frac{1}{1-\alpha}\frac{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)}{r^{\frac{\alpha}{1-\alpha}}}\left(2z_{A_kB_h}^{\frac{1}{1-\alpha}} - z_A^{\frac{1}{1-\alpha}} - z_B^{\frac{1}{1-\alpha}}\right). \tag{A50}$$

Therefore, substituting in for $\frac{d}{dr}\left(s_{A_kB_h} - Y_{j_k} - Y_{i_k}\right)$, $\frac{d}{dr}\left(s_{A_kB_k} - Y_{j_k} - Y_{i_k}\right)$ and $\frac{d}{dr}\left(s_{A_hB_h} - Y_{j_k} - Y_{i_k}\right)$ in Equation (A44) it is sufficient to show that

$$\begin{aligned} &\frac{\rho q_i(\theta)}{2D+\rho q_i(\theta)+\rho q_j(\theta)}\frac{1}{1-\alpha}\frac{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)}{r^{\frac{\alpha}{1-\alpha}}}\left(2z_{A_kB_k}^{\frac{1}{1-\alpha}} - z_A^{\frac{1}{1-\alpha}} - z_B^{\frac{1}{1-\alpha}}\right) \\ &\frac{\rho q_j(\theta)}{2D+\rho q_i(\theta)+\rho q_j(\theta)}\frac{1}{1-\alpha}\frac{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)}{r^{\frac{\alpha}{1-\alpha}}}\left(2z_{A_hB_h}^{\frac{1}{1-\alpha}} - z_A^{\frac{1}{1-\alpha}} - z_B^{\frac{1}{1-\alpha}}\right) \\ > \quad &\frac{1}{1-\alpha}\frac{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)}{r^{\frac{\alpha}{1-\alpha}}}\left(2z_{A_kB_h}^{\frac{1}{1-\alpha}} - z_A^{\frac{1}{1-\alpha}} - z_B^{\frac{1}{1-\alpha}}\right). \end{aligned}$$

Given

$$s_{A_kB_h} - Y_i - Y_j < \frac{\rho q_i(\theta)\left(s_{A_kB_k} - Y_i - Y_j\right)}{2D+\rho q_i(\theta)+\rho q_j(\theta)} + \frac{\rho q_j(\theta)\left(s_{A_hB_h} - Y_i - Y_j\right)}{2D+\rho q_i(\theta)+\rho q_j(\theta)}, \tag{A51}$$

which can be rewritten as

$$\begin{aligned} &\frac{\rho q_i(\theta)}{2D+\rho q_i(\theta)+\rho q_j(\theta)}\frac{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)}{r^{\frac{1}{1-\alpha}}}\left(2z_{A_kB_k}^{\frac{1}{1-\alpha}} - z_A^{\frac{1}{1-\alpha}} - z_B^{\frac{1}{1-\alpha}}\right) \\ &+\frac{\rho q_j(\theta)}{2D+\rho q_i(\theta)+\rho q_j(\theta)}\frac{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)}{r^{\frac{1}{1-\alpha}}}\left(2z_{A_hB_h}^{\frac{1}{1-\alpha}} - z_A^{\frac{1}{1-\alpha}} - z_B^{\frac{1}{1-\alpha}}\right) \\ > \quad &\frac{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)}{r^{\frac{1}{1-\alpha}}}\left(2z_{A_kB_h}^{\frac{1}{1-\alpha}} - z_A^{\frac{1}{1-\alpha}} - z_B^{\frac{1}{1-\alpha}}\right). \end{aligned}$$

Therefore, since $r^{\frac{1}{1-\alpha}}$ and $r^{\frac{\alpha}{1-\alpha}}$ are positive the sufficient condition holds.

Proof of Corollary 2: If condition (27) holds then Lemma 2 tells us that firms match i.e. A_k merges with B_k. The gains from a merger are $s_{A_kB_h} - Y_A - Y_B$. Merger gains are being split more equally if $\left|\left(\pi^M_{A_k}(B_k) - Y_A\right) - \left(\pi^M_{B_k}(A_k) - Y_B\right)\right|$ is smaller. Increasing d increases n. Therefore, this lemma is true iff

$$\frac{\partial\left[\left(\pi^M_{A_k}(B_k) - Y_A\right) - \left(\pi^M_{B_k}(A_k) - Y_B\right)\right]}{\partial n} > 0 \text{ if } \pi^M_{A_k}(B_k) - Y_A < \pi^M_{B_k}(A_k) - Y_B, \tag{A52}$$

and

$$\frac{\partial\left[\left(\pi^M_{A_k}(B_k) - Y_A\right) - \left(\pi^M_{B_k}(A_k) - Y_B\right)\right]}{\partial n} < 0 \text{ if } \pi^M_{A_k}(B_k) - Y_A > \pi^M_{B_k}(A_k) - Y_B. \tag{A53}$$

We know that

$$\pi^M_{A_k}(B_k) - Y_A = \frac{(D + \rho q_A(\theta))\left(s_{A_k B_k} - Y_B - Y_A\right)}{2D + \rho q_A(\theta) + \rho q_B(\theta)}. \tag{A54}$$

$$\pi^M_{B_k}(A_k) - Y_B = \frac{(D + \rho q_B(\theta))\left(s_{A_k B_k} - Y_B - Y_A\right)}{2D + \rho q_A(\theta) + \rho q_B(\theta)}. \tag{A55}$$

Therefore, $\pi^M_{A_k}(B_k) - Y_A < \pi^M_{B_k}(A_k) - Y_B$ implies $q_A(\theta) < q_B(\theta)$. We know $\rho \equiv 1/n$. Therefore,

$$\frac{\partial\left[\left(\pi^M_{A_k}(B_k) - Y_A\right) - \left(\pi^M_{B_k}(A_k) - Y_B\right)\right]}{\partial n} = \frac{\partial}{\partial n}\frac{(q_A(\theta) - q_B(\theta))\left(s_{A_k B_k} - Y_B - Y_A\right)}{2Dn + q_A(\theta) + q_B(\theta)} \tag{A56}$$

$$= \frac{\left[(q_A(\theta) - q_B(\theta))\frac{\partial}{\partial n}\left(s_{A_k B_k} - Y_B - Y_A\right)\right]}{2Dn + q_A(\theta) + q_B(\theta)} - \frac{\left[(q_A(\theta) - q_B(\theta))\left(s_{A_k B_k} - Y_B - Y_A\right)\right][2D]}{\left(2Dn + q_A(\theta) + q_B(\theta)\right)^2}. \tag{A57}$$

which, since $\frac{\partial}{\partial n}\left(s_{A_k B_k} - Y_B - Y_A\right) > 0$,[21] is greater than 0 if $q_A(\theta) < q_B(\theta)$ and less than 0 if $q_A(\theta) > q_B(\theta)$. Q.E.D.

Proof of Proposition 5: $D = r\frac{\lambda^{MP} + \lambda^{NM} + r}{\lambda^{NM} + r}$ and output is $y_i = z_i\left(K_i^{NM*}\right)^\alpha$. The first inequality requires

$$\begin{aligned}
\frac{\pi_i^{NM}}{K_i^{NM*}} &< \frac{\pi_i^{MP}}{K_i^{MP*}}, \\
&< \frac{1}{K_i^{NM*}}\frac{\lambda^{NM}}{\lambda^{NM} + r}\frac{(2D + q_j(\theta))Y_i + q_i(\theta)(s - Y_j)}{2D + q_i(\theta) + q_j(\theta)} + \frac{1}{K_i^{NM*}}\frac{y_i}{\lambda^{NM} + r} \\
&< \frac{1}{K_i^{MP*}}\frac{(2D + q_j(\theta))Y_i + q_i(\theta)(s - Y_j)}{2D + q_i(\theta) + q_j(\theta)},
\end{aligned} \tag{A58}$$

but $K_i^{MP*} = K_i^{NM*}$. Therefore

$$\begin{aligned}
Y_i &< \frac{(2D + q_j(\theta))Y_i + q_i(\theta)(s - Y_j)}{2D + q_i(\theta) + q_j(\theta)}, \\
0 &< s - Y_j - Y_i.
\end{aligned} \tag{A59}$$

Thus it is true as long as the synergy is positive.

$$\frac{\pi_i^{MP}}{K_i^{MP*}} < \frac{\pi_i^{M}}{K_i^{MP*}}, \tag{A60}$$

also reduces almost immediately to

$$0 < s - Y_j - Y_i. \tag{A61}$$

[21] The gains from merging, $\left(s_{A_k B_k} - Y_B - Y_A\right)$, increase with greater matching, n, as assumed.

So it is also true as long as the synergy is positive.

The last inequality requires

$$\frac{\pi_i^M + \pi_j^M}{K_i^{MP*} + K_j^{MP*}} > \frac{z_M(\left(K_i^{M*}\right)^\alpha + \left(K_j^{M*}\right)^\alpha)}{r\left(K_i^{M*} + K_j^{M*}\right)}, \tag{A62}$$

$$\frac{\left(D + q_i(\theta)\right)s + \left(D + q_j(\theta)\right)s}{\left(K_i^{MP*} + K_j^{MP*}\right)\left(2D + q_i(\theta) + q_j(\theta)\right)} > \frac{z_M(\left(K_i^{M*}\right)^\alpha + \left(K_j^{M*}\right)^\alpha)}{r\left(K_i^{M*} + K_j^{M*}\right)},$$

$$\frac{s}{K_i^{MP*} + K_j^{MP*}} > \frac{z_M(\left(K_i^{M*}\right)^\alpha + \left(K_j^{M*}\right)^\alpha)}{r\left(K_i^{M*} + K_j^{M*}\right)},$$

where s is given in equation (15). Thus,

$$\frac{z_M(\left(K_i^{M*}\right)^\alpha + \left(K_j^{M*}\right)^\alpha)}{r\left(K_i^{MP*} + K_j^{MP*}\right)} - \frac{\left(K_i^{M*} + K_j^{M*} - K_i^{MP*} - K_j^{MP*}\right)}{K_i^{MP*} + K_j^{MP*}}$$

$$> \frac{z_M(\left(K_i^{M*}\right)^\alpha + \left(K_j^{M*}\right)^\alpha)}{r\left(K_i^{M*} + K_j^{M*}\right)},$$

$$\frac{z_M(\left(K_i^{M*}\right)^\alpha + \left(K_j^{M*}\right)^\alpha)}{r}\left(\frac{\left(K_i^{M*} + K_j^{M*}\right) - \left(K_i^{MP*} + K_j^{MP*}\right)}{\left(K_i^{MP*} + K_j^{MP*}\right)\left(K_i^{M*} + K_j^{M*}\right)}\right)$$

$$> \frac{\left(K_i^{M*} + K_j^{M*} - K_i^{MP*} - K_j^{MP*}\right)}{K_i^{MP*} + K_j^{MP*}}.$$

$\left(K_i^{M*} + K_j^{M*}\right) > \left(K_i^{MP*} + K_j^{MP*}\right)$ because we assumed $z_M > z_i \forall i$. Thus, this inequality is true as long as

$$\frac{z_M(\left(K_i^{M*}\right)^\alpha + \left(K_j^{M*}\right)^\alpha)}{r} > K_i^{M*} + K_j^{M*}. \tag{A63}$$

That is, as long as the value of all investment is greater than the cost. Using the fact that

$$K_i^{M*} = \sqrt[\alpha-1]{\frac{r}{z_M \alpha}}, \tag{A64}$$

this reduces to

$$\frac{K_i^{M*}}{\alpha} + \frac{K_j^{M*}}{\alpha} > K_i^{M*} + K_j^{M*}, \tag{A65}$$

which is true since $\alpha < 1$.

Proof of Proposition 6: The market reaction is

$$\frac{\pi_i^M - \pi_i^{MP}}{\pi_i^{MP}} = \frac{D\left(s - Y_j - Y_i\right)}{\left(2D + q_j(\theta)\right)Y_i + q_i(\theta)\left(s - Y_j\right)} > 0. \tag{A66}$$

The derivative with respect to λ^{MP} (mergers become less likely) is

$$\frac{\partial \frac{\pi_i^M - \pi_i^{MP}}{\pi_i^{MP}}}{\partial \lambda^{MP}} = \frac{r\left(s - Y_j - Y_i\right)\left[q_j(\theta)Y_i + q_i(\theta)\left(s - Y_j\right)\right]}{\left(\lambda^{NM} + r\right)\left(\left(2D + q_j(\theta)\right)Y_i + q_i(\theta)\left(s - Y_j\right)\right)^2} > 0. \tag{A67}$$

The derivative with respect to λ^{NM} (mergers become more likely) is

$$\frac{\partial \frac{\pi_i^M - \pi_i^{MP}}{\pi_i^{MP}}}{\partial \lambda^{MP}} = \frac{-r\lambda^{MP}\left(s - Y_j - Y_i\right)\left[q_j(\theta)Y_i + q_i(\theta)\left(s - Y_j\right)\right]}{\left(\lambda^{NM} + r\right)^2\left(\left(2D + q_j(\theta)\right)Y_i + q_i(\theta)\left(s - Y_j\right)\right)^2} < 0. \tag{A68}$$

The derivative with respect to θ is

$$\frac{\partial \frac{\pi_i^M - \pi_i^{MP}}{\pi_i^{MP}}}{\partial \theta} = \frac{-D\left(s - Y_j - Y_i\right)\left(q_i'(\theta)\left(s - Y_j\right) + q_j'(\theta) Y_i\right)}{\left(\left(2D + q_j(\theta)\right) Y_i + q_i(\theta)\left(s - Y_j\right)\right)^2}. \tag{A69}$$

The sign of which depends on whether

$$q_i'(\theta)\left(s - Y_j\right) + q_j'(\theta) Y_i \lesseqgtr 0. \tag{A70}$$

Since $q_i'(\theta)$ and $q_j'(\theta)$ have opposite signs and nothing pins down their relative magnitudes, more scarce firms may have larger or smaller market reactions.

$$\frac{\partial \frac{\pi_i^M - \pi_i^{MP}}{\pi_i^{MP}}}{\partial s} = \frac{D\left(2DY_i + \left(q_j(\theta) + q_i(\theta)\right) Y_i\right)}{\left(\left(2D + q_j(\theta)\right) Y_i + q_i(\theta)\left(s - Y_j\right)\right)^2} > 0. \tag{A71}$$

Thus greater synergy increases the market reaction.

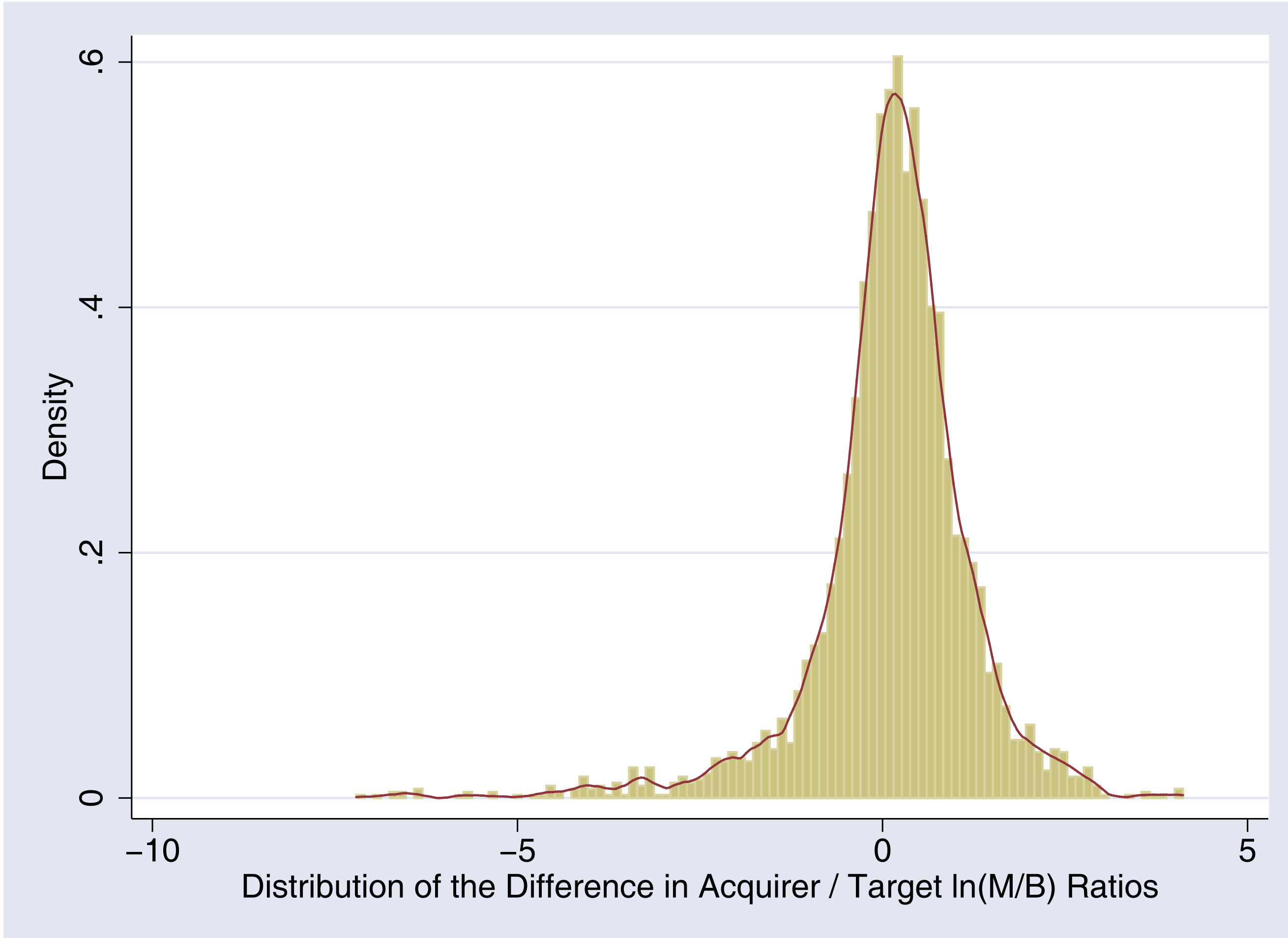

Figure 1: Distribution of M/B Spreads

This graph shows the distribution of the difference between the acquirer M/B ratio and the target M/B ratio. The area to the left of the origin on the x-axis is the 40% of the distribution for which the acquirer's M/B is lower than that of the target.

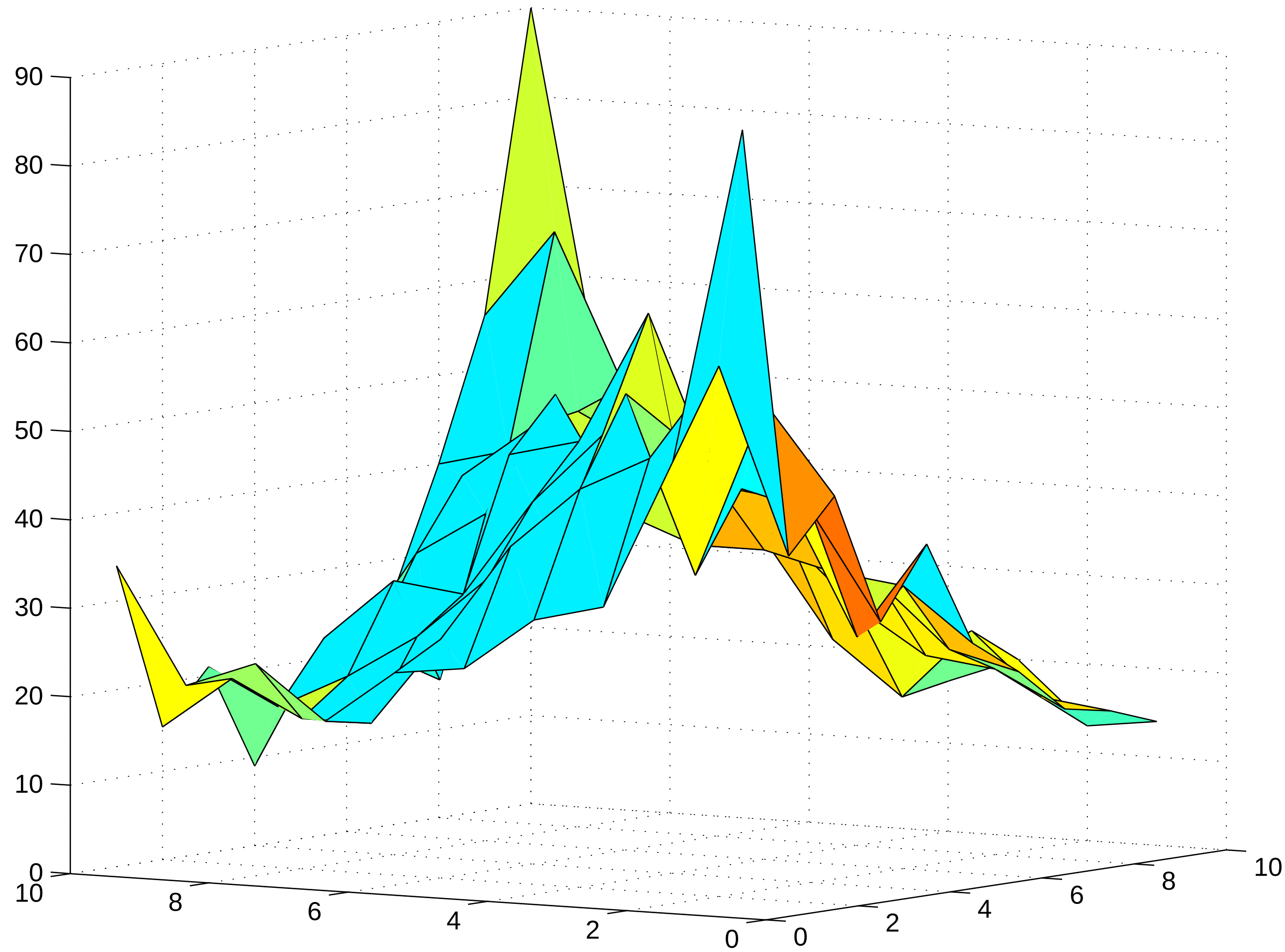

Figure 2: Industry-adjusted Ranking of Acquirer and Target M/B Ratios

This graph shows the bivariate distribution of M/B Ratios in mergers, but adjusts for the industry M/B of the bidder and target. The bottom axes, which run from 0 to 10, are M/B deciles for acquirers (on the left) and targets (on the right). "10" is the highest M/B; "0" is the lowest. The vertical axis is the count of the number of transactions with acquirer and target M/B ratios falling into that bin. A 1-st nearest neighbor smoothing approach is used to produce an empirical density. The fact that the peaks of the distribution occur at the 10/10 and 0/0 bins means that mergers are most commonly 'high buying high' and 'low buying low'; the saddle down the 45-degree line in the x-y plane shows that most mergers involve 'like buying like,' regardless of their valuation level.

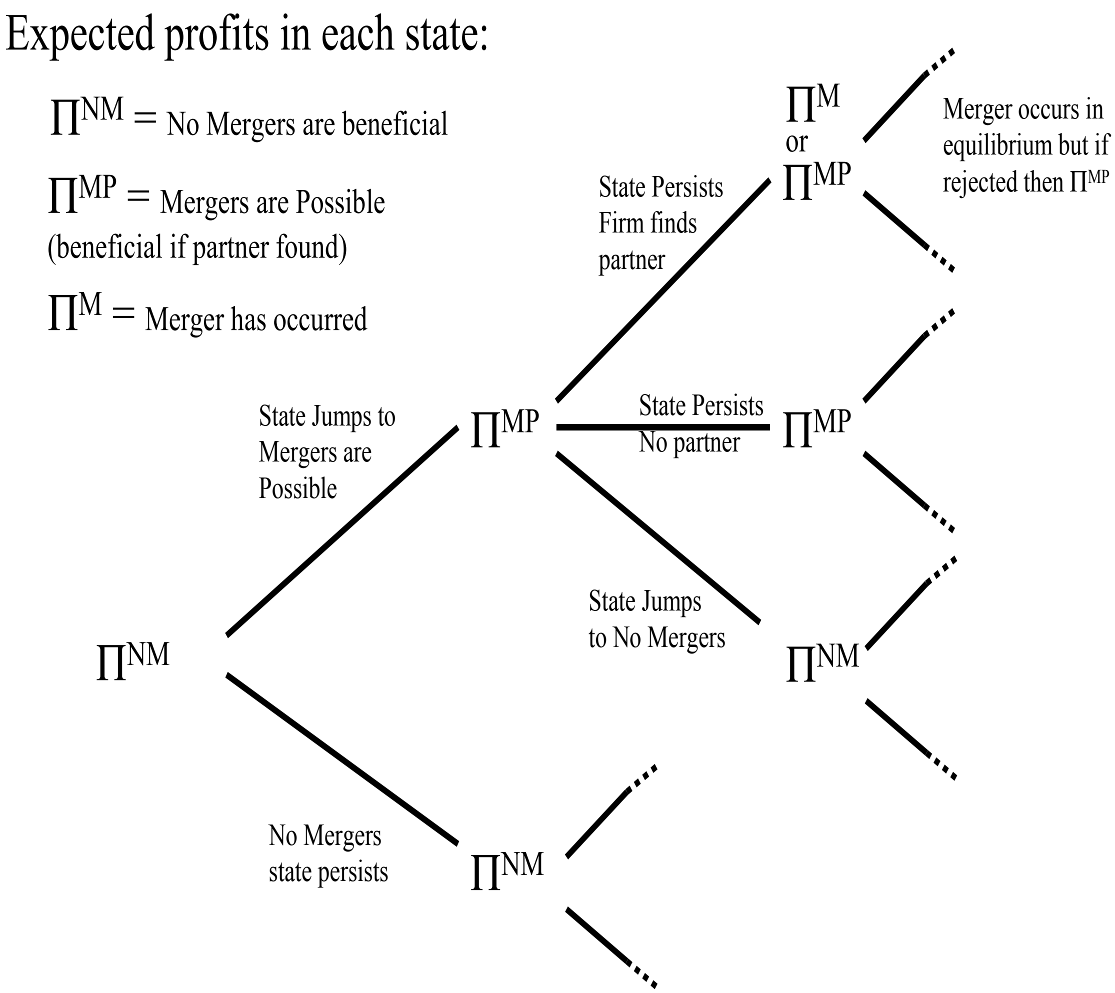

Figure 3: Extensive Form Representation of the Search Model

Table 1: Acquirer and Target M/B Ratios Using NYSE Deciles

Target Decile:	NYSE M/B Decile of Acquirer High	9	8	7	6	5	4	3	2	Low	Total
High	252	95	49	39	26	20	21	8	9	24	543
9	102	79	44	36	23	16	9	2	7	13	331
8	100	51	49	54	25	13	12	12	7	14	337
7	78	76	58	42	30	30	11	9	7	8	349
6	55	41	34	56	45	25	19	23	5	6	309
5	47	35	37	45	62	51	26	16	12	15	346
4	28	30	36	43	53	49	39	30	13	13	334
3	22	26	28	32	41	42	35	24	21	15	286
2	12	26	20	22	42	33	24	24	21	18	242
Low	22	20	22	26	25	33	27	34	26	45	280
Total	718	479	377	395	372	312	223	182	128	171	3,357

Decile breakpoints are based on the distribution of m/b ratios for NYSE traded firms and were obtained from website of Professor Kenneth French (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french`). Each cell counts the number of mergers after 1980 between publicly traded bidders and targets in that decile pairing. The deciles are numbered from 10 to 1 in descending order of M/B.

Pearson's χ^2 test for independence of bidder and target m/b ratios has a value of 854.91, with an associated p-value of 0.00.

The mean acquirer lies in the fourth decile, while the mean target lies in the fifth decile of the m/b distribution. The mean difference is $\frac{8}{10}$ of a decile.

Table 2: Q Levels and Differences in Bidder and Target Q

	Sample Size	Percent of Sample	Mean Scaled M/B Difference
Total Mergers, 1981-2001	3,400	100%	14.58
Mean Acquirer ln(M/B): .8118			
Mean Target ln(M/B): .6816			
Target Exceeds Acquirer M/B	1,274	37%	-89.38
Acquirer Exceeds Target M/B	2,126	63%	77.16
Both Above Respective Industry Median	1,111	33%	11.46
Both Below Respective Industry Median	990	29%	2.38
Target Above, Acquirer Below	419	12%	-138.08
Acquirer Above, Target Below	880	26%	105.7

Scaled M/B Difference is the difference between acquirer ln(mb) and target ln(mb) divided by the standard deviation of the m/b for the acquirer's industry in the year of the acquisition. The units are in percent of a standard deviation; i.e., 100 is one standard deviation.

Table 3: Time-Series of M/B Spreads

Year	Count	Target	Acquirer	Median(T)	Median(A)
1981	37	0.22	0.43	0.21	0.12
1982	39	0.39	0.15	0.43	0.40
1983	52	0.27	0.31	0.51	0.41
1984	90	0.47	0.40	0.44	0.38
1985	104	0.49	0.46	0.64	0.64
1986	97	0.43	0.62	0.69	0.65
1987	126	0.70	0.68	0.52	0.53
1988	113	0.65	0.73	0.57	0.58
1989	110	0.61	0.62	0.56	0.55
1990	76	0.52	0.57	0.41	0.41
1991	89	0.48	0.71	0.65	0.62
1992	62	0.53	0.62	0.74	0.72
1993	97	0.71	0.97	0.77	0.76
1994	178	0.69	0.78	0.71	0.71
1995	239	0.67	0.84	0.83	0.84
1996	260	0.59	0.95	0.81	0.77
1997	358	0.72	0.86	0.99	0.97
1998	347	0.91	1.13	0.77	0.74
1999	365	0.82	1.12	0.87	0.86
2000	352	0.88	0.97	0.55	0.54
2001	208	0.71	0.83	0.50	0.44

This table reports average ln(M/B) for all targets and acquirers in a given year. Count is the number of transactions. Columns headed 'Target,' and 'Acquirer,' are, respectively, annual average ln(M/B) values for targets and acquirers involved in transactions that year. Columns headed 'Median(T)' and 'Median(A)' report the average value of the industry median ln(M/B) for the target and acquirer, respectively.

Table 4: Market/Book Differentials in High and Low Valuation Periods

Panel A: Target Industry Valuation Levels

	(1)	(2)
Years When M/B Above Industry Median	.8922	.0115
Years When Below Industry Median	.817	.0965

Panel B: Valuation Levels for Target and Acquirer Industries

	Target Industry:	
	Low	High
Acquirer Industry Low	-.090	-.251
Acquirer Industry High	.054	.006

In Panel A, Column (1) reports the overall industry dispersion in ln(M/B) ratios, expressed in standard deviations. Column (2) reports the median ln(M/B) spread between bidders and targets. In Panel B, targets and acquirers are in high/low valuation periods according to whether their industry is above/below its median ln(M/B) ratio. The cell values are the average difference in ln(M/B) between bidder and target, weighted by transaction value, averaged over all transactions happening in the relevant valuation period.