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Be Nice, unless it Pays to Fight

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Abstract
This paper considers industries where a firm or group of firms acts as price leader. It shows that entry in such industries can lead to higher prices through a crowding effect. Further, efficiency gains can lead to higher prices by making it too costly to fight. Mergers that bring the merged firms' efficiency close to that of the price leader(s) lead to higher prices if the merged firm does not belong to the group of price leaders. This is a formalization of joint dominance or coordinated effects. Finally, the model is extended to endogenize the identity of the price leader. This is done by allowing firms to make price announcements

Keywords: price leadership, mergers, joint dominance, coordinated effects, endogenous price leadership

JEL classification: D43, L11, L41

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1. Introduction

This paper considers price setting in industries where a firm or group of firms acts as price leader(s). In such industries we find two surprising comparative static results. Entry by a new firm and efficiency gains by existing firms can lead to higher prices. The intuition for these results is as follows. When the price leader (or group of leaders) decides on its price, it understands that charging a higher price leads to more firms entering the industry with which the market has to be shared. Hence we have the classic trade off between price cost margin and output sold by the price leader. Consider now the firm that is just kept out of the market by the price leader. If this firm experiences a small efficiency gain, it is (presumably) optimal for the leader to fight. That is, the leader will reduce its price to keep this firm out. But if the efficiency gain of this marginal firm is big enough, it is too costly for the leader to fight and keep this firm out of the market. Thus the leader will accommodate and raise its price compared to the initial situation (note that, after the efficiency gain, the initial price can be raised without immediately inviting entry).

Next, consider the effect of entry by a firm that was initially not in the market. Again, if this firm is not very efficient, the leader may decide to fight and keep the entrant out of the market. But if the entrant has an efficiency level close to that of the leader, this strategy is too costly and it is more profitable to be nice. As shown below, there is a crowding effect which leads to a higher price. Intuitively, if a given price reduction $\Delta p < 0$ reduces the number of firms n in the market by five, this is more profitable if the reduction is from $n = 10$ to 5 than it is in the case of $n = 105$ to $n = 100$.[1] Hence the more firms there are, crowding the market with efficiency levels close to the leader, the less profitable it becomes for the leader to reduce the price to fight firms at the margin. Similarly, mergers that lead to efficiency gains can cause

[1] A more mundane example is the following. Professional cyclists are willing to spend serious money to reduce the weight of their bike's chain by another 50 grams. This is worthwhile if your bike weighs around 6 kilograms. If your bike weighs 20 kilogram, the same reduction of 50 grams is not so interesting.

increases in the equilibrium price. This can be viewed as a formalization of joint dominance or coordinated effects.

These effects turn out to hold irrespectively of the identity of the price leader(s). Yet, sometimes one may want to argue that a new entrant or a merged firm becomes leader in the industry. To analyze these cases, we extend the model to endogenize the identity of the leader. We do this by allowing firms to pre announce price changes. We then focus on the set of equilibria where firms are only punished for secretly undercutting opponents. That is, if a firm wants to reduce its price and pre announces this, the other firms will not revert to playing the Bertrand Nash equilibrium strategies. This leads to the intuitive idea that the most efficient firm in the industry acts as price leader. Hence a new firm entering the industry becomes the new price leader if it is more efficient than the current leader. Compared to the case where the leader is exogenously fixed, this effect tends to reduce the price as firms with lower costs tend to prefer higher output levels and thus lower prices.

The motivation for this paper is the following question: What is the effect of the efficiency distribution in an industry on how aggressively firms interact? To illustrate, the early SCP literature explaining prices in a certain industry only considered the average cost level in the industry (see, for instance, Ravenscraft (1983) and Scherer and Ross (1990)). In other words, only the first moment of the efficiency distribution is used as an explanatory variable. In standard Cournot and Bertrand models, this is actually correct. To illustrate, in a Cournot model the relevant variable is some average of the firms' efficiency levels. In a Bertrand model with homogenous goods only the second lowest cost level is relevant, not the third lowest cost level etc. However, it is simple to come up with examples where a firm that is far more efficient than its opponents has higher profits under Bertrand competition than under Cournot competition.[2] Within the Bertrand and Cournot frameworks it (usually) does not make sense

[2]For instance, consider the case with 2 firms, a linear demand function $p = 1 - x_1 - x_2$ where x_i is the output level of firm i where firm 1 has constant marginal costs equal to 0 and firm 2 has constant marginal costs equal

to allow firms to switch from Cournot competition to Bertrand competition if this is more profitable for firms. This is the idea we try to formalize in this paper: the aggressiveness of firms' conduct is determined by the efficiency levels of their opponents. If their opponents are weak they will fight, but it is better to be nice to opponents that are equally or more efficient than you are. The implication for empirical research explaining price cost margins in an industry is that higher moments than just the first moment of the efficiency distribution should be included.

This paper is related to three strands of literature. First, the result that prices are higher if firms are more similar (in terms of efficiency) is related to recent contributions in the literature on supergames by Compte et. al. (2002), Motta (2004) and Vasconcelos (2002). Each of these papers formalizes the notion of joint dominance or coordinated effects. They show that if a merger makes the firms in the industry more similar in terms of either production capacities, efficiency levels or number of product lines sold by each firm, it is easier to sustain collusion using trigger strategies. The differences with our paper are the following. First, these papers focus on mergers and thus do not analyze the effects of entry. Second, in the supergame literature if the discount factor is close enough to 1 (one) the monopoly price is sustainable (perfect collusion). But this literature does not ask whether the monopoly price is actually a desirable outcome. Here our assumption of price leadership comes in. From all the sustainable outcomes, the price leader chooses the one that maximizes its profits. That will not be the perfect collusion outcome if that outcome forces the leader to share the market with too many firms. Hence, for each choice of price leader(s) we get a unique outcome. We then ask how this outcome changes with entry, efficiency gains and mergers. Third, in the supergame literature if the (expected) duration between sales increases, it becomes harder to discipline firms and the (collusion) price decreases. Here, a longer (expected) duration between sales makes it more likely that firms

to c. Then there is a range for c such that firm 1 has higher profits under Bertrand competition than under Cournot competition.

can react to each other's price changes before the customer arrives and hence prices tend to be higher. Finally, a disadvantage of the supergame approach is the multiplicity of equilibria. Here we get a unique equilibrium for each choice of price leaders, but different price leaders lead to different outcomes. Once we extend the model to allow for price announcements, we get a unique (endogenous) price leader and thus a unique equilibrium outcome.

Second, the result that entry by a new firm into the market can raise the equilibrium price, is reminiscent of results by Amir and Lambson (2000), Bulow and Klemperer (2002), Rosenthal (1980) and Stiglitz (1989). These papers formalize the price increasing effect of entry in the following ways. Amir and Lambson (2000) assume increasing returns to scale in a Cournot setting. Hence, entry by reducing each firm's output level, increases costs and thereby the price. Bulow and Klemperer (2002) consider a common value setting and derive conditions under which the winner's curse gets worse if there are more bidders.[3] In Rosenthal (1980) firms are active on a number of markets and entry in one market affects firms' behavior in other markets. In Stiglitz (1989) entry by new firms makes the market less transparent. This increases monopoly power for each firm and hence leads to higher prices. None of these papers models the crowding effect of entry.

Third, the pricing game introduced here allows firms (with some probability) to react to opponents' price changes before the consumer arrives. As other dynamic games, this solves the Bertrand paradox. This paradox starts from the observation that Bertrand competition in a duopoly with two firms that produce a homogenous good with the same constant marginal costs yields a price equal to marginal costs. This is called a paradox because it seems counterintuitive that two firms are sufficient to get the perfect competition outcome of price equal to marginal costs. Surely, two firms competing in prices must be able to get to an outcome with a strictly positive price cost margin. Further, this Bertrand game features the following discontinuity.

[3]In their auction setting they find that entry by new bidders can reduce the expected revenue of the auction. This is equivalent to the result that entry leads to higher prices in a market context.

With zero entry costs, we get the perfect competition outcome. If entry costs are positive (no matter how small) we get the monopoly outcome. Both these shortcomings of the Bertrand outcome are resolved in the pricing game here. Two equally efficient firms will not fight because being nice yields higher profits. Moreover, the equilibrium price is continuous in the entry cost. The result that a dynamic extensive form pricing game removes the Bertrand paradox has already been noted by Maskin and Tirole (1988) and Farm and Weibull (1987). The former paper analyzes the price dynamics of a symmetric duopoly game, where in each period only one firm can adjust its price. Maskin and Tirole derive conditions under which, for instance, an Edgeworth cycle is a Markov perfect equilibrium. Our paper does not focus on price dynamics but on comparative statics with respect to the industry cost distribution. The Farm and Weibull paper has a pricing process that is similar to the one presented here, but they assume that all firms are symmetric. Therefore the monopoly price is always an equilibrium in their paper while this is not the case here.

This paper is organized as follows. Section 2 introduces the model of the pricing game and its price leadership equilibrium outcome. Section 3 derives the implications for competition policy by analyzing the effects on the equilibrium price of entry, efficiency gains and mergers for an exogenously given set of price leaders. Section 4 extends the model to endogenize the identity of the price leader. Finally, section 5 concludes the paper. Proofs of all results can be found in the appendix.

2. The model

Consider an industry with N (potential) firms. Each firm produces with constant marginal costs c_i and we order firms such that $c_1 \leq c_2 \leq ... \leq c_N$. Firms produce a homogenous good and demand is given by $X(p)$ where p denotes the price. Let p_i^m denote the monopoly price for firm i. We assume that $p_i^m \equiv \arg\max_p X(p)(p - c_i)$ is well defined and that $X(p)(p - c_i)$ is concave in p for each $i \in \{1, 2, ..., N\}$. A firm that enters the market has to invest a sunk cost $f \geq 0$ to be able to produce and sell.

We assume the following dynamic structure for the game. The firms compete to sell to one consumer. Every period firms can enter or leave the market and change the price they charge. Firms do not know in which period the consumer arrives. After firms have fixed their prices, there is a probability $q \in \langle 0, 1 \rangle$ that the consumer arrives in that period. If the consumer arrives, he chooses randomly one of the firms charging the lowest price in that period and buys the quantity given by his demand function $X(.)$ and the game ends. If the consumer does not arrive, firms can again decide to change prices, enter or leave the industry etc. Note that with $q = 1$ this game boils down to a standard Bertrand pricing game and the Nash equilibrium price equals $p = c_2$. Also in the dynamic game with $q < 1$ it is still the case that $p = c_2$ is a Nash equilibrium price if $f = 0$.

Instead of this equilibrium, however, we consider an equilibrium where some firms act as price leader and set a price p above c_2. To make sure firms do not deviate from this equilibrium with a price above c_2, firms play trigger strategies. If a firm deviates from the equilibrium price p (and the consumer does not arrive in that period), all firms play the static Nash equilibrium from then on. If a firm deviates, it deviates with a price just below p and gets a profit (almost) equal to $(p - c_i)X(p)$. Routine manipulation yields that the incentive compatibility constraints can be written as follows.

$$\begin{aligned} \frac{X(p)}{n}(p - c_1) &\geq \frac{r+q}{1+r}X(p)(p - c_1) + \frac{1-q}{1+r}X(c_2)(c_2 - c_1) \\ \frac{X(p)}{n}(p - c_i) &\geq \frac{r+q}{1+r}X(p)(p - c_i) \end{aligned}$$

for $i > 1$, where n denotes the number of firms charging the price p and r denotes the discount rate. Note that only the most efficient firm 1 can make positive profits in the static Nash equilibrium (if $c_2 > c_1$). Hence, the profits that a firm gets in equilibrium should be greater than a weighted average of the deviation profit and the static Nash profit with weights $\frac{r+q}{1+r}$ and

$\frac{1-q}{1+r}$ resp. The more likely it becomes that the consumer arrives this period (i.e. the higher q), the more this game converges to a static Bertrand game. Therefore it becomes harder to sustain prices $p > c_2$.

If the price leaders decide on a price p, how many firms will there be in the market? We define the function $n(p)$ as follows

$$n(p) \equiv \max\{i | qX(p)(p - c_i) - f > 0\}$$

In words, for firm $n(p) + 1$ it is the case that $qX(p)(p - c_{n(p)+1}) - f \leq 0$. This firm cannot profitably deviate from an equilibrium where a price p is charged and where it is supposed to stay out of the market. By deviating and entering (at cost f) with a price just below p there is a probability q that the consumer arrives in which case this deviating firm has the lowest price and sells $X(p)$. If the consumer does not arrive, all firms switch to the static Nash equilibrium in which case this firm makes zero profits. The same reasoning implies that at a price p firm $n(p)$ cannot be kept out of the market. Hence, $n(p)$ is the number of firms in the market at price p.

Using this notation, we can define the set of prices $\mathbb{P}$ which are incentive compatible (IC) for firm 1.

$$\mathbb{P} = \left\{ p | \frac{1}{n(p)} \geq \frac{r+q}{1+r} + \frac{1-q}{1+r} \frac{X(c_2)(c_2 - c_1)}{X(p)(p - c_1)} \right\}$$

The following lemma shows that a price which satisfies the incentive compatibility constraint for the most efficient firm 1 satisfies all incentive compatibility constraints.

Lemma 1 *For each $p \in \mathbb{P}$ it is the case that*

$$\frac{X(p)}{n(p)}(p - c_i) \geq \frac{r+q}{1+r} X(p)(p - c_i)$$

for $i = \{2, 3, ..., n(p)\}$ and firm $n(p)$ can profitably enter in equilibrium.

The intuition is twofold. First, the most efficient firm has the biggest incentive to increase output by charging a lower price. Second, the most efficient firm can still make a positive profit in the static Nash equilibrium while all other firms make zero profits in the Bertrand outcome. Hence if the most efficient firm 1 does not want to deviate from p then no other firm will. It is routine to verify that the definition of $n(p)$ together with the incentive compatibility constraint of $n(p)$ implies that $n(p)$'s equilibrium profits are strictly positive.

The only sustainable prices in the industry are the IC ones in the set $\mathbb{P}$. The question is, which of these prices will be chosen? We assume that a number of firms in the industry act as price leader. They choose the price p which solves

$$\max_{p \in \mathbb{P}} \sum_{i=1}^{n(p)} \alpha_i \frac{X(p)}{n(p)} (p - c_i)$$

In words, the price leaders maximize a weighted sum of profits of the firms in the industry, where firm i has weight $\alpha_i \geq 0$. We normalize these weights such that they add up to 1, $\sum_{i=1}^{N} \alpha_i = 1$. If firm i is the (only) price leader in the industry, it chooses the price that maximizes its own profits (taking other firms' entry behavior into account) and we have $\alpha_i = 1$ and $\alpha_j = 0$ for all $j \neq i$. If there is more than one price leader in the industry, they maximize the sum of their profits where α_i can be interpreted as reflecting the bargaining power of firm i among the price leaders. A firm with higher weight α_i gets a price that is closer to the one that maximizes its own profits. Note that it may happen that one of the price leaders does, in fact, not enter if $\alpha_i > 0$ but $i > n(p)$ at the chosen price p.

The objective function is a reasonable one in the case where there is only one price leader ($\alpha_i = 1$). For the case with two or more price leaders it is, admittedly, a rather crude formalization of the group's objective function. Let us mention three points on this issue. First, a Pareto inferior outcome for the price leaders cannot maximize the weighted sum of their profits. Hence such outcomes are excluded by the chosen objective function, which seems reasonable. Second, it turns out to be a convenient functional form to prove the main results below. Finally, we will derive conditions under which the main results here also hold in case of Nash bargaining by the leaders and even if we only use Pareto efficiency to rank the possible outcomes. In this sense, the results are robust to the way the group bargaining between the leaders is formalized.

The main assumption implicit in the chosen objective function of the price leaders is that side payments are not allowed. If side payments would be possible, only the most efficient firm would produce and distribute its profits among the other firms in the industry. As usual in the literature, such transfers are ruled out for two reasons. First, side payments between firms to stick to an agreement are illegal. Further, such transfers will leave physical evidence for the competition authority to prove explicit collusion. This is often too risky for firms. Second, if only the most efficient firm produces in equilibrium, any firm can come up and claim part of the profits by asserting that it can produce in the industry. Since it does not have to produce in equilibrium, the assertions cannot be verified. Hence if a non-producing firm's efficiency level is not verifiable, such a scheme with side payments cannot be implemented.

We first take the vector $\alpha = (\alpha_1, ..., \alpha_N)$ as given. In the literature on price leadership, the price leader is determined in an industry by the following factors. In Rotemberg and Saloner (1990) the better informed firm (about demand conditions in the industry) becomes price leader. In Kirman and Schueller (1990) a firm is price leader in his home country (in an international context where a country has at most one home producer of the product). In Deneckere and Kovenock (1992) the firm with the biggest capacity is price leader and in Deneckere, Kovenock and Lee (1992) the firm with the largest segment of loyal consumers is price leader. In the simple model we consider, none of these considerations is present but they could be added to determine the vector α. To illustrate, $\alpha_i = 1$ could happen because firm i has the most accurate information about demand conditions. We focus on the effects of efficiency gains, entry and mergers for given α. Lateron we consider a model where α is endogenized.

3. Competition policy

In this section we analyze the effects of entry, efficiency gains and mergers on the equilibrium price chosen by the price leaders. Clearly, the identity of the price leaders, formalized above by the vector α, affects the equilibrium price. However, the main results that entry and efficiency gains can lead to higher prices hold for any α. Note that entry can be viewed as a special case of efficiency gains. A firm that could not enter before, may be able to enter after it has experienced an efficiency gain. However, an efficiency gain by firm i does not always lead to (actual) entry as the price leader(s) may decide to fight i to keep it out of the market. Also if firm i does enter, it may happen that the price leaders will respond by increasing their price such that other firms can enter as well while they did not experience efficiency gains. When we discuss the effects of entry below, we mean entry by a firm that is either new or experienced an efficiency gain.

We derive the results by comparing the relative profitability for the price leaders of two arbitrary prices p and $p' > p$. Define $n = n(p)$, $n' = n(p') > n$ and $\nu = n' - n$. In words, ν is the number of additional firms that are active at the high price p' compared to the low price p. Then the following function compares the relative profitability of the two prices

$$\Delta(p,p') = \left[\sum_{i=1}^{n} \alpha_i X(p)(p - c_i) - \frac{n}{n+\nu} \sum_{i=1}^{n'} \alpha_i X(p')(p' - c_i)\right]$$

Note that the correct comparison of the relative profitability is Δ/n. But there is only a switch from one price to the other if Δ changes sign. Thus scaling factors are not relevant and this specification of Δ turns out be convenient.

We say that an efficiency gain by firm j makes the low (high) price more attractive for the price leaders if it raises (reduces) Δ. However, raising or reducing Δ is not sufficient to get a price change for two reasons. First, a price change only happens if Δ changes sign. An increase in Δ is necessary to switch from negative to positive, but it is not sufficient. Second, we do not consider the effects of entry or efficiency gains on the set of IC prices $\mathbb{P}$. This is done because of three reasons. First, the effect of entry on the set of sustainable prices is well known from the supergame literature. With symmetric firms, an increase in the number of firms makes collusion harder to sustain by raising the gain of deviating. This effect is present here as well. But, note that with asymmetric firms it is not clear that entry necessarily shrinks the set $\mathbb{P}$. To see this, consider an industry where $c_1 < c_2$ and where a new firm e enters with costs $c_e = c_1$. Now a deviation from the equilibrium by firm 1 leads to Bertrand Nash profits equal to zero for this firm, whereas these profits were strictly positive before entry. Hence it can happen that entry enlarges the set of IC prices through this effect. Second, by ignoring the effect of efficiency gains on the set $\mathbb{P}$ the analysis becomes more transparent as we focus on the effects that are new here compared to the existing literature on repeated games. Finally, lemma 2 in the appendix shows that if efficiency gains lead to a switch in the sign of $\Delta(p, p')$ then there exist values of r and q small enough such that p and p' are indeed IC given that profits in each case exceed the Bertrand Nash profits for the most efficient firm.

Summarizing, in this section we simplify the analysis by deriving necessary conditions for a change in price but not sufficient conditions. In an example, however, it is straightforward to keep track of the sufficient conditions as well. Below we give an example where efficiency gains indeed lead to higher prices.

3.1. Efficiency gains and entry

Consider the effects of an efficiency gain for firm j from c_j to $\hat{c}_j < c_j$. We introduce the following notation. The expression $c_j \succ p$ denotes the case where firm j cannot profitably enter in equilibrium at price p and $p \succ c_j$ denotes the case where j can enter at price p. Note that for $f = 0$, the ordering induced by $\succ$ is identical to $>$ but with $f > 0$ these orderings are no

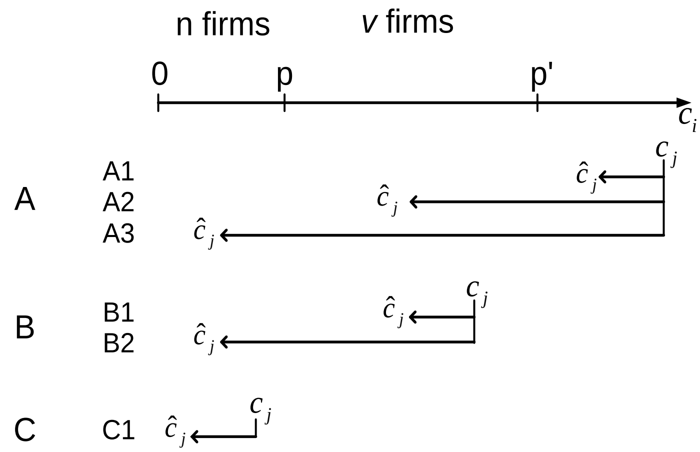

Figure 1: *Six ways in which firm j can reduce its marginal costs from c_j to $\hat{c}_j$ for given efficiency distribution and prices p and $p' > p$.*

longer the same. That is, with $f > 0$ it is not the case that $p > c_i$ implies that i can profitably enter since the entry costs f have to be recouped as well.

We are interested in the effect of the efficiency gain on $\Delta(p, p')$. To simplify notation, we drop the arguments p and p' and let Δ denote the value before the efficiency gain and $\hat{\Delta}$ the value after the efficiency gain. We first compare Δ and $\hat{\Delta}$ for the case where j is not among the price leaders ($\alpha_j = 0$). Then we characterize what changes if $\alpha_j > 0$. To describe the effect of efficiency gains, we distinguish a number of cases. These are depicted in figure 1 and enumerated in table 1. The following result summarizes the effect of an efficiency gain on Δ.

Proposition 1 *If $\alpha_j = 0$ then an efficiency gain for firm j from c_j to $\hat{c}_j \in [0, c_j\rangle$ does not make the low price more attractive ($\hat{\Delta} \leq \Delta$) unless we are in the situation where $c_j \succ p' \succ \hat{c}_j \succ p$.*

Rather surprisingly we do not find that efficiency gains always lead to lower prices. In fact, there is only one case where the efficiency gain does lead to lower prices. That happens when firm j could not enter at the high price p' before the efficiency gain and after the efficiency gain j can enter at p' but not at the low price p. This makes the low price relatively more attractive for the price leaders. By charging the low price, the price leaders can keep an additional entrant (j) out of the market. This makes the low price more attractive after the efficiency gain. In all other cases, the comparison between p and p' either does not change or the higher price becomes more attractive after entry. This happens in the cases A3 and B2 in table 1. The intuition for this upward pressure on the price is the following crowding effect.

If by pricing low (i.e. p instead of p') you can remove, say, $\nu = 4$ opponents from the market, then it may pay to fight if the number of firms is reduced from 14 to 10. But if the same price reduction leads to 100 opponents instead of 104, this price reduction is less attractive and it is more profitable to be nice and charge the high price p'. This is what happens in case A3. At price p firm j still enters after the efficiency gain. Hence the reduction in opponents due to the price reduction $p' - p$ has to be shared with an additional firm. This crowding makes the low price less attractive. In case B2 there is the following additional effect working in the direction of higher prices. Before the efficiency gain firm j could be removed from the market by charging the low price. After j's efficiency gain, the firm enters at the low price as well. Hence fewer firms are deterred by charging the low price after the efficiency gain, making the low price less attractive.

Table 1: different cases of efficiency gains: $c_j \to \hat{c}_j$

Figure 1	Case		Effect
A	A1	$c_j > \hat{c}_j \succ p' > p$	$\hat{\Delta} = \Delta$
	A2	$c_j \succ p' \succ \hat{c}_j \succ p$	$\hat{\Delta} > \Delta$
	A3	$c_j \succ p' > p \succ \hat{c}_j$	$\hat{\Delta} < \Delta$
B	B1	$p' \succ c_j > \hat{c}_j \succ p$	$\hat{\Delta} = \Delta$
	B2	$p' \succ c_j \succ p \succ \hat{c}_j$	$\hat{\Delta} < \Delta$
C	C1	$p' > p \succ c_j > \hat{c}_j$	$\hat{\Delta} = \Delta$

In proposition 1 we have assumed that the firm experiencing the efficiency gain is not among the price leaders in the industry. The following proposition summarizes what additional effect we get if $\alpha_j > 0$ compared to the case with $\alpha_j = 0$ in proposition 1. In other words, the next proposition only gives a partial effect. Adding the effects in the two propositions yields the total effect if price leader j experiences a gain in efficiency. More formally, proposition 2 characterizes $(\hat{\Delta} - \Delta)_{\alpha_j > 0} - (\hat{\Delta} - \Delta)_{\alpha_j = 0}$. Adding this to the result in table 1, which is $(\hat{\Delta} - \Delta)_{\alpha_j = 0}$ yields the overall effect $(\hat{\Delta} - \Delta)_{\alpha_j > 0}$.

Proposition 2 *Firm j being among the price leaders ($\alpha_j > 0$) tends to raise the price (compared to $\alpha_j = 0$ in proposition 1) if $p' \succ \hat{c}_j \succ p$. If $p \succ \hat{c}_j$, $X(p)(p - \hat{c}_j) \geq \frac{n+1}{n+1+\nu} X(p')(p' - \hat{c}_j)$ and ν big enough then j being a leader tends to reduce the price (compared to proposition 1).*

If firm j is among the price leaders, we get the following downward pressure on the price. As firm j's costs go down, firm j's preference for a low price goes up. More efficient firms tend to prefer higher output levels and thus lower prices. However, this is not always the case. First, if after the efficiency gain firm j can enter at the high price but still not at the low price, then clearly the efficiency gain leads to an increased preference for the high price among the leaders. This happens in cases A2 and B1 where $p' \succ \hat{c}_j \succ p$. If the efficiency gain is big enough to ensure entry at the low price, there are still two caveats. First, it may be that at cost level $\hat{c}_j$, firm j prefers the high price. This happens if $X(p)(p - \hat{c}_j)/(n+1) < X(p')(p' - \hat{c}_j)/(n+1+\nu)$. Second, in case B2 firm j can only enter at the high price p' before the efficiency gain. After the efficiency gain, j can enter at the low price as well. Yet, its profit at the high price is bigger now than it was before the efficiency gain. The latter effect works in the direction of raising Δ. However, if the market at the high price has to be shared with many additional opponents (i.e. ν is high) then this effect is small. The dominant effect is then that j with the low cost $\hat{c}_j$ makes a profit at the low price, while it could not produce at p before the efficiency gain. This makes the low price relatively more profitable after the efficiency gain.

Table 2: additional effects if $\alpha_j > 0$ compared to $\alpha_j = 0$ in table 1

Figure 1	Case	Effect	Condition
A	A1	$(\hat{\Delta}-\Delta)_{\alpha_j>0} - (\hat{\Delta}-\Delta)_{\alpha_j=0} = 0$	
	A2	$(\hat{\Delta}-\Delta)_{\alpha_j>0} - (\hat{\Delta}-\Delta)_{\alpha_j=0} < 0$	
	A3	$(\hat{\Delta}-\Delta)_{\alpha_j>0} - (\hat{\Delta}-\Delta)_{\alpha_j=0} \lesseqgtr 0$	iff $X(p)(p-\hat{c}_j) \lesseqgtr \frac{n+1}{n+\nu+1} X(p')(p'-\hat{c}_j)$
B	B1	$(\hat{\Delta}-\Delta)_{\alpha_j>0} - (\hat{\Delta}-\Delta)_{\alpha_j=0} < 0$	
	B2	$(\hat{\Delta}-\Delta)_{\alpha_j>0} - (\hat{\Delta}-\Delta)_{\alpha_j=0} > 0$	for ν big enough
C	C1	$(\hat{\Delta}-\Delta)_{\alpha_j>0} - (\hat{\Delta}-\Delta)_{\alpha_j=0} > 0$	

Summarizing, the general picture is as follows. An efficiency gain by a firm j that is not one of the price leaders tends to reduce the price if it pays for the leaders to fight j to keep it out of the market. If j has become so efficient that fighting it becomes too costly, it is better to be nice and the price leaders may raise the price due to the crowding effect. If firm j is among the price leaders, the efficiency gain leads to an increased preference for lower prices. The bigger j's efficiency gain, the stronger the preference for a low price. This effect works against the former effect where j is not a price leader. Hence, in general, the overall effect is ambiguous.

Up until now we have focussed on necessary conditions for price changes due to efficiency gains. Now we consider an example showing that lower costs can lead to higher equilibrium prices taking all effects into account. We assume in the example that the most efficient firm acts as (sole) price leader. Below we present a model where we endogenize the identity of the price leader. The outcome is then that indeed the most efficient firm ends up as price leader.

Example 1 *Consider two industries where in both the demand function is given by* $X(p) = 1 - p$. *In industry* I *the (constant marginal) cost distribution of the three firms is* $c_1^I = 0, c_2^I = 0.35, c_3^I = 0.4$ *and in industry* II *it is* $c_1^{II} = 0, c_2^{II} = 0.1, c_3^{II} = 0.4$. *The entry cost equals* $f = 0$ *in both industries. We assume that in both industries the most efficient firm acts as price leader* $(\alpha_1 = 1)$. *The equilibrium price is determined as follows. In industry* I, *firm* 1 *considers three possible prices:* 0.35 *and be the only firm in the market,* 0.4 *and share the market with firm* 2 *and thirdly a price equal to the monopoly price* 0.5 *and share the market with firms* 2 *and* 3. *The following inequalities show that* $p^I = 0.35$ *is the equilibrium price. Note that firm* 1*'s* *profits equal* $\frac{X(p)}{n}(p - c_1) = \frac{1-p}{n}p$.

$$\frac{1-0.35}{1}0.35 > \frac{1-0.4}{2}0.4 > \frac{1-0.5}{3}0.5$$

In industry II*, however, we have the following inequalities.*

$$\frac{1-0.4}{2}0.4 > \frac{1-0.1}{1}0.1 > \frac{1-0.5}{3}0.5$$

Hence the equilibrium price in industry II *equals* $p^{II} = 0.4$ *which is higher than the equilibrium price in industry* I*. The intuition is that it is too costly for firm* 1 *to keep firm* 2 *out of the market in industry* II*, while this is not very costly in industry* I*. Hence firm* 1 *is more aggressive in industry* I *and the equilibrium price is lower than in industry* II*. Finally, we can find values for* r *and* q *low enough such that these prices are indeed IC. Note that both Cournot and Bertrand competition predict that the price in industry* II *is lower than in* I*. This illustrates the difference between these standard models and the intuition introduced here.*

The results derived above that cost reductions or entry can lead to higher equilibrium prices are in such contrast with conventional wisdom that one can ask whether the theory presented here has empirical support. It is beyond the scope of this paper to show empirically that the theory presented here is relevant in some industries. However, we do want to illustrate that the price reducing effects of efficiency gains and entry are not so strong empirically as standard Cournot and Bertrand models suggest. For instance, Geroski (1989) estimating the effect of entry on price cost margins concludes that 'entry shows an enormous amount of variation across industries and over time, while margins are rather predictable over time for any given industry'. Put differently, more entry is not always associated with lower price cost margins (and Geroski corrects for the reverse causality problem). Klette (1999) finds no significant relationship between mark ups and industry concentration nor between mark ups and import penetration.

Concerning the effects of efficiency gains on the price, consider the analysis of Ashenfelter and Sullivan (1987) of the effects of changes in the excise tax on cigarettes on the cigarette price. They write 'Our failure to uniformly find that excise tax increases (decreases) act to increase (decrease) cigarette prices ... calls into question virtually any hypothesis about firm behavior in this industry'. Indeed, the standard Cournot and Bertrand models can shed no light on such findings. However, the model introduced here can give intuition for this result. Rewriting the equation for $\Delta(p, p')$ above to introduce excise taxes, one gets the following comparison between two prices p and $p' > p$ with p' the optimal price before the rise in excise tax t and p an arbitrary price below p':

$$\Delta\left(p, p'; t\right) \equiv \sum_{i=1}^{n} \alpha_i X\left(p + t\right)\left(p - c_i\right) - \frac{n}{n+\nu} \sum_{i=1}^{n+\nu} \alpha_i X\left(p' + t\right)\left(p' - c_i\right)$$

The following result gives a sufficient condition for the rise in t to make the lower price p relatively more profitable compared to p'.

Corollary 1 *If $X''(p) \leq 0$ for all $p \geq 0$ then $\frac{d\Delta(p,p';t)}{dt} > 0$.*

The idea of the proof is that choking off demand (by raising t) is particularly bad for profits if the price cost margin is high. Hence the rise in t makes the lower price p more profitable compared to the high price p'. The assumption that the demand function is (weakly) concave excludes the case where high prices lead to smaller losses in output due to an increase in the price by $dt > 0$.[4]

This clearly does not constitute a proof that the price leadership model analyzed in this paper is the explanation for the findings cited above. Yet, it does suggest that there are empirical observations that are not easily understood using standard I.O. models. The model introduced here can help to understand these phenomena.

[4]If $X''(p) > 0$, a high price would make demand less responsive to the tax rise. This would work in the direction of making p' more profitable after t went up.

3.2. Decision rules for the group of price leaders

The most surprising result above is that efficiency gains or entry can lead to higher prices. We show here that this result does not depend on the assumption that the price leaders maximize a weighted average of their profits. In particular, we consider two alternatives: Nash bargaining by the price leaders and the Pareto efficiency criterion. In both cases we still find the results above. This will further illustrate the mechanisms driving these results.

Under Nash bargaining we make the following assumptions. We define the Nash bargaining function as

$$\mathcal{B}(p) \equiv \ln \left[\left(\frac{X(p)}{n(p)}(p - c_1) - X(c_2)(c_2 - c_1) \right)^{\alpha_1} \left(\frac{X(p)}{n(p)}(p - c_2) \right)^{\alpha_2} \ldots \left(\frac{X(p)}{n(p)}(p - c_L) \right)^{\alpha_L} \right]$$

where $\alpha_i \in [0, 1]$ denotes firm i's bargaining power, $\sum_{i=1}^{L} \alpha_i = 1$ and $L \leq N$ is the least efficient firm in the set of leaders (more formally, $L = \max\{i | \alpha_i > 0\}$). The leaders then choose the price p to maximize the (logarithm of the) product of profits minus disagreement utilities, where we assume that the leaders play the Bertrand equilibrium in case of disagreement (outside option). Note that the profits for firms $i = 2, 3, \ldots$ are zero in the disagreement point. Also note that we assume that the bargaining takes place after the entry cost f has been sunk. With the objective function based on a weighted average of the leaders' profits, it is possible that one of the leaders cannot enter in equilibrium. With Nash bargaining this is not optimal. If firm L cannot enter at a price p and its profits are zero, we find $\mathcal{B}(p) = -\infty$. Hence there are prices $p > c_L$ which yield a higher value for the Nash bargaining function.

Let $\Delta(p, p')$ now be defined as

$$\Delta(p, p') = \mathcal{B}(p) - \mathcal{B}(p')$$

with $p' > p$. We consider the effects of the following two situations (corresponding to B2 and A3 resp. in figure 1). First, there is a firm j with $p' \succ c_j \succ p$ that experiences an efficiency gain such that $p' > p \succ \hat{c}_j$. Second, there are $y \geq 1$ firms that initially could not enter at the high price p' which can now enter at the low price p. Because of the assumption of Nash bargaining, these firms (j in the former case and the y new firms in the latter case) do not belong to the group of price leaders (if they were, both prices p and p' are irrelevant as they could not have been chosen initially). As above, we use the following notation $\Delta = \Delta(p, p')$ before the efficiency gain and $\hat{\Delta}$ denotes the relative values of the Nash product at p and p' after the efficiency gain. With the case of entry, we define

$$\begin{aligned} &\Delta(p, p'; y) \equiv \\ &\ln\left[\left(\frac{X(p)}{n(p)+y}(p-c_1) - X(c_2)(c_2-c_1)\right)^{\alpha_1}\left(\frac{X(p)}{n(p)+y}(p-c_2)\right)^{\alpha_2}...\left(\frac{X(p)}{n(p)+y}(p-c_L)\right)^{\alpha_L}\right] \\ - \ &\ln\left[\left(\frac{X(p')}{n(p')+y}(p'-c_1) - X(c_2)(c_2-c_1)\right)^{\alpha_1}\left(\frac{X(p')}{n(p')+y}(p'-c_2)\right)^{\alpha_2}...\left(\frac{X(p')}{n(p')+y}(p'-c_L)\right)^{\alpha_L}\right] \end{aligned}$$

Proposition 3 *If an efficiency gain by firm j (with $a_j = 0$) causes $n = n(p)$ to go up by 1 while $n(p') = n + \nu$ remains unchanged then*

$$\hat{\Delta} < \Delta$$

If $\alpha_1 > 0$ then we assume that c_2 is close enough to c_1. Under this assumption an increase in y causes Δ to fall

$$\frac{\partial \Delta(p, p'; y)}{\partial y} < 0$$

The intuition for these results is as follows. First, if due to an efficiency gain for firm j it is the case that $n = n(p)$ increases while $n(p') = n(p) + \nu$ is unchanged, then clearly the Nash bargaining function $\mathcal{B}$ at price p is reduced after the efficiency gain while the function $\mathcal{B}$ at p' is unchanged (note that we use here that $\alpha_j = 0$). Hence the higher price becomes relatively more attractive after the efficiency gain. When the number of firms $n(p) + y$ that can produce at price p go up (via entry by $dy > 0$ new firms), the crowding effect makes the high price p'

relatively more attractive. We need an additional condition here for the following reason. Since we are comparing two arbitrary prices p and $p' > p$, it may be the case that

$$\frac{X(p)}{n(p)+y}(p-c_1) > \frac{X(p')}{n(p')+y}(p'-c_1) > X(c_2)(c_2-c_1)$$

Then there is a value of y such that

$$\frac{X(p')}{n(p')+y}(p'-c_1) = X(c_2)(c_2-c_1)$$

in which case the Nash bargaining function equals $-\infty$ at the high price. Clearly, in such a case an increase in y will not make the high price relatively more attractive for the leaders (if firm 1 is among the leaders). By assuming that (for given y) the difference in cost levels $c_2 - c_1$ is small enough, we can exclude this case.

Now turn to the Pareto criterion. We assume that the two prices p and $p' > p$ that we consider are both below the monopoly price for firm 1. Then by the assumption made above that $X(p)(p - c_1)$ is concave in p we find that $X(p')(p' - c_1) > X(p)(p - c_1)$. Now we denote the number of firms that can be active at price p by $n(p) = n + y$ while the number of firms that can be active at the high price p' is given by $n(p') = n + y + \nu + z$. Then $dy > 0$ and $dz = -dy$ corresponds to dy firms experiencing efficiency gains from c_j satisfying $p' \succ c_j \succ p$ to $\hat{c}_j$ satisfying $p' > p \succ \hat{c}_j$, while $dy > 0$ and $dz = 0$ corresponds to entry by new firms that could initially not enter at the high price p'.

We say that for given values of y and z the high price Pareto dominates the low price, if it is the case that

$$\frac{X(p')}{n+y+\nu+z}(p'-c_i) > \frac{X(p)}{n+y}(p-c_i)$$

for all leaders i. We can show the following result.

Proposition 4 *Starting from $y = z = 0$, there exists $\bar{\delta}_y \leq \nu$ such that for all $dy \geq \bar{\delta}_y$ and $dz = -dy$ it is the case that the high price p' Pareto dominates the low price p. Again starting from $y = z = 0$, there exists $\bar{\delta}_y$ such that for all $dy > \bar{\delta}_y$ and $dz = 0$ it is the case that the high price Pareto dominates the low price.*

In the first case (where $dz = -dy$) it is clear that for $y = \nu$ the result holds. If raising the price from p to p' does not lead to additional firms with whom the market has to be shared, then clearly the higher price is Pareto dominant. In the second case, the crowding effect makes the high price Pareto dominant. If enough new firms enter at the price p, the price reduction $p' - p$ is no longer profitable if it only removes ν firms from the market. Hence the most surprising result above that efficiency gains and entry lead to higher prices is robust to the objective function chosen for the price leaders.

3.3. Mergers and joint dominance

With the model above we get the following results on mergers and joint dominance. We assume that a merger between firms i and j gives the merging firms an efficiency gain, in particular the merged firm has efficiency level $c_{i\&j} < c_i, c_j$. Further, a merger does not affect the number of outlets of the merging firms. If there are n active firms in the market and two firms merge, the merging firms get a market share of $2/n$ instead of $1/(n-1)$.[5] For instance, if the firms involved have well known brand names or specific locations for their shops, this is a reasonable assumption.

If neither of the two merging firms is a price leader, the main effect of the merger is that two firms experience cost reductions. Only in the case where it becomes optimal for the leaders

[5]If one would make the assumption that the market share for the firm after the merger equals $1/(n-1)$, this has two implications. First, bigger efficiency gains are needed to make a merger profitable for the merging firms, since the efficiency gain has to compensate the loss in market share. Second, there is the effect that a merger leads to less firms in the industry which tends to reduce prices because the crowding effect is reduced.

to price both outlets of the merged firm out of the market, will there be a price reduction after the merger. In the other cases, where the efficiency gain for the merging firms is so big that it becomes too costly for the price leaders to keep these firms out of the industry, a price reduction is ruled out. In such a case, if the price changes, it will go up. Hence, this is a formalization of the joint dominance doctrine. As firms converge to the efficiency levels of the price leaders, it becomes too costly to fight and the price tends to increase.

However, there is also a force going against this price increase if the merging firms are among the price leaders.[6] In that case, as suggested by proposition 2, there is a tendency for the price to go down since more efficient price leaders prefer higher output levels and thus lower prices. Hence, if a competition authority wants to argue that a merger in an industry leads to joint dominance and hence higher prices, the clearest case can be made if two conditions are satisfied. First, it has to be argued that the merger brings efficiency gains which make the merged firm's efficiency level comparable to that of the price leaders in the industry. This reduces the incentive to fight the merged firm. Second, the merged firm should not be among the price leaders in the industry (or its α should be small). If it were a price leader with strong bargaining power, the increase in efficiency would work in the direction of reducing the equilibrium price.

Here the analysis differs from papers like Compte et al. (2002), Motta (2004) and Vasconcelos (2002). These papers focus on the possibilities to collude perfectly. Then more symmetry in the industry implies that for more values of the discount factor perfect collusion is implementable. However, we argue that it is not always optimal for firms to have perfect collusion since the market then has to be shared with (too) many firms. Allowing for this consideration

[6] There is an issue here what happens to the α's when two firms merge. One might argue that the merged firm increases its clout because of the merger and hence the α's should change to reflect that. However, since the vector α is exogenous here, any assumption on how it should change after a merger is rather ad hoc. In the next section we endogenize α. The main conclusion from that is that the most efficient firm acts as price leader. Hence a merger only leads to a change in price leadership if the merged entity becomes more efficient that the current leader. This will tend to reduce the price, since more efficient firms tend to prefer higher output levels and thus lower prices.

leads to two forces working in the direction of lower prices after a merger which are not present in the mentioned papers. First, it may be optimal to reduce the price to keep the merged firm out of the market. Second, if the merged firm is one of the price leaders, then the increased efficiency gives a preference for lower prices.

4. Endogenous price leader: pre announcements

In case of efficiency gains and mergers, it matters whether the firms involved are (one of the) price leader(s) or not. Above the set of price leaders is exogenous. This section changes the game above to get an endogenous price leader. In particular, here we allow firms to make pre announcements of price changes. That is, a firm can state at the start of a period the price it will charge that period. Other firms can react to such pre announcements. With this game we can formalize the idea that the price leader is the most efficient firm in the industry. Hence only an entrant or firm that experiences efficiency gains such that it becomes more efficient than the current leader, leads to a change in price leadership.

The game we consider has the same dynamic structure as above. Every period there is a probability q that the consumer arrives. If the consumer arrives, he buys from one of the firms charging the lowest price and the game ends. If the consumer does not arrive, the game continues. The timing within a period is changed, since we now allow for pre announcements of prices. In particular, at the start of period $t = 0, 1, 2, \ldots$ firms can pre announce the price they will charge in period t. We consider two cases regarding the commitment value of the price announcements. First, we consider the case where the announced price is a binding maximum price for the firm that announced it. Second, we consider the case where the announcement is not binding at all. The former case is often referred to as a public announcement, the latter case as a private announcement (see, for instance, Motta (2004)). After the announcements have been made, firms decide whether to enter the industry and what price they charge in period t

(if they are active).[7] Then with probability q the consumer arrives. If the consumer does not arrive, the game moves to period $t+1$ and firms can again make announcements about their prices etc.

As said, we first consider the case of public announcements. The announcement is called 'public' to stress that the consumer will know the announcement once he arrives. One can think here of newspaper advertisements quoting the price of a product. Thus, if firm i announces price p_{it}^a for period t, it is not allowed in that period to charge a price above this announcement. In that case the consumer will complain and in most OECD countries the consumer will be entitled to buy the product at the quoted price.[8] However, a firm is free to charge a price below its announced price, as no consumer will complain about that. In other words, the price announcement is only binding as a maximum price. Further, a firm is not obliged to make an announcement. Thus all equilibria above (for different sets of price leaders determined by the vector α) are still equilibria in this game. However, we choose to focus on equilibria where firms only revert to the Bertrand Nash punishment from period $t+1$ onward if a firm undercuts its opponents without announcing it beforehand. In particular, if a firm would have gained the whole market in period t (had the consumer arrived) at a price that was not pre announced that period, then all firms play Bertrand Nash from then onward. Focussing on this class of equilibria yields a unique equilibrium price.

In other words, charging a low price or undercutting previous period's price is not punished in itself. It is only punished if the price was not pre announced. This seems a reasonable class of equilibria to consider for the following reasons. First, it is not unreasonable to assume that firms under implicit collusion allow each other to reduce their prices as long as they announce it beforehand. What firms want to prevent is secret undercutting of the price, there is no

[7]Note that in reality these announcements are often formulated as 'in three months time, I will reduce my price by 10%' where the period (three months in this case) should be long enough for other firms to react to the announcement.

[8]Unless it is obvious that there was a misprint in the advertisement.

(or, at least, less) need to prevent any price change. Second, this system is more flexible than a system which just fixes a price and punishes any deviation from this price. In that case, the firms cannot respond to, for instance, reductions in their own costs or reductions in demand calling for price reductions.[9] Third, such pre announcements of price changes are known to occur in reality. Examples go back to Stigler (1947) which includes examples of pre announcements in the cigarette and anthracite industries. Adams and Brock (2001) describe the price announcements in the US car and airline industries. Recent European antitrust cases where, for instance, trade associations made 'price recommendations' include the Dutch concrete industry, German fire insurers and Belgian manufacturers and importers of tobacco.[10] Finally, as we will see, focussing on this set of equilibria formalizes the intuitive notion that the most efficient firm acts as price leader in an industry.

To characterize the equilibrium price in this case, we first need some notation.

Notation 1 $p_i^* = \arg\max_{p \in \mathbb{P}} \frac{X(p)}{n(p)} (p - c_i)$ *where* $n(p) = \max\{i | qX(p)(p - c_i) - f > 0\}$

In words, firm i chooses from the set of IC prices $\mathbb{P}$ the price that maximizes its profits, taking into account the number of firms that will enter at that price. Choosing from the set $\mathbb{P}$ is reasonable because choosing a price that is not IC for one or more firms can never lead to an equilibrium in the dynamic game. In that case, at least one firm will deviate from the chosen price.

To check subgame perfection, we also need to consider out of equilibrium behavior. To describe this we need some further notation. Let $\mathbb{I}_t$ denote the set of firms that are in the market at time t (i.e. have sunk their entry cost f before time t) and $\mathbb{O}_t$ the set of firms that are out of the market at t in the sense that they have not (yet) sunk f when period

9 It turns out that we have a similar asymmetry as Maskin and Tirole (1988). Price reductions are easier to coordinate than price increases. However, since our paper is concerned with comparative statics in the industry structure and not with price dynamics this issue is left for further research.

10 The cases are Cementhandelaren v. Commission [1972] ECR977, Verband der Sachversicherer v. Commission [1987] ECR405 and Van Landewyck v. Commission [1980] ECR3125.

t starts. Then $\{\mathbb{I}_t | c_i < p\}$ denotes the set of firms that are in the market at time t and have marginal costs below p and $|\{\mathbb{I}_t | c_i < p\}|$ denotes the number of such firms. Similarly, $|\{\mathbb{O}_t | qX(p)(p - c_i) - f > 0\}|$ denotes the number of firms that are outside the market at time t for which the inequality $qX(p)(p - c_i) - f > 0$ holds. Now we can generalize the notation above as follows.

Notation 2 $\bar{p}^*_{it} = \arg\max_{p \in \mathbb{P}} \frac{X(p)}{\bar{n}_t(p)} (p - c_i)$ *where*

$$\bar{n}_t(p) \equiv |\{\mathbb{I}_t | c_i < p\}| + |\{\mathbb{O}_t | qX(p)(p - c_i) - f > 0\}|$$

The main difference between $n(p)$ and $\bar{n}_t(p)$ is the following. If at time t firm i is already in the market then a price p such that $qX(p)(p - c_i) - f < 0$ is not sufficient to keep i out. At any price p with $p > c_i$ firm i will still be active in the market. $\bar{n}_t(p)$ takes this into account while $n(p)$ does not. Hence, the optimal price p^*_i is in general not equal to $\bar{p}^*_{it}$. Further, if the prices are not equal, then the profits $\max_p \frac{X(p)}{n(p)} (p - c_i)$ exceed the profits $\max_p \frac{X(p)}{\bar{n}_t(p)} (p - c_i)$ since at any given price there are (weakly) more firms to share the market with in the latter case.

4.1. Public price announcements

The following proposition describes the subgame perfect strategies that lead to the most efficient firm 1 acting as price leader ($\alpha_1 = 1$). We use the following additional notation: $\mathbf{p}^t_a$ denotes the set of announced prices in period t, p^{t-1} is the lowest price charged in period $t - 1$ and $\tilde{p}^t \equiv \min\{\mathbf{p}^t_a \cup p^{t-1}\}$ denotes the lowest price in the set of announced prices in period t and previous period's price p^{t-1}. Then $\hat{p}^t \equiv \max\{p \in \mathbb{P} | p \leq \tilde{p}^t\}$ denotes the price weakly below $\tilde{p}^t$ that is in the set of IC prices $\mathbb{P}$. The idea here is the following. If the lowest announced price in a period t is not IC, which price should firms charge? In equilibrium this is solved by the assumption that firms charge the highest IC price below the announced price. Finally, we assume that at the start of the game, $t = 0$, the previous period's price is defined as $p^{-1} = +\infty$.

Proposition 5 *If we focus on subgame perfect equilibria where firms are only punished (by reverting to Bertrand Nash from then on) for undercutting last period's price to a price that is not pre announced, the unique subgame perfect equilibrium price equals* $p = p_1^*$.

Strategy profiles leading to this equilibrium outcome prescribe for firm i *in any round* t *of the pricing game to do the following:*

- *If* $p^{t-1} > \bar{p}_{it}^*$ *then pre announce* $\bar{p}_{it}^*$ *otherwise make no pre announcement,*
- *if* $i \in \mathbb{I}_t$ *and*
  - *if* $p^{t-1} \notin \mathbf{p}_a^{t-1} \cup \{p^{t-2}\}$ *then play Bertrand Nash from now on,*
  - *if* $p^{t-1} \in \mathbf{p}_a^{t-1} \cup \{p^{t-2}\}$ *and* $\hat{p}^t > c_i$ *then charge* $\hat{p}^t$*; otherwise charge* c_i*,*
- *if* $i \in \mathbb{O}_t$ *and* $qX(\hat{p}^t)(\hat{p}^t - c_i) > f$ *then enter the industry, otherwise stay out; if* i *enters it charges either* $p = \hat{p}^t$ *or undercuts* $\hat{p}^t$ *whichever yields higher expected discounted profits.*

If all firms adopt the strategy described in the proposition than the lowest announced price at the start of the game ($t = 0$) is p_1^*. This can be seen as follows. No firm is active at $t = 0$ and hence all firms i enter with $qX(p_1^*)(p_1^* - c_i) > f$. That is, $n(p_1^*)$ firms enter the market. For each of these firms the optimal strategy is to charge $p = p_1^*$, because by definition of the set $\mathbb{P}$ it is not optimal for these firms to undercut p_1^*.

Next we check that the strategies described, are indeed optimal and then we check uniqueness. First, consider a firm $i \neq 1$. Can this firm profitably deviate from the strategy above? Clearly, when firm 1 announces p_1^* it cannot be optimal to charge a price above p_1^*; in that case sales will be zero and firm i does better by staying out of the market. If i enters, secretly undercutting p_1^* cannot be optimal since $p_1^* \in \mathbb{P}$. Further, announcing a price $p < p_1^*$ is not optimal either as each firm $i > 1$ (weakly) prefers higher prices than p_1^* (see proof of proposition 5 in the appendix). Hence given that i enters, it is optimal to charge p_1^*. As above, entry is then only profitable for firms $i \leq n(p_1^*)$. Finally, following the punishment strategy (if a firm deviates) is optimal as well, given that all other firms follow it.

Now consider firm 1. Is it optimal for 1 to announce and charge p_1^*? First, if 1 announces p_1^*, it is also optimal to charge p_1^* since $p_1^* \in \mathbb{P}$. Consider the case where 1 announces a price $p < p_1^*$ (note that in this case of public announcements, firm 1 is obliged to charge a price equal to (or below) p). Then all other firms that enter charge this price (or a lower price that is IC if $p \notin \mathbb{P}$). Since the price charged by the other firms is IC, it is optimal for 1 to charge that price as well. But then by definition of p_1^*, profits for 1 must be lower than when announcing and charging p_1^*. Next, consider the case where 1 announces a price $p > p_1^*$. Again all other firms that enter charge this price (or a lower price that is IC if $p \notin \mathbb{P}$). Now it may happen that firm i enters because at this price p (or the lower one in $\mathbb{P}$) it is the case that $qX(p)(p - c_i) - f > 0$. If, further, it is the case that next period's optimal price for firm 1 lies below i's marginal costs, $\bar{p}_1^* \leq c_i$, then it is optimal for i to slightly undercut all other firms. Since it will be out of the market next period anyway, undercutting today is the optimal strategy for i. If $\bar{p}_1^* > c_i$, it may be optimal for i to charge the same price as the other firms. Since announcing and charging a price above p_1^* invites more entry that is subsequently harder to get rid of (because now a price $p \leq c_i$ is needed to push i out of the market), by definition of p_1^* this cannot be optimal for 1 either. Hence the strategies described above are subgame perfect.

The reason why we get a unique equilibrium in this case is the following. Each firm can announce its profit maximizing price and charge it without being punished. Hence, firm 1 can announce its profit maximizing price and charge it. As mentioned, if firm 1 prefers p_1^* above a lower price $p' < p_1^*$ than every firm prefers p_1^* above the lower price. The intuition is that the most efficient firm prefers the highest output level and thus the lowest price. Hence, no firm will undercut (either secretly or openly) the price set by the price leader 1. And no one can stop firm 1 from announcing and charging its profit maximizing price. This is different in the analysis with exogenous price leaders. To illustrate, if $\alpha_2 = 1$ then firm 2 charges $p_2^* \geq p_1^*$ and if any firm deviates from this price, all firms play Bertrand Nash from then onward. Hence even if firm 1 prefers a strictly lower price $p_1^* < p_2^*$, there is nothing it can do; it is optimal for 1 to stick to price p_2^*. In this case, there is no unique equilibrium because every price in the set $\mathbb{P}$ then becomes subgame perfect. We have eliminated all of these equilibria but one by endogenizing the identity of the price leader.

From this analysis it follows that an efficiency gain for firm $j > 1$ leads to a change in the identity of the price leader if after the gain in efficiency firm j is the most efficient firm in the industry. Similarly, entry by a new firm or a merger between firms leads to a new price leader if the new entity is more efficient than the most efficient firm 1. This possibility of a change in price leadership leads to the following downward pressure on prices, compared to the case with exogenous price leadership. If there is a new price leader, it is more efficient than the previous price leader and hence prefers higher output and lower prices. If the price changes, it goes down compared to the case where the price leadership does not change. Note that it is not the case that an efficiency gain or entry that leads to a change in price leadership leads to a lower price. There is still the crowding effect described above which tends to raise prices. Thus we can only state that the change in leadership reduces the price compared to the case where the identity of the leader is exogenously fixed.

4.2. Private price announcements

Now consider the case where the price announcements of firms are private in the sense that they are directed at competitors only. That is, consumers do not know of these private announcements and hence the firm is free to charge a price above the price that it announced. Hence, in this case the price announcements are not binding at all. In this sense they are cheap talk. However, other firms can still react to the announcements.

In this case, the equilibrium strategies described above are no longer optimal. Although, in principle, any firm can deviate from the strategies described above, the hardest case to resolve is where firm 1 deviates.[11] This is the case we focus on here. When the price announcements are not binding, firm 1 can deviate from the equilibrium above in the following way. From period 0 onward, firm 1 announces a price equal to c_2 every period. Given the strategies above, no firm will enter and firm 1 charges the monopoly price every period $p_1^m \equiv \arg\max_p X(p)(p - c_1)$. In order to prevent this deviation, we need to make the following assumption. There exists $\bar{p}^2 \in \mathbb{P}$ such that the following two inequalities are satisfied

$$\begin{aligned} qX(\bar{p}^2)(\bar{p}^2 - c_2) &\geq f & (1) \\ \frac{X(p_1^*)}{n(p_1^*)}(p_1^* - c_1) &\geq \frac{r+q}{1+r}X(p_1^m)(p_1^m - c_1) + \frac{1-q}{1+r}\frac{X(\bar{p}^2)}{2}(\bar{p}^2 - c_1) & (2) \end{aligned}$$

Clearly, if $\frac{X(p_1^*)}{n(p_1^*)}(p_1^* - c_1) > \frac{X(\bar{p}^2)}{2}(\bar{p}^2 - c_1)$ there exist r and q small enough such that these inequalities hold.

If the inequalities hold for $\bar{p}^2$ we can adjust the strategies of the firms such that the strategies described in proposition 5 form again a subgame perfect equilibrium in the case of private announcements. In particular, consider the following change in firm 2's strategy. If in period $t = 0$[12] firm 1 announces a price and charges a higher price in that period, then firm 2 enters in the next period and charges $\bar{p}^2 \in \mathbb{P}$. Since $\bar{p}^2 \in \mathbb{P}$, it is IC for 1 to charge this price as well (if firm 1 did not charge $\bar{p}^2$, firm 2 would revert to Bertrand Nash from then onward). Given that inequality (1) holds and $\bar{p}^2 \in \mathbb{P}$ it follows (in the same way as in lemma 1) that firm 2 can profitably enter at price $\bar{p}^2$. Once 2 enters, both firms charge $\bar{p}^2$. Hence (2) is the IC constraint for firm 1 that makes such announcements unprofitable. If firm 1 plays the equilibrium strategy its profits equal the left hand side of inequality (2). If it deviates and announces a price such

[11] The reason is that if 1 deviates the (weakly) less efficient firm 2 must punish 1. If firm $i > 1$ deviates, firm 1 can do the punishing which is more easily made subgame perfect.

[12] The incentive to announce one price and charge a higher one is highest in period 0 when there are no other firms in the market. Once other firms have entered, they will charge their marginal cost level c_i if 1 announces a price below that. This implies that charging a higher price than the one announced becomes (weakly) less profitable for firm 1.

that no firm enters, its expected discounted profits are a weighted average of the monopoly profit in the first period and the profits $\frac{X(\bar{p}^2)}{2}(\bar{p}^2 - c_1)$ from then onward.

Thus for r and q small enough, the equilibrium in proposition 5 is also an equilibrium with private price announcements.

5. Conclusion

The main idea of this paper is to formalize that the aggressiveness of firms' conduct is determined by the efficiency distribution in the industry. Price leaders fight firms that are a lot less efficient than they are themselves while they are nice to firms that have efficiency levels close to theirs. This leads to the following two comparative statics effect. In case of entry by new firms at cost levels close to leaders', there is a crowding effect. This crowding effect tends to raise the equilibrium price. If the marginal entrant experiences efficiency gains, the leaders will fight the entrant in case of small efficiency gains. If the efficiency gains become substantial it is too costly to keep this firm out and the leaders will raise the price.

Using this intuition, we have formalized the idea of coordinated effects or joint dominance. A merger between two firms that leads to efficiency gains such that the new entity has a cost level that is comparable to that of the leaders tends to raise the price if the merged firm is not a price leader itself.

Finally, we have endogenized the identity of the price leader by allowing firms to pre announce price changes. This reduces the plethora of equilibria in a supergame to just one. In the unique equilibrium the most efficient firm acts as price leader. This firm charges the price that maximizes its profits taking other firms' entry behavior into account.

Although the theory is formulated here in terms of a pricing game, the underlying mechanism is more widely applicable. In all situations where aggressive play is not immediately rewarded and hence leaves time for opponents to react before payoffs are realized, firms will consider whether they should be nice or whether it pays to fight. In all such cases, it is not necessarily the case that more players or more efficient players lead to a more aggressive outcome. Other examples in economics that have a similar structure are advertising and R&D races.

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Appendix A. Proof of results

This appendix contains the proofs of the results in the main text. The following claim was made informally in section 3.

Lemma 2 *Consider two prices: p and $p' > p$. Assume that before entry $\Delta(p,p') > 0$ while after entry $\Delta(p,p') < 0$ and assume that $\frac{X(p)}{n}(p - c_1) > X(c_2)(c_2 - c_1)$ and $\frac{X(p')}{n+\nu+1}(p' - c_1) > X(\hat{c}_2)(\hat{c}_2 - c_1)$ where $\hat{c}_2 = min\{c_2, \hat{c}_e\}$ and $\hat{c}_e \geq c_1$ is the cost level of the entrant. Then there exist r and q close enough to zero such that p is IC before entry and p' is IC after entry.*

Proof of lemma Since $\frac{X(p)}{n(p)}(p - c_1) > X(c_2)(c_2 - c_1)$ there exist r and q close enough to zero such that

$$\frac{X(p)}{n(p)}(p - c_1) > \frac{r+q}{1+r}X(p)(p - c_1) + \frac{1-q}{1+r}X(c_2)(c_2 - c_1)$$

and similarly after entry. *Q.E.D.*

Proof of lemma 1 Writing the IC constraint for firm $i > 1$ as

$$\frac{1}{n} \geq \frac{r+q}{1+r} + 0$$

we see that the constraint holds for every $p \in \mathbb{P}$. In equilibrium, firm $n(p)$'s expected discounted profits equal

$$q\frac{1+r}{r+q}\frac{X(p)}{n(p)}(p - c_{n(p)}) - f \geq qX(p)(p - c_{n(p)}) - f > 0$$

where the first inequality follows from the IC constraint for firm $n(p) > 1$ and the second inequality follows from the definition of $n(p)$. *Q.E.D.*

Proof of proposition 1 In the case of efficiency gain by firm j with $\alpha_j = 0$, the effect on

$$\Delta(p,p') = \left[\sum_{i=1}^{n}\alpha_i X(p)(p - c_i) - \frac{n}{n+\nu}\sum_{i=1}^{n'}\alpha_i X(p')(p' - c_i)\right]$$

is driven only by the effect on $\frac{n}{n+\nu}$. In case A1 both n and ν are unchanged and hence Δ does not change after entry. In A2 entry raises ν and hence Δ goes up after entry. In A3 entry raises n but leaves ν unchanged. Hence Δ falls after entry. etc. *Q.E.D.*

Proof of proposition 2 Let us first define the relevant differences:

$$\begin{aligned}
(\hat{\Delta}(p,p') - \Delta(p,p'))_{\alpha_j>0} &= \sum_{i=1}^{\hat{n}} \alpha_i X(p)(p-\hat{c}_i) - \frac{\hat{n}}{\hat{n}+\hat{\nu}} \sum_{i=1}^{\hat{n}+\hat{\nu}} \alpha_i X(p')(p'-\hat{c}_i) \\
&- [\sum_{i=1}^{n} \alpha_i X(p)(p-c_i) - \frac{n}{n+\nu} \sum_{i=1}^{n+\nu} \alpha_i X(p')(p'-c_i)] \\
&= \sum_{i=1,i\neq j}^{n} \alpha_i X(p)(p-c_i) - \frac{\hat{n}}{\hat{n}+\hat{\nu}} \sum_{i=1,i\neq j}^{n+\nu} \alpha_i X(p')(p'-c_i) \\
&- [\sum_{i=1,i\neq j}^{n} \alpha_i X(p)(p-c_i) - \frac{n}{n+\nu} \sum_{i=1,i\neq j}^{n+\nu} \alpha_i X(p')(p'-c_i)] \\
&+ \iota(p,\hat{c}_j)\alpha_j X(p)(p-\hat{c}_j) - \frac{\hat{n}}{\hat{n}+\hat{\nu}} \iota(p',\hat{c}_j)\alpha_j X(p')(p'-\hat{c}_j) \\
&- [\iota(p,c_j)\alpha_j X(p)(p-c_j) - \frac{n}{n+\nu} \iota(p',c_j)\alpha_j X(p')(p'-c_j)]
\end{aligned}$$

where $\hat{c}_i = \begin{cases} \hat{c}_j, & \text{if } i = j; \\ c_i, & \text{otherwise} \end{cases}$, with a slight abuse of notation $\hat{n}$ ($\hat{\nu}$) denotes the number of firms that can enter at price p (cannot enter at p but can at p') after entry, and $\iota(p,c_j) = \begin{cases} 1, & \text{if } p \succ c_j; \\ 0, & \text{otherwise.} \end{cases}$ Hence we can write

$$\begin{aligned}
(\hat{\Delta}(p,p') - \Delta(p,p'))_{\alpha_j>0} &- (\hat{\Delta}(p,p') - \Delta(p,p'))_{\alpha_j=0} = \\
\iota(p,\hat{c}_j)\alpha_j X(p)(p-\hat{c}_j) &- \frac{\hat{n}}{\hat{n}+\hat{\nu}} \iota(p',\hat{c}_j)\alpha_j X(p')(p'-\hat{c}_j) \\
-[\iota(p,c_j)\alpha_j X(p)(p-c_j) &- \frac{n}{n+\nu} \iota(p',c_j)\alpha_j X(p')(p'-c_j)]
\end{aligned}$$

Then we find that in case A1 $(\hat{\Delta}(p,p') - \Delta(p,p'))_{\alpha_j>0} - (\hat{\Delta}(p,p') - \Delta(p,p'))_{\alpha_j=0} = 0$ because firm j cannot enter at either price, even after the efficiency gain. In case A2 we find that $(\hat{\Delta}(p,p') - \Delta(p,p'))_{\alpha_j>0} - (\hat{\Delta}(p,p') - \Delta(p,p'))_{\alpha_j=0} = -\frac{\hat{n}}{\hat{n}+\hat{\nu}}\alpha_j X(p')(p'-\hat{c}_j) < 0$. In case A3 we find that $(\hat{\Delta}(p,p') - \Delta(p,p'))_{\alpha_j>0} - (\hat{\Delta}(p,p') - \Delta(p,p'))_{\alpha_j=0} = \alpha_j X(p)(p-\hat{c}_j) - \frac{\hat{n}}{\hat{n}+\hat{\nu}}\alpha_j X(p')(p'-\hat{c}_j) \lesseqgtr 0$ if and only if $X(p)(p-\hat{c}_j) \lesseqgtr \frac{n+1}{n+1+\nu} X(p')(p'-\hat{c}_j)$ since under A3 it is the case that $\hat{n} = n+1$ and $\hat{\nu} = \nu$. Under B1 we get $(\hat{\Delta}(p,p') - \Delta(p,p'))_{\alpha_j>0} - (\hat{\Delta}(p,p') - \Delta(p,p'))_{\alpha_j=0} = \frac{n}{n+\nu}\alpha_j X(p')(p'-c_j) - \frac{n}{n+\nu}\alpha_j X(p')(p'-\hat{c}_j) < 0$ because $\hat{c}_j < c_j$. Under B2 with ν big enough, we get approximately $(\hat{\Delta}(p,p') - \Delta(p,p'))_{\alpha_j>0} - (\hat{\Delta}(p,p') - \Delta(p,p'))_{\alpha_j=0} = \alpha_j X(p)(p-\hat{c}_j) > 0$. Finally, C1 yields

$(\hat{\Delta}(p,p')-\Delta(p,p'))_{\alpha_j>0}-(\hat{\Delta}(p,p')-\Delta(p,p'))_{\alpha_j=0}=\alpha_j X(p)(c_j-\hat{c}_j)-\alpha_j\frac{n}{n+\nu}X(p')(c_j-\hat{c}_j)>0$ because $\frac{X(p)}{n}>\frac{X(p')}{n+\nu}$. *Q.E.D.*

Proof of Corollary 1

Differentiating $\Delta(p,p';t)$ with respect to t yields

$$\frac{d\Delta(p,p';t)}{dt}=\sum_{i=1}^{n}\alpha_i X'(p+t)(p-c_i)-\frac{n}{n+\nu}\sum_{i=1}^{n+\nu}\alpha_i X'(p'+t)(p'-c_i)$$

Hence $\frac{d\Delta(p,p';t)}{dt}>0$ if and only if

$$\left|\frac{X'(p'+t)}{X'(p+t)}\right|>\frac{\frac{\sum_{i=1}^{n}\alpha_i(p-c_i)}{n}}{\frac{\sum_{i=1}^{n+\nu}\alpha_i(p'-c_i)}{n+\nu}}\tag{A.1}$$

Using that $p'>p$ was the optimal price before $dt>0$, we know that

$$\frac{n}{n+\nu}X(p'+t)\sum_{i=1}^{n+\nu}\alpha_i(p'-c_i)>X(p+t)\sum_{i=1}^{n}\alpha_i(p-c_i)$$

Using $X(p'+t)<X(p+t)$ we find

$$1>\frac{X(p'+t)}{X(p+t)}>\frac{\frac{\sum_{i=1}^{n}\alpha_i(p-c_i)}{n}}{\frac{\sum_{i=1}^{n+\nu}\alpha_i(p'-c_i)}{n+\nu}}$$

Hence a sufficient condition for inequality (A.1) to hold is

$$\left|\frac{X'(p'+t)}{X'(p+t)}\right|\geq 1$$

which is satisfied if $X''(p)\leq 0$ for all p. *Q.E.D.*

Proof of proposition 3

If n goes up, while $n+\nu$ remains unchanged, clearly Δ falls after the efficiency gain since $\mathcal{B}(p)$ falls while $\mathcal{B}(p')$ remains unchanged. Now consider the case where $dy>0$. Then we find that

$$\begin{aligned}\frac{\partial\Delta(p,p';y)}{\partial y} &= \frac{-1}{n+y}+\frac{1}{n+y+\nu}-\frac{\alpha_1 X(c_2)(c_2-c_1)}{X(p)(p-c_1)-(n+y)X(c_2)(c_2-c_1)}\times\\ &\quad\left(1-\frac{X(p)(p-c_1)-(n+y)X(c_2)(c_2-c_1)}{X(p)(p-c_1)-(n+y+\nu)X(c_2)(c_2-c_1)}\right)\\ &< 0\end{aligned}$$

if either $\alpha_1 = 0$ or (in case $\alpha_1 > 0$) if $c_2 - c_1$ close enough to zero. *Q.E.D.*

Proof of proposition 4

If $dy > 0$ big enough while $dy + dz = 0$, it follows that $\frac{X(p')}{n+y+\nu+z}(p' - c_i) > \frac{X(p)}{n+y}(p - c_i)$. By assumption we have $X(p)(p' - c_1) > X(p)(p - c_1)$ and from this it follows that $X(p)(p' - c_i) > X(p)(p - c_i)$ for each i. Since we can write

$$\frac{X(p')}{n+y+\nu+z}(p'-c_i)-\frac{X(p)}{n+y}(p-c_i) = \frac{1}{n+y+\nu+z}\left(X(p')(p'-c_i) - \frac{n+y+\nu+z}{n+y}X(p)(p-c_i)\right)$$

we find that for given z there exists a value of y big enough such that

$$\frac{X(p')}{n+y+\nu+z}(p'-c_i) - \frac{X(p)}{n+y}(p-c_i) > 0$$

Q.E.D.

Proof of proposition 5

In the text we show that the strategies form a subgame perfect equilibrium. Here we show that no firm $i > 1$ prefers a price strictly below p_1^*. Consider a price $p \in \mathbb{P}$ with $p < p_1^*$ then we find the following inequalities for each $c_i \geq c_1$:

$$\begin{aligned} 0 &> \frac{X(p)}{n(p)}(p-c_1) - \frac{X(p_1^*)}{n(p_1^*)}(p_1^*-c_1) \\ &= \frac{X(p)}{n(p)}p - \frac{X(p_1^*)}{n(p_1^*)}p_1^* - c_1\left(\frac{X(p)}{n(p)} - \frac{X(p_1^*)}{n(p_1^*)}\right) \\ &\geq \frac{X(p)}{n(p)}p - \frac{X(p_1^*)}{n(p_1^*)}p_1^* - c_i\left(\frac{X(p)}{n(p)} - \frac{X(p_1^*)}{n(p_1^*)}\right) \\ &= \frac{X(p)}{n(p)}(p-c_i) - \frac{X(p_1^*)}{n(p_1^*)}(p_1^*-c_i) \end{aligned}$$

where the first inequality follows from the fact that p_1^* is the profit maximizing price for firm 1 and the second inequality follows from $c_i \geq c_1$ and $\frac{X(p)}{n(p)} - \frac{X(p_1^*)}{n(p_1^*)} > 0$. Hence we find that for each firm i it is the case that

$$\frac{X(p_1^*)}{n(p_1^*)}(p_1^*-c_i) > \frac{X(p)}{n(p)}(p-c_i)$$

Q.E.D.