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Can Coasean Bargaining Justify Pigouvian Taxation?

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Abstract

The fact that according to the celebrated Coase theorem rational parties always try to exploit all gains from trade is usually taken as an argument against the necessity of government intervention through Pigouvian taxation in order to correct externalities. However, we show that the hold-up problem, which occurs if non-verifiable investments have external effects and parties cannot be prevented from always trying to exploit all gains from trade, may in fact be solved by taxation. Thus, in our framework Coasean bargaining is not a substitute for Pigouvian taxation, instead it is the very reason for government intervention.

Keywords: Hold-up problem; Bargaining; Contracts; Taxation; Externalities

JEL classification: D62; H21; H23; L14

1 Introduction

A standard textbook argument known by every student of public economics goes as follows.¹ If the activity of party A has an uncompensated external effect on party B 's utility, then party A will not choose the socially efficient activity level. Party A can be made to internalize the externality by Pigouvian taxation. However, opponents of government intervention typically argue that Pigouvian taxation is not necessary. According to the celebrated Coase Theorem, rational parties always exploit all possible gains from trade, provided there is symmetric information. They will hence write a contract that induces party A to choose the efficient activity level and divide the gains from trade by appropriate transfer payments. Thus, if one does not make the assumption that the government has better information than the parties themselves (which many economists consider to be unrealistic), Coasean bargaining makes Pigouvian taxation unnecessary.

In this paper, we will argue that recent contract theoretic research on the so-called hold-up problem has identified important circumstances under which the above argument does not hold.² Indeed, we will show that the very fact that parties always engage in Coasean bargaining in order to exploit all gains from trade, which lies at the heart of the hold-up problem, can justify government intervention. Pigouvian taxation can help to solve the hold-up problem which in the presence of unverifiable investments with direct externalities cannot be solved by contractual agreements.

In order to convey the intuition for our argument, consider the following simple example, which is based on Maskin and Moore (1999). There are two risk-neutral parties, a producer and a buyer. At date 1, the producer can decide whether or not to exert effort (i.e., invest) while producing one unit of an indivisible good, which can be traded with the buyer at date 2. If the producer does not exert effort at date 1, the buyer's willingness-to-pay for the good is v_l . If the producer exerts effort at a personal disutility cost $c > 0$, then the buyer's valuation is v_h , where $v_h > v_l > 0$. The producer has no further costs. Notice that the producer's effort has a direct externality on the buyer's valuation $v \in \{v_l, v_h\}$. If the producer were not compensated,

¹See e.g. Mas-Colell, Whinston, and Green (1995, p. 354) for a modern textbook treatment. See also the seminal contributions by Pigou (1932) and Coase (1960).

²For discussions of the hold-up problem, see e.g. Hart (1995) and Tirole (1999). See also Schmitz (2001) for a non-technical survey of the literature.

she would never exert effort. However, exerting effort is socially efficient provided that $v_h - v_l > c$, which we assume in what follows.

Let effort (and hence the buyer's valuation) be observable by the two parties, but unverifiable to outsiders such as the courts. Even though effort is thus non-contractible, it might be possible for the parties to ex ante write a contract which induces the producer to invest by giving the buyer an option to buy the good at date 2, with a strike price of v_h . The idea is that the buyer would only exercise the option if the producer had exerted effort, and the producer would exert effort because she would then get the total surplus $v_h - c$. Yet, there is a problem. Suppose that the producer had exerted effort, so that the buyer's valuation is v_h . If the buyer exercises the option, he receives zero. But if he does not exercise, there would still remain gains from trade to be exploited, since $v_h > 0$. Thus, in accordance with the Coase Theorem, the parties would start to bargain. Let the outcome of the negotiations be given by the Nash bargaining solution, so that each party would get $\frac{1}{2}v_h$ at date 2. If the producer had not exerted effort, a similar reasoning shows that each party would get $\frac{1}{2}v_l$. Hence, the producer would not exert effort if $\frac{1}{2}(v_h - v_l) < c$. In fact, Maskin and Moore (1999) prove that there is no ex ante contract the parties could write which induces effort if this inequality holds. This is the hold-up problem in its most severe form.

We will now show that the hold-up problem, which is caused by the direct externality of the producer's effort and the fact that parties always exploit all gains from trade (which makes threats of ex post inefficient outcomes incredible), can in fact be solved by government intervention. Notice that we do *not* assume that the government can observe otherwise non-verifiable information. Hence, we do not condition the tax on the producer's effort or the buyer's true valuation.³ Instead, consider the following tax scheme. The buyer must pay $t(v_h - z)$ to the government, where $t \in (0, 1)$ denotes the tax rate. Note that any payment z that he makes to the producer is deductible;⁴

³Since the producer's effort has a positive externality, a naive Pigouvian solution would be to directly subsidize effort. This is not feasible because effort is unverifiable, so that a solution must be sought which circumvents this problem.

⁴Transfer payments are usually assumed to be verifiable in the hold-up literature. Of course, whenever a taxpayer can deduct expenditures, there is a danger of fraudulent tax evasion (the buyer might overstate the true expenses, perhaps colluding with the seller). For simplicity, we assume that this could be detected by auditing and is deterred by sufficiently high penalties. (Our findings hold qualitatively as long as tax audits are not

i.e., the buyer's net payoff is $xv - tv_h - (1-t)z$, where $x \in \{0, 1\}$ denotes the parties' trade decision. Assume that the parties have written no contract ex ante, which will turn out to be optimal. Then they will always negotiate at date 2. We continue to assume that the outcome of the negotiations is given by the Nash bargaining solution. The producer's and the buyer's threatpoint payoffs (what they get if negotiations fail) are 0 and $-tv_h$, respectively. Thus, the parties will always agree to trade ($x = 1$) and choose the payment that maximizes the Nash product $(z - 0)(v - tv_h - (1-t)z + tv_h)$,⁵ so that

$$z = \frac{v}{2(1-t)}.$$

Anticipating the bargaining outcome, the producer will exert effort if and only if

$$\frac{1}{2(1-t)}(v_h - v_l) \geq c.$$

Therefore, if $t = \frac{1}{2}$, the producer will invest whenever it is socially efficient to do so and thus the hold-up problem has been solved.⁶

To the best of our knowledge, this is the first paper which argues that the hold-up problem, which is caused by unverifiable investments with direct externalities and, ironically, by the fact that Coasean bargaining always exploits all gains from trade, can be solved by a simple form of Pigouvian taxation. The hold-up problem has played a prominent role in recent contract-theoretic research. In particular, Maskin and Moore (1999), Che and Hausch (1999), and Segal and Whinston (2002) have shown that there are no contractual arrangements that induce efficient investments in the presence of direct externalities, provided that the parties cannot commit not to exploit future gains from trade and each party receives a positive fraction of the renegotiation surplus. In the absence of direct externalities, the parties can solve the hold-up problem by writing appropriate contracts, as has been demonstrated by Edlin and Reichelstein (1996).⁷ Hence, the standard argument that it is the presence of externalities which calls for

completely ineffective.)

⁵For an exposition of the Nash product and the Nash bargaining solution, see e.g. Muthoo (1999). It should be noted that even though Nash bargaining is a standard assumption in the contract theoretic literature, it might be more difficult to provide a strategic justification in a framework without commitment (cf. Muthoo, 1990).

⁶Note that in equilibrium the tax revenue is zero.

⁷See also Rogerson (1992) and Hermalin and Katz (1993), who show that the first best can be achieved if the parties can commit not to exploit future gains from trade, even

intervention by the government is true in the present context. Moreover, the fact that in accordance with the Coase theorem the parties will always exhaust any gains from trade does not *solve* the hold-up problem, instead it *causes* the hold up problem. In this sense, our paper shows that the very Coase theorem can justify government interventions that are usually associated with Pigou.

The remainder of the paper is organized as follows. In section 2, we show that our argument outlined above can be generalized to a framework with uncertainty where effort and valuation are continuous variables. In section 3, we discuss heterogenous bargaining powers and the role of further costs of the producer. While it may no longer achieve the first best, our simple taxation scheme can still implement significant welfare improvements. Concluding remarks follow in section 4. Finally, some technical details have been relegated to the appendix.

2 The model

Consider two risk neutral parties, a producer A and a buyer B . At date 0, the parties can write a contract. At date 1, the producer chooses an unobservable relationship-specific effort level $e \geq 0$. Her personal effort costs are given by the increasing and convex function $c(e)$, which satisfies the usual Inada conditions $c'(0) = 0$ and $\lim_{e \rightarrow \infty} c'(e) = \infty$. At date 2, the buyer's gains from trade $v \geq 0$, which stochastically depend on e , are realized.⁸ At date 3, trade can occur and payments can be made. If the contract does not already lead to trade, the parties will negotiate according to the generalized Nash bargaining solution, where the producer's bargaining power is given by $\alpha \in (0, 1)$.

The buyer's valuation $v \in [v_l, v_h]$ is observable by the two parties, but un-

in the presence of private information. Schmitz (2002) shows that their results can be extended to the case in which renegotiation is possible, provided that there are no direct externalities. For the case of symmetric information, see also Aghion, Dewatripont, and Rey (1994), Chung (1991), Nöldeke and Schmidt (1995), and De Fraja (1999), who derive first best results for alternative forms of renegotiation.

⁸One can imagine that the producer's effort influences the quality of the specific good to be traded, which also depends on random events. In this section we assume that the good has only value for the buyer and the producer incurs no further (opportunity) costs at date 3.

verifiable to outsiders. Let the buyer's valuation be distributed according to the cumulative distribution function $F(v|e)$. We assume that the producer's effort increases the buyer's valuation in the sense of first-order stochastic dominance, so that $F_e(v|e) < 0$. Moreover, we assume that $F_{ee}(v|e) > 0$, so that the expected value $E[v|e]$ is concave in e . The probability of trade is denoted by $x \in [0, 1]$. In a first-best world, trade would always occur ($x^{FB} = 1$), and the effort level would be $e = e^{FB}$, where

$$e^{FB} = \arg \max E[v|e] - c(e) \quad (1)$$

maximizes the expected social surplus. We say that the hold-up problem is solved if the parties are induced to make these first-best decisions.

Let $t \in (0, 1)$ denote the tax rate and suppose that the buyer must pay $t(\bar{v} - z)$ to the government; i.e., any payment z that he makes to the producer is deductible. Hence, the parties' payoffs are as follows:

$$\begin{aligned} u_A &= z - c(e) \\ u_B &= xv - t\bar{v} - (1 - t)z \end{aligned}$$

In general, a contract between the two parties can specify a trade decision and a transfer payment as a function of verifiable messages sent after the state of the world has been realized.⁹ Thus, a contract is given by $[X(s_A, s_B), Z(s_A, s_B)]$, where s_A and s_B denote A 's and B 's reporting strategies, that can depend on the true valuation v . It is straightforward to see that the parties would write a contract solving the hold-up problem if ex post negotiations were impossible.¹⁰ However, as we have discussed in the introduction, a contract may be valueless when we take into consideration that the parties will always exploit any remaining gains from trade through Coasean bargaining. Since ex post efficiency ($x = 1$) is thus achieved independently of any ex ante contract, the only use of such a contract could be to improve effort incentives.

⁹An example for such a contract is the option contract mentioned in the introduction. In the case of the option contract, only the buyer sends a message (namely, whether or not he exercises the option).

¹⁰For example, consider the following contract. After v has been realized, the seller and the buyer both report a value. If the reports match, trade occurs, and the buyer pays the reported value. Otherwise, no trade occurs. It is an equilibrium that both parties tell the truth and the seller invests e^{FB} . More sophisticated mechanisms in order to get rid of inefficient equilibria can easily be constructed following Moore (1992).

Proposition 1 a) Let \tilde{e} denote the effort level that the producer chooses in the absence of an ex ante contract. Then an effort level \hat{e} can be contractually induced if and only if $\hat{e} \in [0, \tilde{e}]$.

b) The parties write no contract at date 0, i.e. $[X, Z] \equiv [0, 0]$.

c) The producer chooses the effort level $\tilde{e} = \arg \max E \left[\frac{\alpha}{1-t} v | e \right] - c(e)$.

Proof. See the appendix. ■

Intuitively, in the absence of a contract the producer will receive a fraction of the buyer's realized valuation through Coasean bargaining at date 3. Thus, she has at least some incentive to exert effort at date 1 in order to increase the buyer's valuation. On the other hand, if the contract simply prescribed trade at an ex ante specified price, ex post efficiency would be achieved without further bargaining at date 3, but the producer had no incentives to exert effort, because this would only benefit the buyer. The proof of Proposition 1a) shows that even more sophisticated contracts can only reduce the incentives to exert effort when ex post inefficiencies are always negotiated away at date 3. This result is in the spirit of Che and Hausch (1999) and Segal and Whinston (2002) who argue that parties cannot solve the hold-up problem by writing contracts, because this could only decrease effort further below the first-best level. Our model differs from their models (which do not consider taxation), because here utility is not transferable on a 1:1 basis from one party to the other party.¹¹ In particular, in our framework the no-contract effort level \tilde{e} may well be above the first-best level e^{FB} .¹² In this case, the parties *could* in principle write a contract inducing $e = e^{FB}$. However, Proposition 1b) says that such a contract can never be profitable for both parties simultaneously and thus will never be written.

Specifically, assume that the parties write no ex ante contract, so that trade will only occur through ex post bargaining. Since $[X, Z] \equiv [0, 0]$, party A 's and party B 's threatpoint payoffs at date 3 (i.e., after the effort costs $c(e)$ are sunk) are given by 0 and $-t\bar{v}$, respectively. Thus, the payment according to the generalized Nash bargaining solution is characterized by

$$z = \arg \max z^\alpha (v - (1-t)z)^{1-\alpha},$$

¹¹In general, the irrelevance of contracting might no longer hold if utilities are non-transferable; cf. Bensaïd and Gary-Bobo (1993).

¹²It should be noted that overinvestments are also possible in the model of Muthoo (1998), yet for different reasons (his focus is on the communication technology in outside option bargaining).

i.e. by $z = \frac{\alpha}{1-t}v$. The expected payoff of the producer at date 1 is hence given by

$$E \left[\frac{\alpha}{1-t}v | e \right] - c(e). \quad (2)$$

The first-best effort level can be induced by an appropriate tax rate, as is stated in the following result.

Corollary 1 *The government can solve the hold-up problem by choosing the tax rate $t = 1 - \alpha$.*

Proof. This follows immediately from the fact that trade always occurs, i.e. ex post efficiency is achieved through negotiations at date 3, and the producer chooses e in order to maximize (2), which coincides with the definition of e^{FB} given in (1) if $\frac{\alpha}{1-t} = 1$. ■

In contrast, \tilde{e} is smaller (larger) than e^{FB} whenever the tax rate t is smaller (larger) than $1 - \alpha$. A strictly positive tax rate is thus necessary in order to solve the hold-up problem whenever the buyer has some bargaining power.¹³ While we follow the literature in assuming that the parties' bargaining powers cannot be changed, the government can influence the bargaining outcome (and hence the investment incentives) by taxation, the important effect of which is to change the slope of the Pareto frontier that is the boundary of the parties' feasible payoff pairs at date 3. Notice that the government can choose $\bar{v} = v_h$, so that the buyer's tax never becomes negative.¹⁴ Finally, note also that the result that in the absence of an ex ante contract the first best will be achieved in case of $t = 1 - \alpha$ does not depend on the particular way in which the producer's effort level influences the buyer's valuation.¹⁵

¹³Of course, if the producer had all bargaining power ($\alpha = 1$), the first best would be achieved without taxation, since there would be no hold-up problem in the first place.

¹⁴Alternatively, the government could set $\bar{v} = E[v|e^{FB}]$, so that the expected tax revenue would be zero. The choice of \bar{v} has obviously no influence on the incentives. We might also set $\bar{v} = 0$, so that tz would simply be a subsidy, if the government can use different funds to finance the subsidy.

¹⁵For example, the producer's effort could be a multidimensional variable. What is important is that the right hand side of (1) coincides with (2) for $t = 1 - \alpha$.

3 Discussion

3.1 Bargaining powers

It is a standard assumption in the literature on the hold-up problem to model negotiations with the regular Nash bargaining solution, i.e. with $\alpha = \frac{1}{2}$.¹⁶ In this case, the solution to the hold-up problem given in section 2 is remarkably simple to implement. A tax rate of 50% is sufficient, independent of the characteristics of the particular hold-up problem under consideration, such as the effort cost function or the distribution function of the valuations. However, if the parties' bargaining powers differ, the solution offered in the previous section only implements the first best if $t = 1 - \alpha$. One might argue that the government does not know the precise bargaining powers of any two parties that are in the hold-up dilemma. Moreover, one might (not just in our model, but quite generally) doubt whether a tax system can be sufficiently fine-tuned in order to solve specific externality problems when there are heterogenous parties facing such problems. Therefore, it might be interesting to observe that even if the government only knows the distribution of the bargaining powers and cannot fine-tune the tax rate, a significant welfare improvement can be achieved by simply using a rule-of-thumb tax scheme.

Specifically, suppose that the bargaining power α is uniformly distributed on the unit interval, but the government can only set one tax rate t independent of α . For simplicity, assume that there is a deterministic relationship between the producer's effort and the buyer's valuation, $v = \sqrt{e}$. Moreover, let $c(e) = e$, so that the total surplus generated by the parties is $\sqrt{e} - e$. Given tax rate t , the producer chooses $\tilde{e} = \arg \max \frac{\alpha}{1-t} \sqrt{e} - e$. Hence, $\tilde{e} = \frac{\alpha^2}{(2-2t)^2}$ and the expected total surplus is $\int_0^1 \left(\frac{\alpha}{2-2t} - \frac{\alpha^2}{(2-2t)^2} \right) d\alpha = \frac{1}{12} \frac{2-3t}{(1-t)^2}$, which is maximized by $t = \frac{1}{3}$. The resulting surplus is $\frac{3}{16}$, while the first best surplus is $\frac{1}{4}$ and the second best surplus without taxation is $\frac{1}{6}$. In other words, in this example even an easily implementable rule-of-thumb tax scheme would lead to a realization of 75% of the first best surplus, while in the absence of taxation only 66% would be realized due to the hold-up problem.

¹⁶See e.g. Hart (1995), Aghion and Tirole (1994), Hart, Shleifer, and Vishny (1997), or Tirole (1999).

3.2 Further costs

In section 2 we considered a model in which trade was always efficient. While this is done in many contributions to the literature on the hold-up problem,¹⁷ one can also consider the additional problem that arises when trade is only ex post efficient in some states of the world. Therefore, let us now assume that the producer incurs further costs k if and only if she trades with the buyer, so that the payoffs are given by

$$\begin{aligned} u_A &= z - xk - c(e), \\ u_B &= xv - t\bar{v} - (1-t)z. \end{aligned}$$

Now trade is efficient whenever $v - k \geq 0$, and the first best effort level is characterized by

$$e^{FB} = \arg \max E[\max\{v - k, 0\}|e] - c(e).$$

For simplicity, assume that the parties negotiate ex post according to the regular Nash bargaining solution. The Nash product $(z - xk)(xv - (1-t)z)$ is now maximized by $z = x \frac{v+(1-t)k}{2(1-t)}$, so that the parties choose $x = 1$ whenever $v - (1-t)k \geq 0$. Hence, setting $t > 0$ means that the parties will want to trade too often at date 3. If there is no uncertainty, so that the buyer's valuation depends deterministically on the producer's effort by a function $v = v(e)$, this is no problem. In this case, the government could simply set $t = \frac{1}{2}$ if $v(e^{FB}) \geq k$, and $t = 0$ otherwise.¹⁸ However, in the presence of uncertainty there is now a trade-off between improving the producer's investment incentives and inducing the ex post efficient trade decision. The producer chooses

$$\tilde{e} = \arg \max E \left[\max \left\{ \frac{v}{2(1-t)} - \frac{k}{2}, 0 \right\} | e \right] - c(e),$$

¹⁷See e.g. Rubinstein and Wolinsky (1992), Aghion and Tirole (1994), Hart (1995), Hart, Shleifer, and Vishny (1997), Nöldeke and Schmidt (1998), Tirole (1999), or Gul (2001).

¹⁸Notice that the government can calculate $v(e^{FB})$, even though it cannot observe the actually chosen effort e and thus $v(e)$.

so that in general the first best will not be achieved.¹⁹ Yet, Pigouvian taxation can still generate significant welfare improvements.

As an illustration, let v be distributed on the unit interval according to the distribution function

$$F(v) = \begin{cases} v & \text{if } e = 0, \\ v^2 & \text{if } e = 1. \end{cases}$$

Assume that $c(0) = 0$, $c(1) = \frac{1}{20}$, and $k = \frac{1}{2}$. It is easy to check that the first best surplus is given by $\int_{1/2}^1 (v - \frac{1}{2}) 2v dv - \frac{1}{20} \approx .158$. The producer's payoff is $\int_{(1-t)/2}^1 \left(\frac{v}{2(1-t)} - \frac{1}{4} \right) 2v dv - \frac{1}{20}$ if she exerts effort, and $\int_{(1-t)/2}^1 \left(\frac{v}{2(1-t)} - \frac{1}{4} \right) dv$ otherwise. Straightforward calculations show that the smallest tax rate which induces the producer to exert effort is given by $t \approx .074$. If the government sets this tax rate, the total surplus is $\int_{(1-t)/2}^1 (v - \frac{1}{2}) 2v dv - \frac{1}{20} \approx .157$, while it would be .125 without taxation. Thus, without taxation less than 79% of the expected first-best surplus could be realized, while more than 99% are realized with taxation.

4 Conclusion

It is true that externalities *per se* do not automatically make intervention by the government through Pigouvian taxation necessary in order to maximize the social surplus. If the activities that have external effects are verifiable, the parties can negotiate contracts which induce an internalization of the externalities, as is suggested by the Coase Theorem. But if investments with direct externalities are unverifiable, contractual arrangements may have no value. Indeed, the very reason that contracts fail to induce first-best behavior is the fact that private parties will always exhaust all future gains from trade. A simple form of Pigouvian taxation can solve or at least alleviate the resulting hold-up problem. If hold-up problems do have the importance that is suggested by recent contributions in contract theory, then the possibility to reduce the welfare losses caused by hold-up problems with the help

¹⁹Note that analogous arguments hold when v is fixed, but the buyer influences $k \in [k_l, k_h]$ by exerting effort, the producer pays a tax τz , and $t = 0$. Then $\tau = 1/2$ implies the first best provided that $v \geq 2k_h$. Otherwise, the first best can generally not be achieved. If both v and k are variable, the parties trade if $(1 - \tau)v \geq (1 - t)k$ and then the payment is $z = v/(2(1 - t)) + k/(2(1 - \tau))$. Whether t or τ should be positive depends on the relative importance of the parties' efforts.

of government intervention should not be neglected.

Appendix

Proof of Proposition 1.

Suppose that the parties have written a contract $[X(s_A, s_B), Z(s_A, s_B)]$ at date 0. At date 3, after the parties A and B have chosen their strategies s_A and s_B , respectively, the parties will negotiate in order to exploit the remaining gains from trade whenever $X(s_A, s_B) < 1$. According to the generalized Nash bargaining solution, the outcome of the negotiations can be characterized by maximizing the Nash product

$$(\zeta - Z(s_A, s_B))^\alpha ((1 - X(s_A, s_B))v - (1 - t)(\zeta - Z(s_A, s_B)))^{1-\alpha},$$

where we have used the fact that the parties' threatpoint payoffs are given by $Z(s_A, s_B)$ and $X(s_A, s_B)v - (1 - t)Z(s_A, s_B) - t\bar{v}$, respectively. Hence, the payment that the producer ultimately receives is given by $\zeta = Z(s_A, s_B) + \frac{1 - X(s_A, s_B)}{1 - t}\alpha v$. The parties' date 3 payoffs are thus given by

$$\omega_A(s_A, s_B, v) = Z(s_A, s_B) + \frac{1 - X(s_A, s_B)}{1 - t}\alpha v, \quad (3)$$

$$\begin{aligned} \omega_B(s_A, s_B, v) &= v - (1 - t)Z(s_A, s_B) - t\bar{v} - (1 - X(s_A, s_B))\alpha v \\ &= v - t\bar{v} - (1 - t)\omega_A(s_A, s_B, v). \end{aligned}$$

In equilibrium, the producer will choose $s_A(v)$ such that for all $(v, \tilde{v}) \in [v_l, v_h]^2$

$$\omega_A(s_A(v), s_B(v), v) \geq \omega_A(s_A(\tilde{v}), s_B(v), v),$$

and the buyer will analogously choose $s_B(v)$ such that for all $(v, \tilde{v}) \in [v_l, v_h]^2$

$$v - t\bar{v} - (1 - t)\omega_A(s_A(v), s_B(v), v) \geq v - t\bar{v} - (1 - t)\omega_A(s_A(v), s_B(\tilde{v}), v),$$

which is equivalent to

$$\omega_A(s_A(v), s_B(\tilde{v}), v) \geq \omega_A(s_A(v), s_B(v), v).$$

Hence, we must have

$$\omega_A(s_A(\tilde{v}), s_B(v), v) \leq \omega_A(s_A(v), s_B(v), v) \leq \omega_A(s_A(v), s_B(\tilde{v}), v) \quad (4)$$

if v is the buyer's true valuation, and analogously

$$\omega_A(s_A(v), s_B(\tilde{v}), \tilde{v}) \leq \omega_A(s_A(\tilde{v}), s_B(\tilde{v}), \tilde{v}) \leq \omega_A(s_A(\tilde{v}), s_B(v), \tilde{v}) \quad (5)$$

if \tilde{v} is the buyer's true valuation. The two chains of inequalities (4) and (5) imply together with (3) that

$$\begin{aligned}
& \frac{\alpha}{1-t} (1 - X(s_A(v), s_B(\tilde{v}))) (\tilde{v} - v) \\
&= \omega_A(s_A(v), s_B(\tilde{v}), \tilde{v}) - \omega_A(s_A(v), s_B(\tilde{v}), v) \\
&\leq \omega_A(s_A(\tilde{v}), s_B(\tilde{v}), \tilde{v}) - \omega_A(s_A(v), s_B(v), v) \\
&\leq \omega_A(s_A(\tilde{v}), s_B(v), \tilde{v}) - \omega_A(s_A(\tilde{v}), s_B(v), v) \\
&= \frac{\alpha}{1-t} (1 - X(s_A(\tilde{v}), s_B(v))) (\tilde{v} - v).
\end{aligned}$$

Thus, dividing by $(\tilde{v} - v)$ and letting \tilde{v} converge to v , we obtain

$$\frac{d\omega_A(s_A(v), s_B(v), v)}{dv} = \frac{\alpha}{1-t} (1 - X(s_A(v), s_B(v))) \quad (6)$$

almost everywhere. Now consider the producer's incentives to exert effort at date 1. Her marginal revenue from exerting effort is

$$\begin{aligned}
& \frac{dE[\omega_A(s_A(v), s_B(v), v)|e]}{de} \\
&= \frac{d}{de} \left[\omega_A(s_A(v_h), s_B(v_h), v_h) - \int_{v_l}^{v_h} \frac{d\omega_A(s_A(v), s_B(v), v)}{dv} F(v|e) dv \right] \\
&= - \int_{v_l}^{v_h} \frac{d\omega_A(s_A(v), s_B(v), v)}{dv} F_e(v|e) dv \\
&= - \int_{v_l}^{v_h} \frac{\alpha}{1-t} (1 - X(s_A(v), s_B(v))) F_e(v|e) dv,
\end{aligned}$$

where the second line follows from partial integration and the last line follows from (6).

Given the contract $[X(s_A, s_B), Z(s_A, s_B)]$, the producer will hence choose $e = \hat{e}$, which is uniquely determined by

$$- \int_{v_l}^{v_h} \frac{\alpha}{1-t} (1 - X(s_A(v), s_B(v))) F_e(v|\hat{e}) dv = c'(\hat{e}).$$

Note that $\hat{e} = 0$ if $X \equiv 1$. The no-contract effort level is given by

$$- \int_{v_l}^{v_h} \frac{\alpha}{1-t} F_e(v|\tilde{e}) dv = c'(\tilde{e}).$$

It is straightforward to see that any $X \neq 0$ can only make \hat{e} smaller than \tilde{e} . Moreover, any effort level $e \in [0, \tilde{e}]$ can obviously be induced by a contract that specifies a fixed trade level $X \in [0, 1]$, which completes the proof of part a).

Consider a contract $[X(s_A, s_B), Z(s_A, s_B)]$ that induces investment \hat{e} and let $\hat{x}(v) \equiv X(s_A(v), s_B(v))$, $\hat{z}(v) \equiv Z(s_A(v), s_B(v))$. The parties will agree

on such a contract whenever there exists a z_0 such that the producer can be made better off,

$$E \left[\hat{z}(v) + \frac{1 - \hat{x}(v)}{1 - t} \alpha v \mid \hat{e} \right] - c(\hat{e}) + z_0 \geq E \left[\frac{\alpha}{1 - t} v \mid \tilde{e} \right] - c(\tilde{e}),$$

and simultaneously the buyer can be made better off,

$$E [v - t\bar{v} - (1 - t)\hat{z}(v) - (1 - \hat{x}(v))\alpha v \mid \hat{e}] - (1 - t)z_0 \geq E [(1 - \alpha)v - t\bar{v} \mid \tilde{e}].$$

Such a z_0 exists if and only if

$$\begin{aligned} & E \left[\frac{\alpha}{1 - t} v \mid \tilde{e} \right] - c(\tilde{e}) - E \left[\hat{z}(v) + \frac{1 - \hat{x}(v)}{1 - t} \alpha v \mid \hat{e} \right] + c(\hat{e}) \\ & \leq \frac{1}{1 - t} (E [v - t\bar{v} - (1 - t)\hat{z}(v) - (1 - \hat{x}(v))\alpha v \mid \hat{e}] - E [(1 - \alpha)v - t\bar{v} \mid \tilde{e}]) \end{aligned}$$

which is equivalent to

$$E \left[\frac{1}{1 - t} v \mid \tilde{e} \right] - c(\tilde{e}) \leq E \left[\frac{1}{1 - t} v \mid \hat{e} \right] - c(\hat{e}). \quad (7)$$

Let $\tilde{e} = \arg \max E \left[\frac{1}{1 - t} v \mid e \right] - c(e)$. Note that $\tilde{e} < \hat{e}$ due to $\alpha < 1$ and concavity. Moreover, we know from part a) that for any contract we have $\hat{e} \leq \tilde{e}$. Hence, there exists no contract so that (7) holds with strict inequality, which proves part b) of the proposition. Part c) then follows immediately. With regard to the discussion following the proposition, note that the marginal revenue from exerting effort in the first-best solution is given by

$$\frac{dE[v|e]}{de} = \frac{d}{de} \left[v_h - \int_{v_l}^{v_h} F(v|e) dv \right] = - \int_{v_l}^{v_h} F_e(v|e) dv,$$

so that the first-best effort level is determined by

$$- \int_{v_l}^{v_h} F_e(v|e^{FB}) dv = c'(e^{FB}).$$

Hence, \tilde{e} is smaller (larger) than e^{FB} whenever $\frac{\alpha}{1-t}$ is smaller (larger) than 1. ■

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