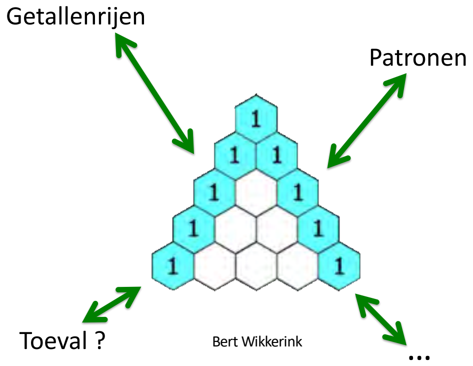
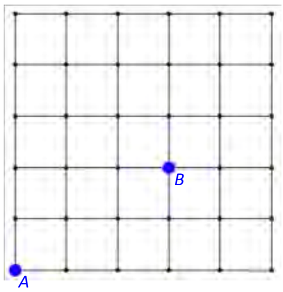
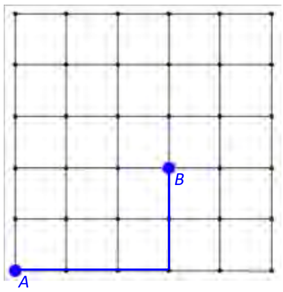
Getallen en een driehoek

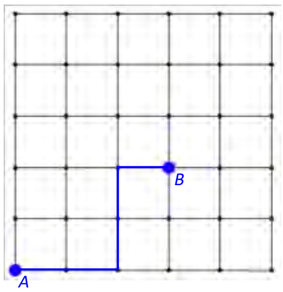


Hoe zat het ook alweer met Pascal?

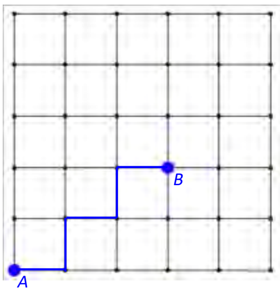




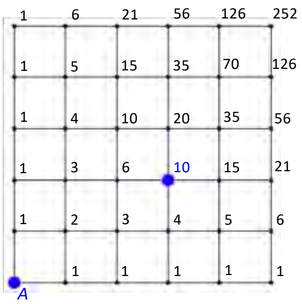
Een mogelijkheid …



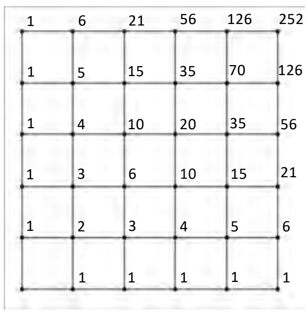
Een mogelijkheid … en nog een …

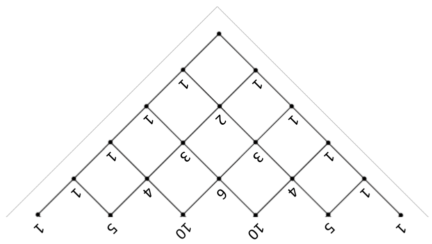


Een mogelijkheid … en nog een … en nog een …

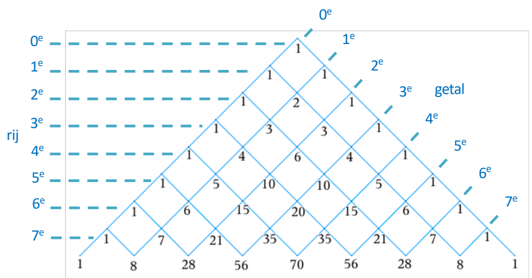


Drie naar rechts, twee omhoog: $\binom{5}{3} = \frac{5!}{3! \cdot 2!} = 10$





De driehoek van Pascal



n-de rij, k-de getal: $\begin{pmatrix} n \\ k \end{pmatrix} = \frac{n!}{k! \cdot (n-k)!} = n \text{ nCr } k$

Getallenrijen

1 3 5 7 9

1 2 4 8 16

1 1 2 3 5 8 13

De Cakegetallen

Hoeveel stukken kun je maximaal krijgen als je een cake 1 keer door snijdt?

En hoeveel stukken als je 2 keer snijdt?

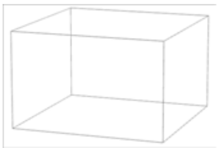
En als je 3 keer snijdt?

En 4 keer?

De Cakegetallen

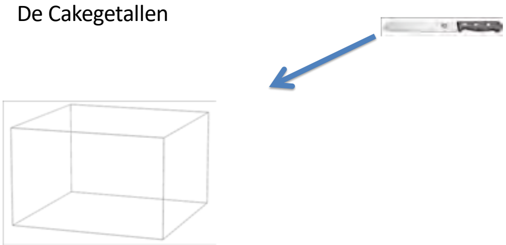
1

De Cakegetallen



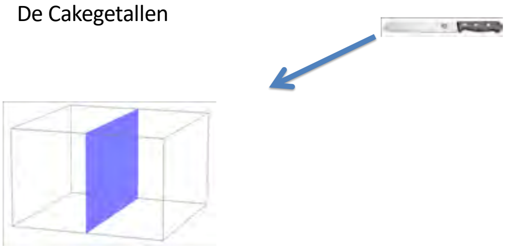
1

De Cakegetallen



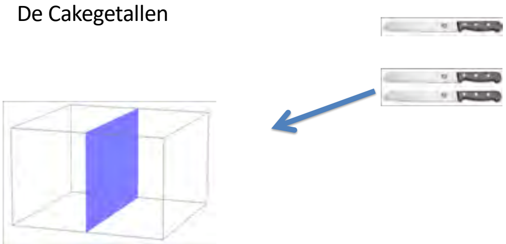
1 ?

De Cakegetallen



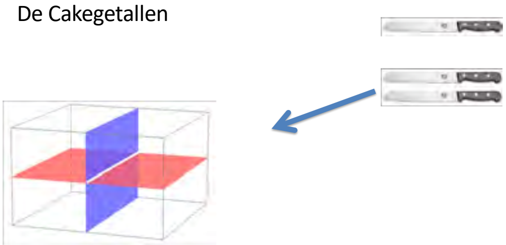
1 2

De Cakegetallen



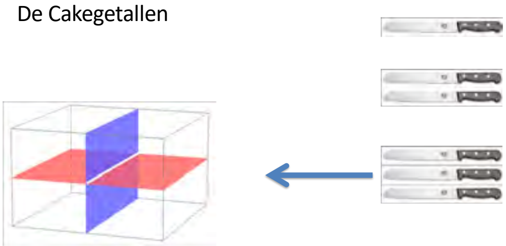
1 2 ?

De Cakegetallen



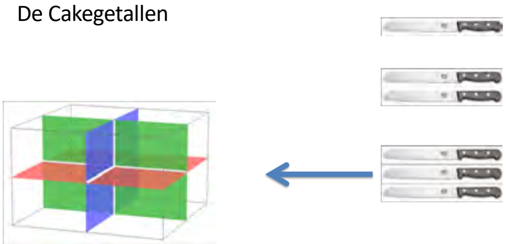
1 2 4

De Cakegetallen



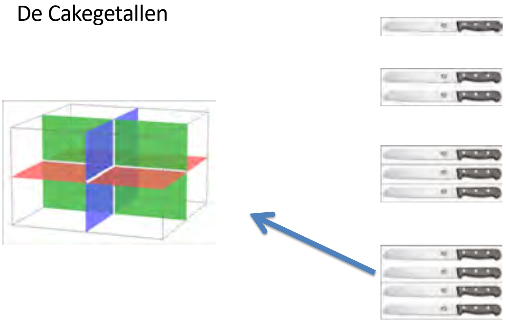
1 2 4 ?

De Cakegetallen



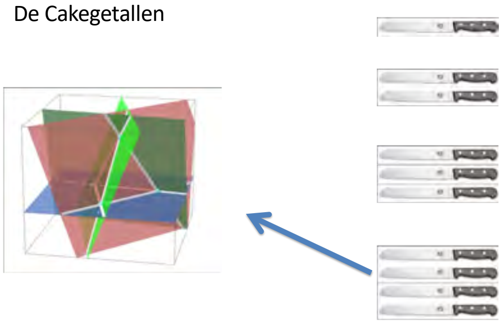
1 2 4 8

De Cakegetallen



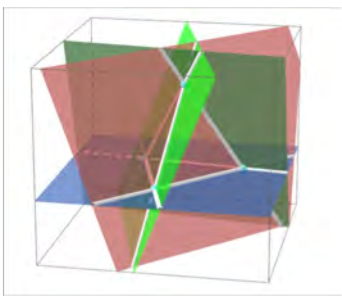
1 2 4 8 ?

De Cakegetallen

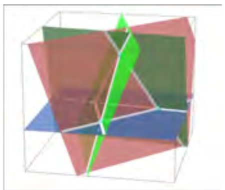


1 2 4 8 ?

De Cakegetallen



De Cakegetallen



1 2 4 8 15

De Cakegetallen

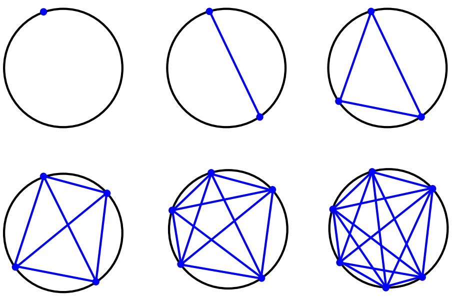
1 2 4 8 15 …

Wat is het volgende getal?

Welke formule past bij deze rij?

Maar eerst …

De Cirkelgetallen



De Cakegetallen

1 2 4 8 15 ...

De Cirkelgetallen

1 2 4 8 16 31 ...

Meer rijen met hetzelfde patroon

De banketstaafgetallen	1	2	3	...			
De pizzagetallen	1	2	4	7	...		
De cakegetallen	1	2	4	8	15	...	
De cirkelgetallen	1	2	4	8	16	31	...

Formules zoeken

De Cakegetallen

aantal snedes	0	1	2	3	4	5	6	7
aantal stukken	1	2	4	8	15	...	...	...

De Cakegetallen

Kijken naar verschillen

	aantal snedes	0	1	2	3	4	5	6	7
	aantal stukken	1	2	4	8	15	...	...	...
1e	verschil		1	2	4	7	...	...	...

De Cakegetallen

Kijken naar verschillen

	aantal snedes	0	1	2	3	4	5	6	7
	aantal stukken	1	2	4	8	15	...	...	...
1e	verschil		1	2	4	7	...	...	...
2e	verschil			1	2	3	...	...	...

De Cakegetallen

Kijken naar verschillen

	aantal snedes	0	1	2	3	4	5	6	7
	aantal stukken	1	2	4	8	15	…	…	…
1e	verschil		1	2	4	7	…	…	…
2e	verschil			1	2	3	…	…	…
3e	verschil				1	1	…	…	…

De Cakegetallen

Kijken naar verschillen

	aantal snedes	0	1	2	3	4	5	6	7
	aantal stukken	1	2	4	8	15	…	…	…
1e	verschil		1	2	4	7	…	…	…
2e	verschil			1	2	3	…	…	…
3e	verschil				1	1	1	…	…

De Cakegetallen

Kijken naar verschillen

	aantal snedes	0	1	2	3	4	5	6	7
	aantal stukken	1	2	4	8	15	…	…	…
1e	verschil		1	2	4	7	…	…	…
2e	verschil			1	2	3	4	…	…
3e	verschil				1	1	1	…	…

De Cakegetallen

Kijken naar verschillen

	aantal snedes	0	1	2	3	4	5	6	7
	aantal stukken	1	2	4	8	15	…	…	…
1e	verschil		1	2	4	7	11	…	…
2e	verschil			1	2	3	4	…	…
3e	verschil				1	1	1	…	…

De Cakegetallen

Kijken naar verschillen

	aantal snedes	0	1	2	3	4	5	6	7
	aantal stukken	1	2	4	8	15	26	…	…
1e	verschil		1	2	4	7	11	…	…
2e	verschil			1	2	3	4	…	…
3e	verschil				1	1	1	…	…

De Cakegetallen

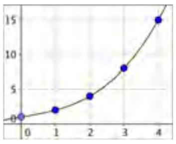
Kijken naar verschillen

	aantal snedes	0	1	2	3	4	5	6	7
	aantal stukken	1	2	4	8	15	26	42	64
1e	verschil		1	2	4	7	11	16	22
2e	verschil			1	2	3	4	5	6
3e	verschil				1	1	1	1	1

Derde verschil constant: derdegraads formule ?

Deze heeft de vorm: $y = ax^3 + bx^2 + cx + d$

x	y
0	1
1	2
2	4
3	8
4	15



Regressieanalyse geeft de formule: $y = \frac{1}{6}x^3 + \frac{5}{6}x + 1$

Hoe zit dat met de driehoek van Pascal?

1 2 3 ...

1 2 4 7 ...

1 2 4 8 15 ...

1 2 4 8 16 31 ...

Pizzagetallen

1							1
1	1						2
1	2	1					4
1	3	3	1				7
1	4	6	4	1			11
1	5	10	10	5	1		16
1	6	15	20	15	6	1	22

$$f(n) = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} = \frac{n!}{0!\,(n-0)!} + \frac{n!}{1!\,(n-1)!} + \frac{n!}{2!\,(n-2)!}$$

$$= \frac{n!}{n!} + \frac{n\cdot(n-1)!}{(n-1)!} + \frac{n\cdot(n-1)\cdot(n-2)!}{2\cdot(n-2)!}$$

$$= 1 + n + \tfrac{1}{2}n(n-1)$$

$$= \tfrac{1}{2}n^2 + \tfrac{1}{2}n + 1$$

Cakegetallen

1							1
1	1						2
1	2	1					4
1	3	3	1				8
1	4	6	4	1			15
1	5	10	10	5	1		26
1	6	15	20	15	6	1	42

$$f(n) = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} = \frac{n!}{0!\,(n-0)!} + \frac{n!}{1!\,(n-1)!} + \frac{n!}{2!\,(n-2)!} + \frac{n!}{3!\,(n-3)!}$$

$$= \cdots$$

$$= \tfrac{1}{6}n^3 + \tfrac{5}{6}n + 1$$

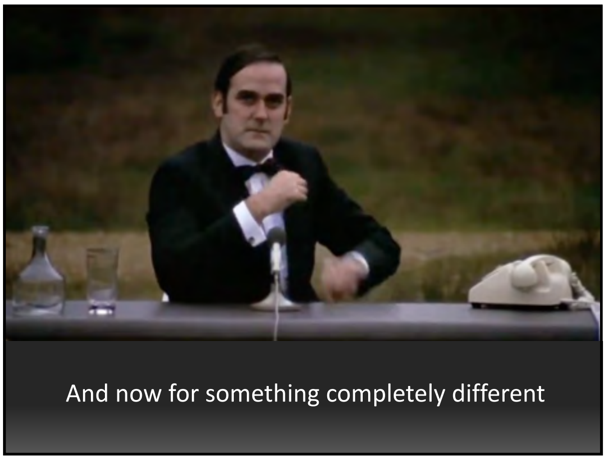
Cirkelgetallen

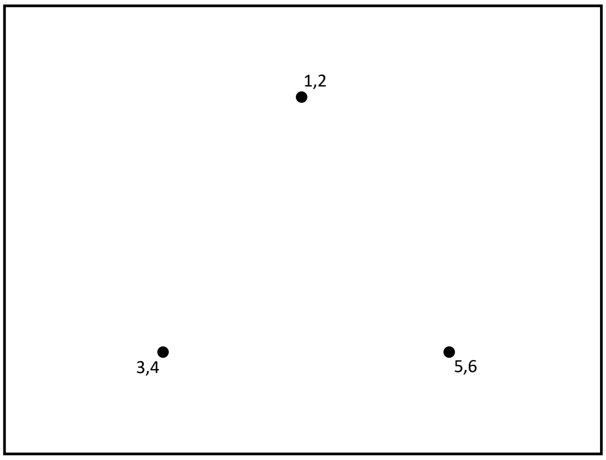
1							1
1	1						2
1	2	1					4
1	3	3	1				8
1	4	6	4	1			16
1	5	10	10	5	1		31
1	6	15	20	15	6	1	57

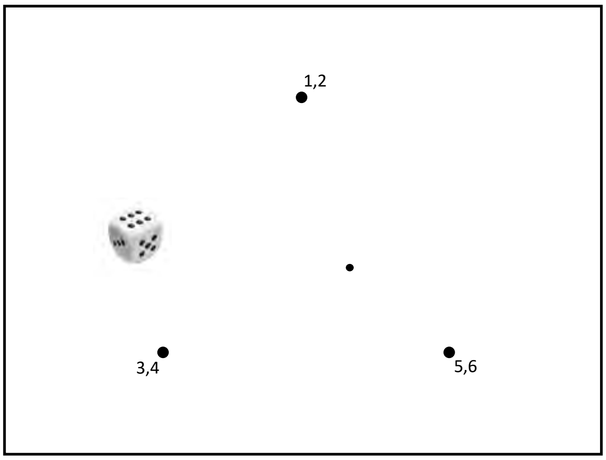
$$f(n) = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \binom{n}{4} = \cdots$$

$$= \cdots$$

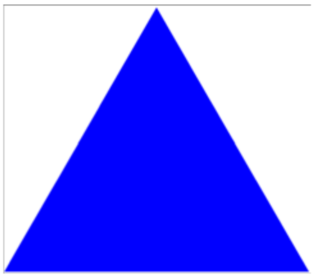
$$= \tfrac{1}{24}n^4 - \tfrac{1}{12}n^3 + \tfrac{11}{24}n^2 + \tfrac{7}{12}n + 1$$


And now for something completely different


1,2
3,4
5,6


1,2
3,4
5,6

Driehoek van Sierpinski (1915)



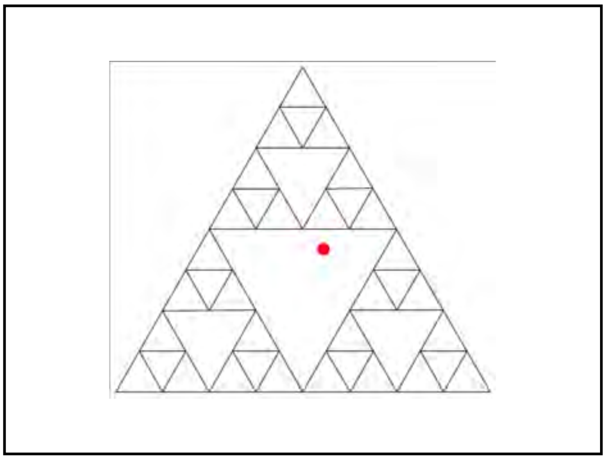
San Clemente-basiliek in Rome, 12e eeuw

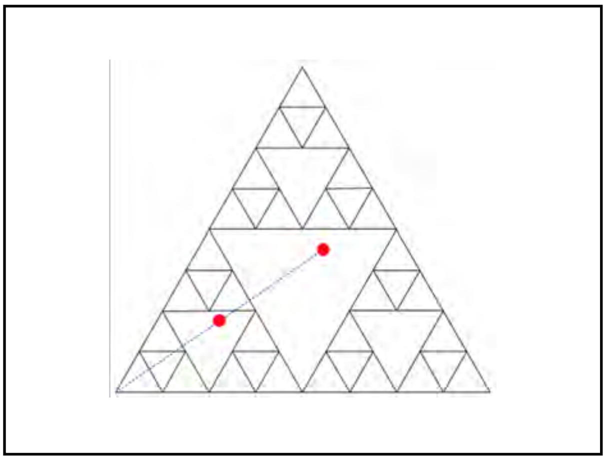
Santa Maria in Trastevere, 12e eeuw

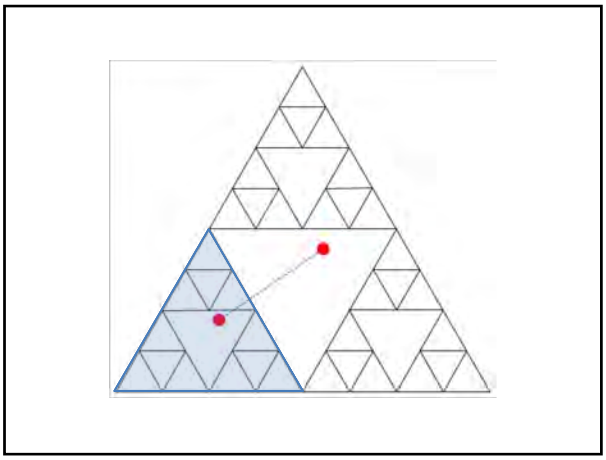
Westminster Abbey, 13e eeuw

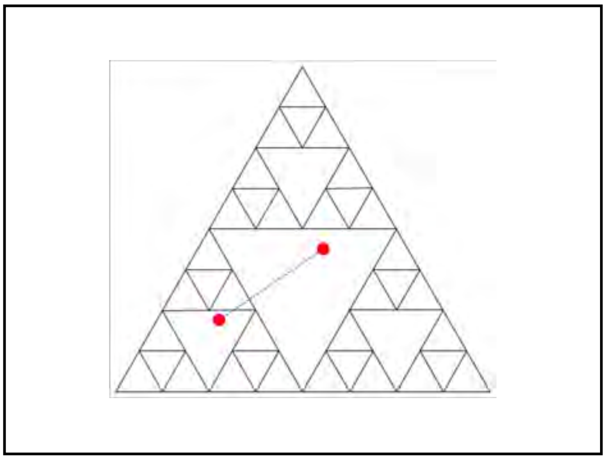
Salvador Dali: "Gezicht van de oorlog", 20e eeuw

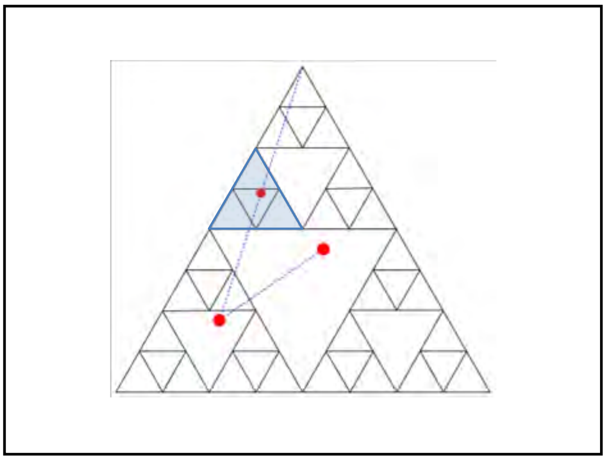

Toeval is logisch
J. Cruijff

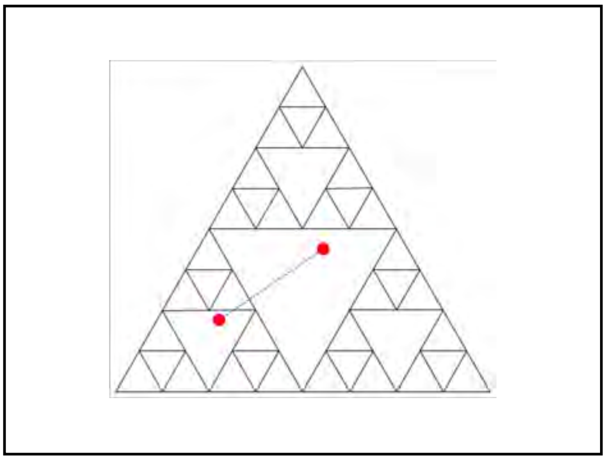


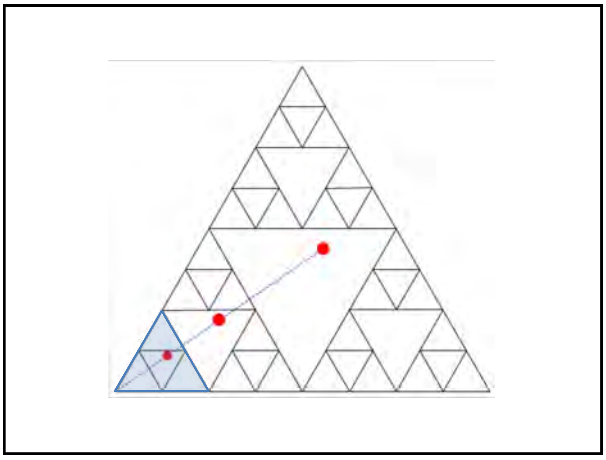


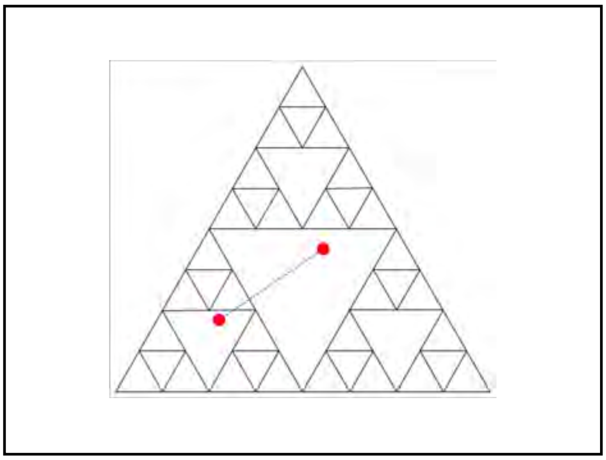


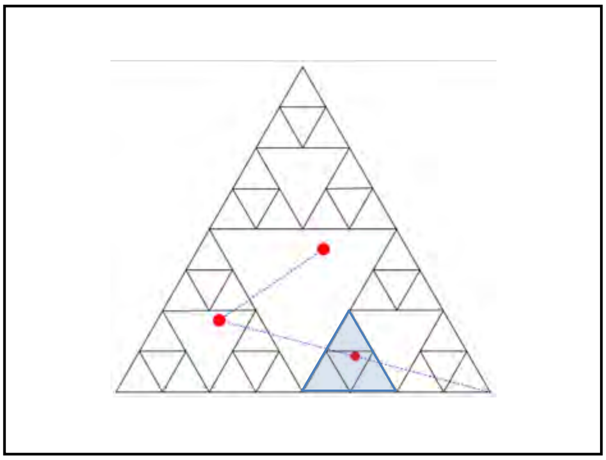


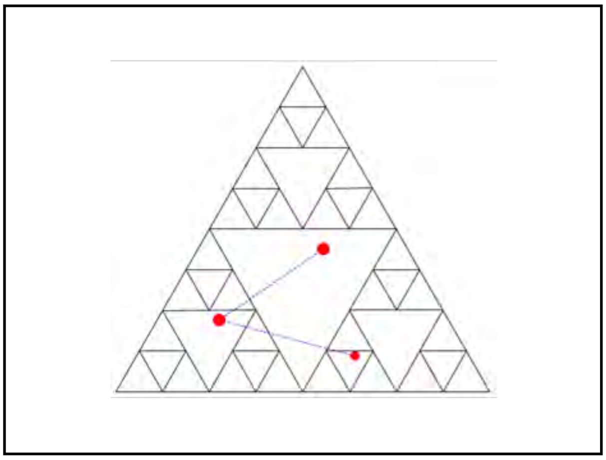


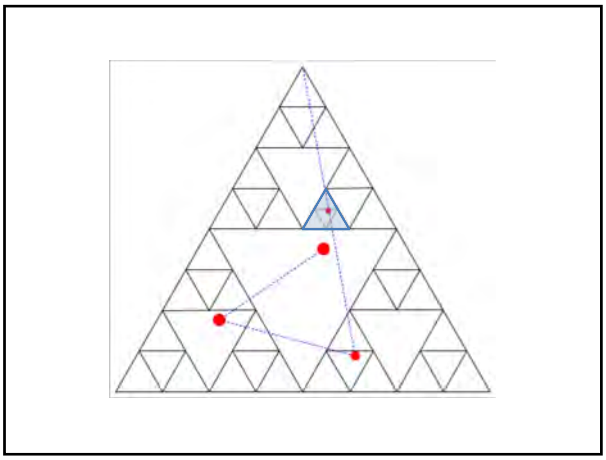


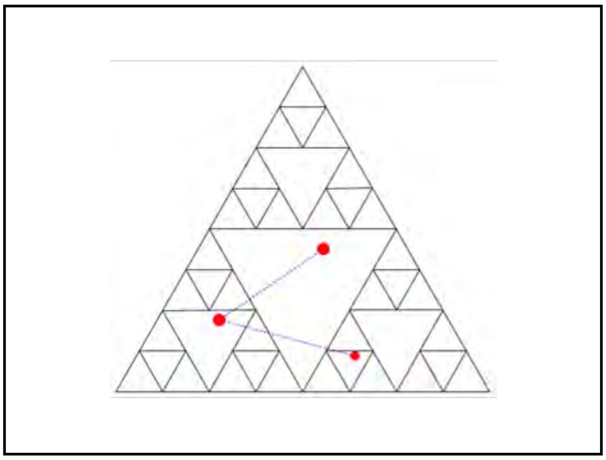


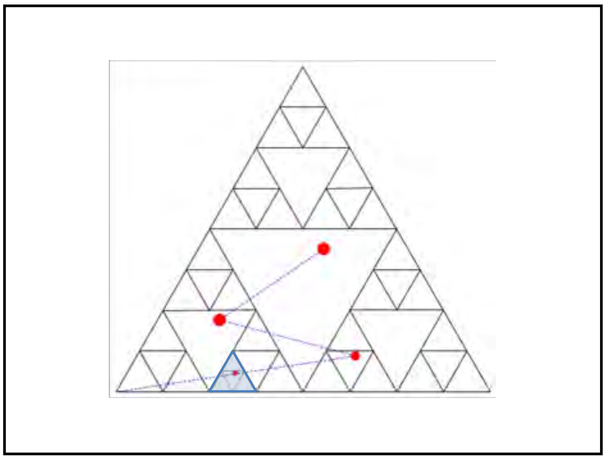


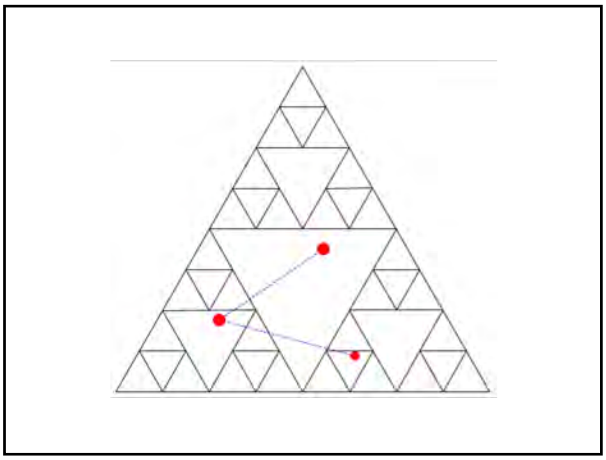


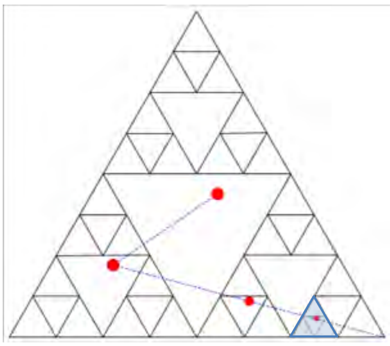






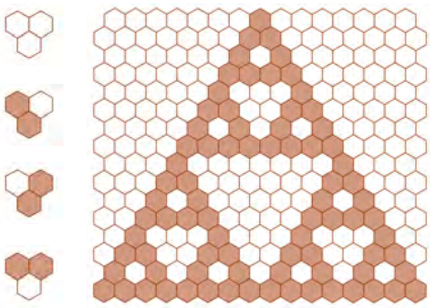




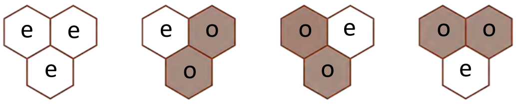


Een andere benadering

Een andere benadering

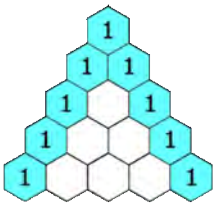


Rekenregels

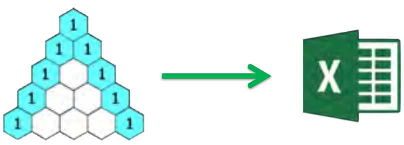


e + e = e   e + o = o   o + e = o   o + o = e

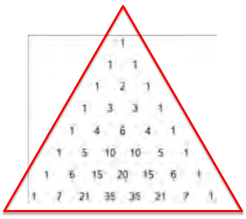
Terug naar Pascal



Pascal met Excel



Nog meer rijen!



Fibonacci in de driehoek van Pascal

1					
1	1				
1	2	1			
1	3	3	1		
1	4	6	4	1	
1	5	10	10	5	1

$f(0)=\binom{0}{0}=1$

	1				
1					
1	1				
1	2	1			
1	3	3	1		
1	4	6	4	1	
1	5	10	10	5	1

$f(0)=\binom{0}{0}=1$

$f(1)=\binom{1}{0}+\binom{0}{1}=1$

	1	1			
1					
1	1				
1	2	1			
1	3	3	1		
1	4	6	4	1	
1	5	10	10	5	1

$f(0)=\binom{0}{0}=1$

$f(1)=\binom{1}{0}+\binom{0}{1}=1$

$f(2)=\binom{2}{0}+\binom{1}{1}+\binom{0}{2}=2$

	1	1	2		
1					
1	1				
1	2	1			
1	3	3	1		
1	4	6	4	1	
1	5	10	10	5	1

$f(0) = \binom{0}{0} = 1$

$f(1) = \binom{1}{0} + \binom{0}{1} = 1$

$f(2) = \binom{2}{0} + \binom{1}{1} + \binom{0}{2} = 2$

$f(3) = \binom{3}{0} + \binom{2}{1} + \binom{1}{2} + \binom{0}{3} = 3$

	1	1	2	3		
1						
1	1					
1	2	1				
1	3	3	1			
1	4	6	4	1		
1	5	10	10	5	1	

$f(0) = \binom{0}{0} = 1$

$f(1) = \binom{1}{0} + \binom{0}{1} = 1$

$f(2) = \binom{2}{0} + \binom{1}{1} + \binom{0}{2} = 2$

$f(3) = \binom{3}{0} + \binom{2}{1} + \binom{1}{2} + \binom{0}{3} = 3$

$f(4) = \binom{4}{0} + \binom{3}{1} + \binom{2}{2} + \binom{1}{3} + \binom{0}{4} = 5$

	1	1	2	3	5	
1						
1	1					
1	2	1				
1	3	3	1			
1	4	6	4	1		
1	5	10	10	5	1	

$f(0) = \binom{0}{0} = 1$

$f(1) = \binom{1}{0} + \binom{0}{1} = 1$

$f(2) = \binom{2}{0} + \binom{1}{1} + \binom{0}{2} = 2$

$f(3) = \binom{3}{0} + \binom{2}{1} + \binom{1}{2} + \binom{0}{3} = 3$

$f(4) = \binom{4}{0} + \binom{3}{1} + \binom{2}{2} + \binom{1}{3} + \binom{0}{4} = 5$

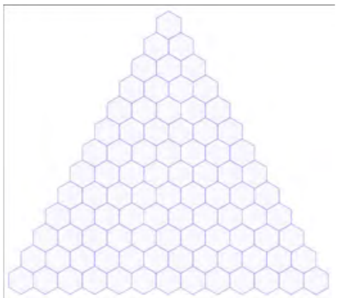
$f(5) = \binom{5}{0} + \binom{4}{1} + \binom{3}{2} + \binom{2}{3} + \binom{1}{4} + \binom{0}{5} = 8$

...

$f(n) = \sum_{i=0}^{n} \binom{n-i}{i}$ (rekenmachine)

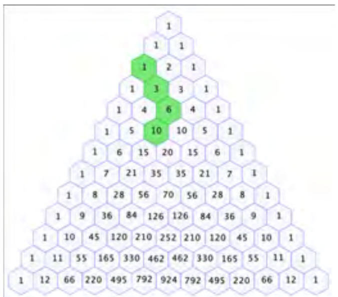
	1	1	2	3	5	8
1						
1	1					
1	2	1				
1	3	3	1			
1	4	6	4	1		
1	5	10	10	5	1	

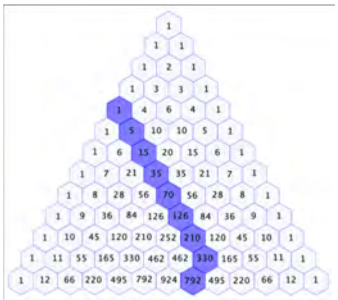

De punten moeten dáár
op de i gezet worden,
waar ze horen
J. Cruijff

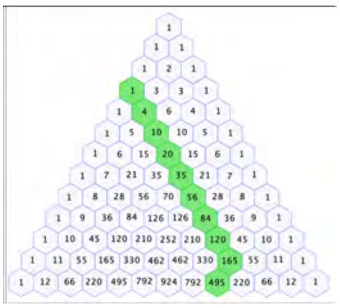


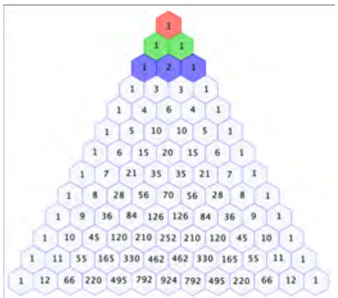
H

E

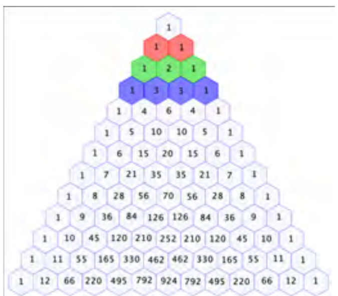

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1



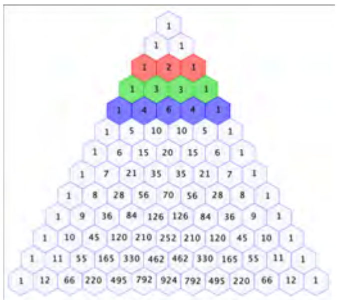




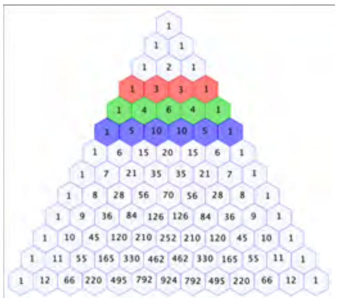
$$\frac{(1)\times(1\times2\times1)}{(1\times1)^2} = 2$$



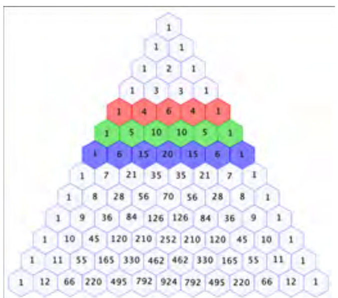
$$\frac{(1\times1)\times(1\times3\times3\times1)}{(1\times2\times1)^2} = 2{,}25$$



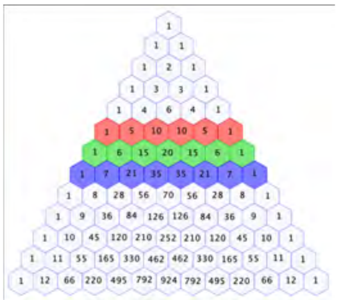
$$\frac{(1\times2\times1)\times(1\times4\times6\times4\times1)}{(1\times3\times3\times1)^2}\approx 2,37$$



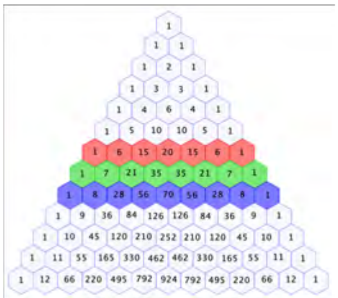
$$\frac{(1\times3\times3\times1)\times(1\times5\times10\times10\times5\times1)}{(1\times4\times6\times4\times1)^2}\approx 2,4414$$



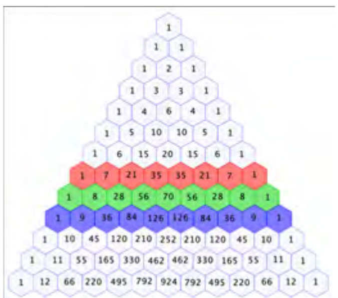
$$\frac{(1\times4\times6\times4\times1)\times(1\times6\times15\times20\times15\times6\times1)}{(1\times5\times10\times10\times5\times1)^2}\approx 2,48832$$



$$\frac{(1\times5\times10\times10\times5\times1)\times(1\times7\times21\times35\times35\times21\times7\times1)}{(1\times6\times15\times20\times15\times6\times1)^2}\approx 2{,}52163$$



$$\frac{(1\times6\times15\times20\times15\times6\times1)\times(1\times8\times28\times56\times70\times56\times28\times8\times1)}{(1\times7\times21\times35\times35\times21\times7\times1)^2}\approx 2{,}54670$$



$$\frac{P_{n-1}\cdot P_{n+1}}{P_n^2}\approx 2{,}71828\ldots$$

$$\lim_{n\to\infty}\frac{P_{n-1}\cdot P_{n+1}}{P_n^{\ 2}}=e$$

Bewijs:

$$P_n=\prod_{k=0}^{n}\frac{n!}{k!(n-k)!}$$

$$\frac{P_{n+1}}{P_n}=\cdots=\frac{(n+1)^n}{n!}$$

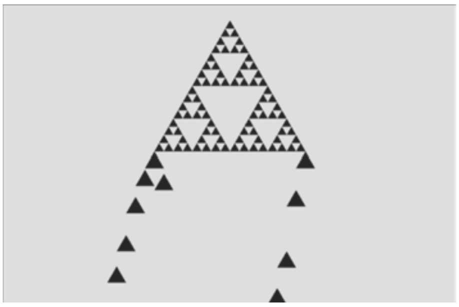
$$\frac{P_{n-1}\cdot P_{n+1}}{\left(P_n\right)^2}=\cdots=\left(1+\frac{1}{n}\right)^n$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e$$


1170 - 1250


1623 - 1662


1707 - 1783



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