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# Non-Linear Dynamics and Chaos

## Complex Systems Studies Workshop

Thomas Gomez

PhD Candidate at U.S.E. and MI

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# Definitions



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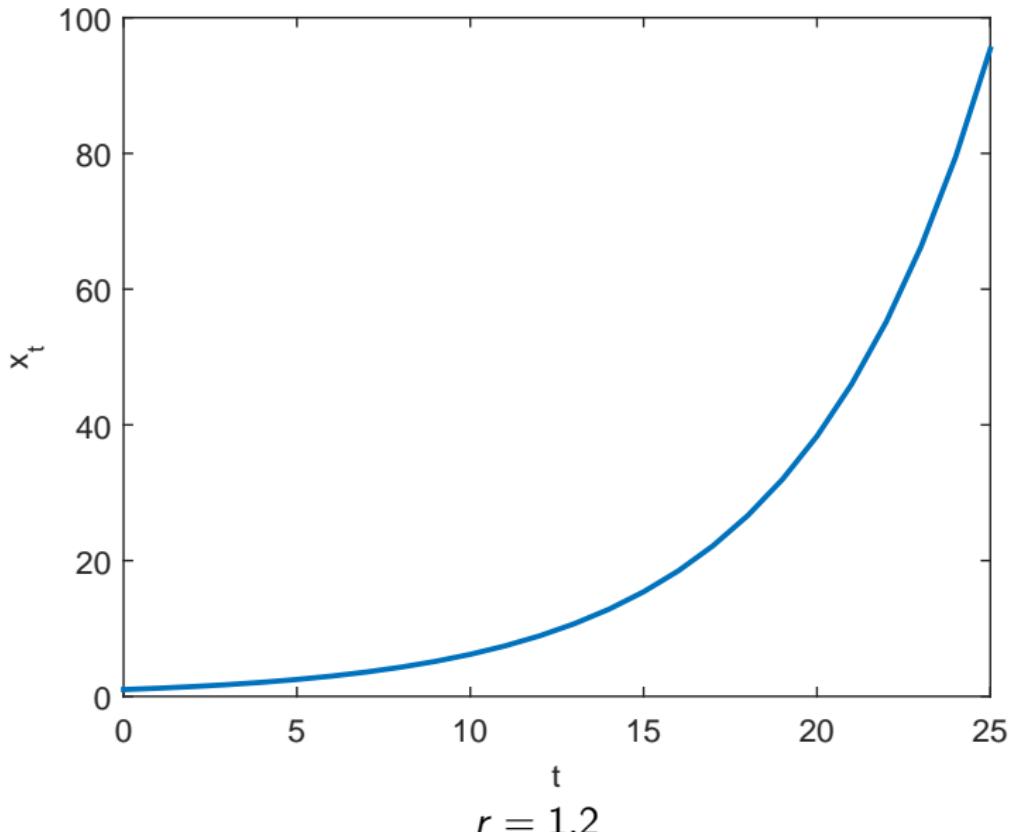


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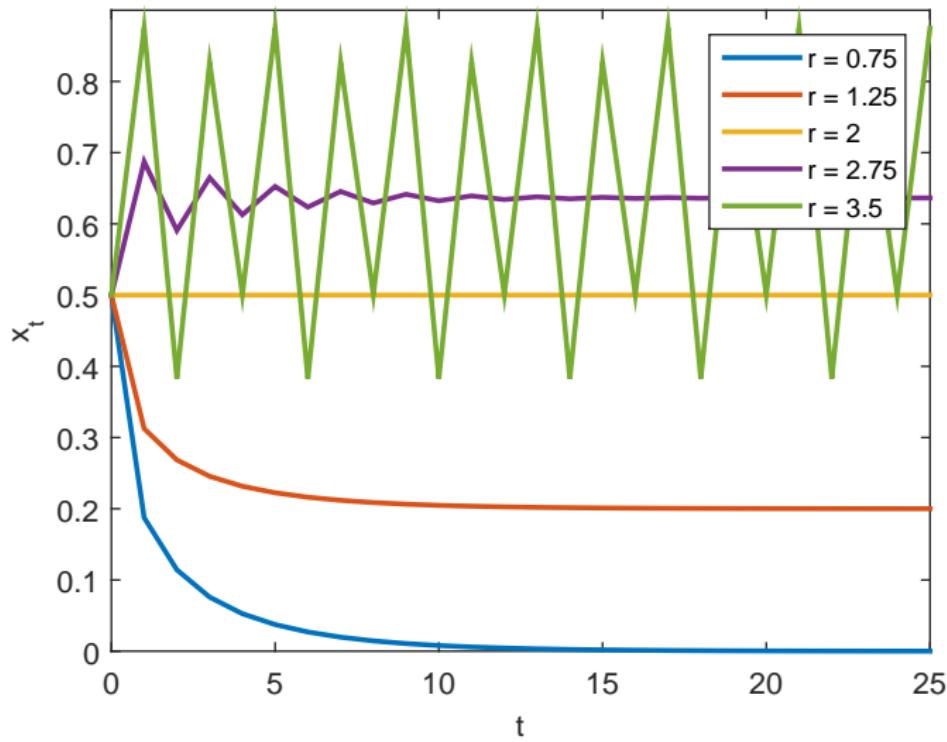
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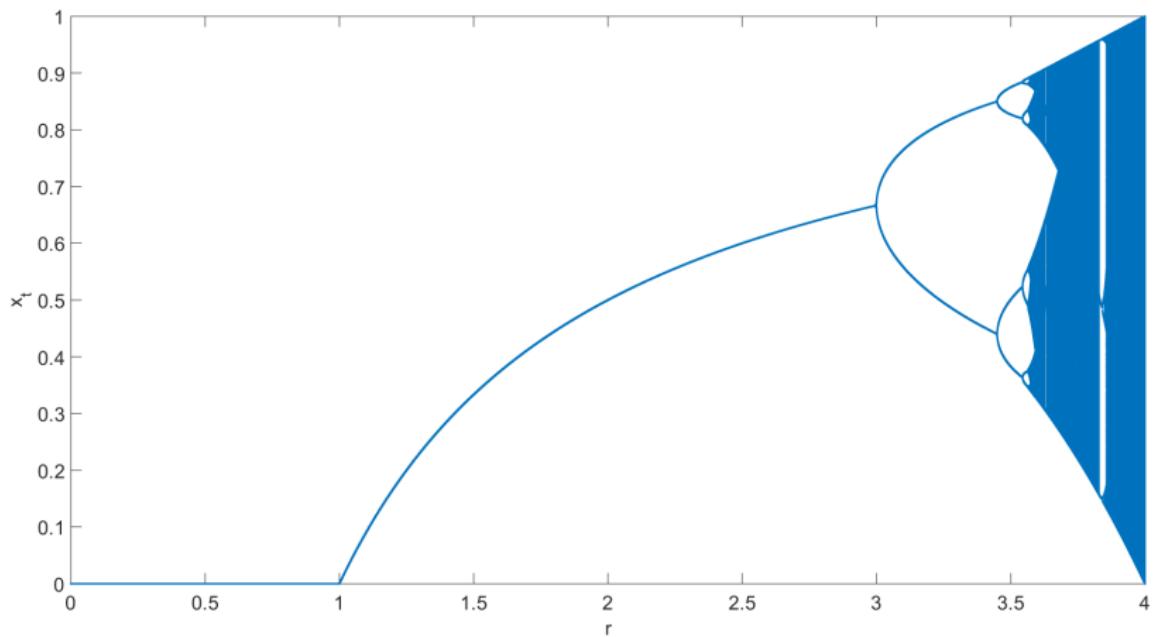


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  - Deterministic

# Dynamics



# Bifurcation Diagram



# Chaos



- Apparent randomness

# Chaos



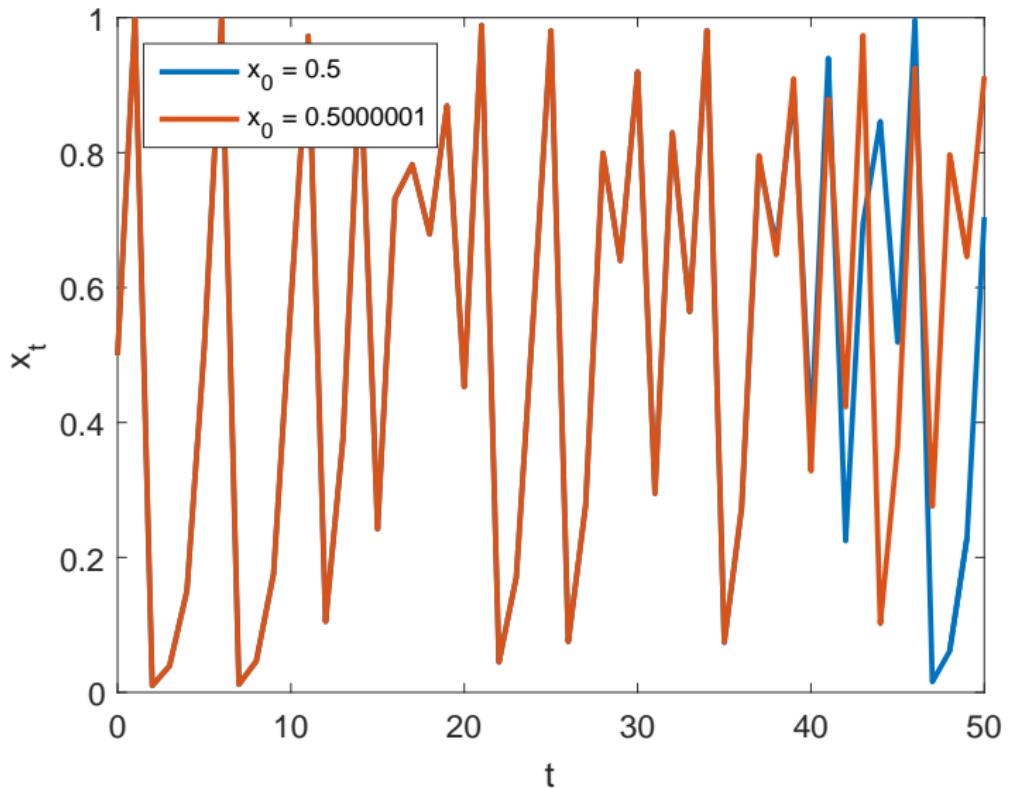
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- Long-term unpredictability

# Chaos



- Apparent randomness
- Deterministic
- Sensitive dependence on initial conditions (*butterfly effect*)
- Long-term unpredictability
- Structure

# Attractor



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- Embedding

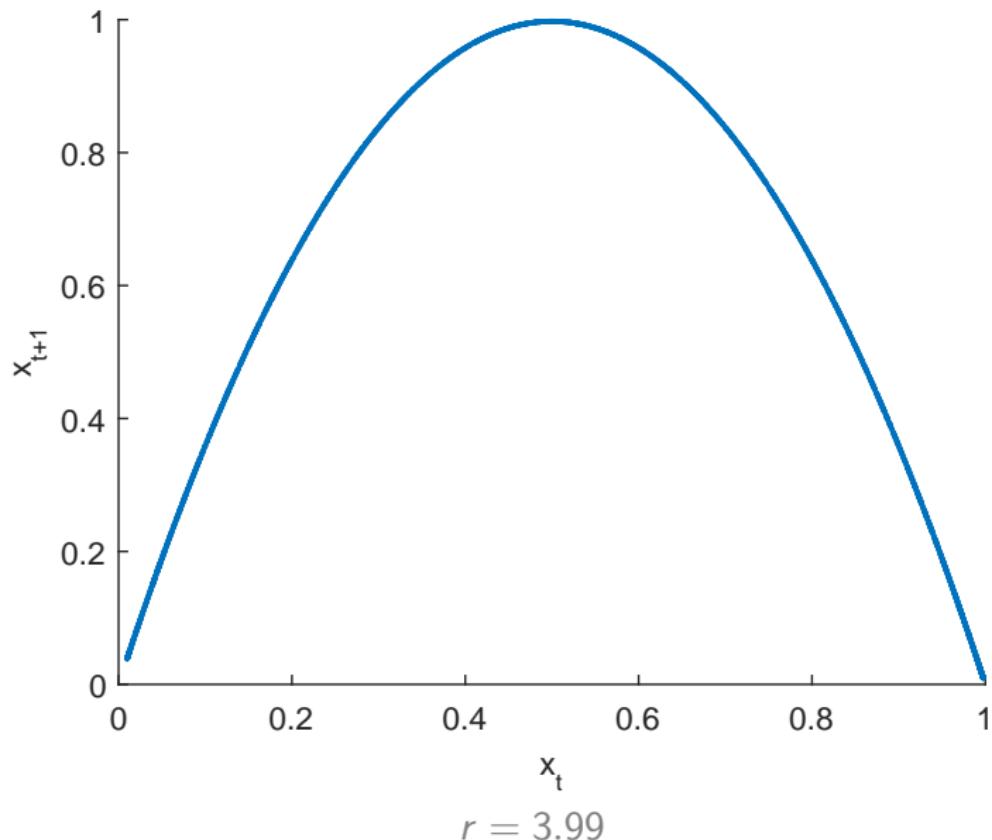
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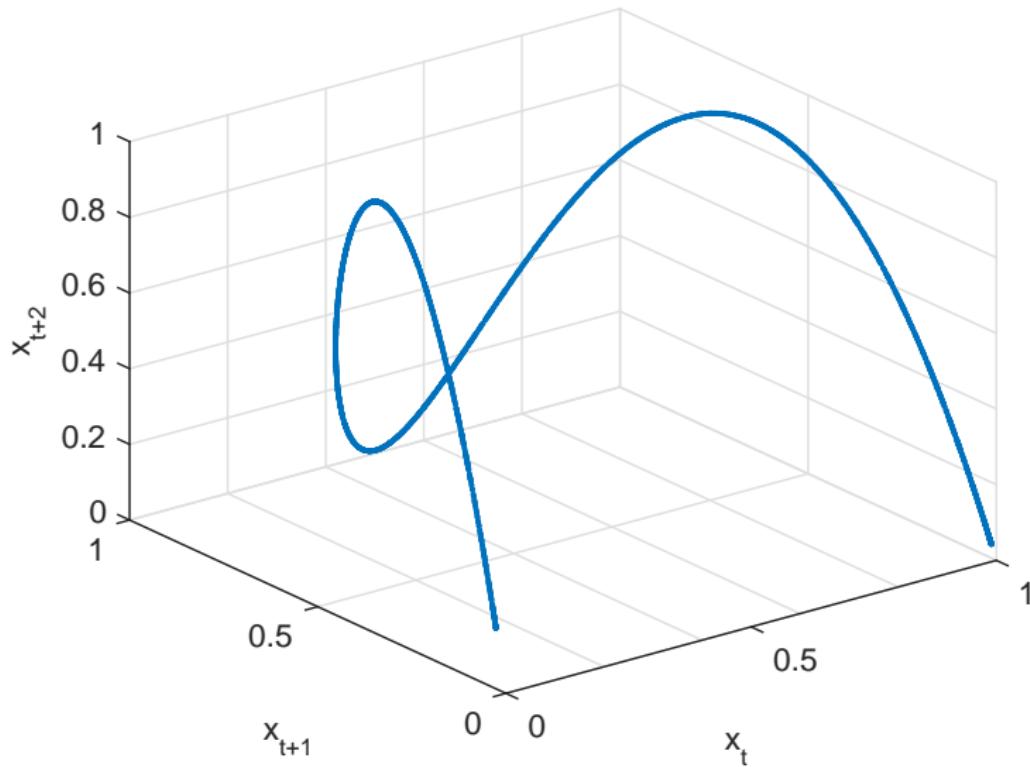
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- Embedding
- Return Map

## Attr



## Attr



$$r = 3.99$$

# Attractor



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- Embedding
- Return Map

# Attractor



- Embedding
- Return Map
- Dimension (box-counting algorithm)



Thank you!

Questions?