Moving towards understanding:
Reasoning about graphs in primary mathematics education

Carolien Duijzer

Supervisors and co-supervisors:
Marja van den Heuvel-Panhuizen
Paul Leseman
Michiel Veldhuis
Michiel Doorman
Chapter 2
(Study 1)

Micro-development

Chapter 3
(Study 2)

Intervention
Reasoning about motion graphs, through physical experiences
(i.e., embodied learning environment)

Macro-development

Chapter 4
(Study 2)

Chapter 5
(Study 3)

Graphing motion

Early algebra

Review
Embodied learning environments

Domain of linear equations
Part I

Theoretical framing
Question 1

Embodied learning environment supporting students’ understanding of graphing motion, what is reported on:

1. Embodied configuration?
2. Presumed factors mediating learning?
3. Relationship between configuration and the factors mediating learning?
4. Efficacy of embodied learning environments.
Embodied cognition theories differ in how they conceptualize the relationship between (lower-order) sensorimotor processes and (higher-order) abstract cognitive processes: simple to radical embodiment
simple to radical embodiment?
Operationalizing embodied learning environments for graphing motion

A. Indirectly and Directly involving the physical body
   • From observing and influencing other (human) movements to making movements oneself.

*Levels of bodily involvement*

B. Immediate and Non-immediate cognitive activities
   • the immediacy of the embodiment of cognitive activities can differ between learning situations

*Levels of immediacy*
Observing and influencing others/objects’ motion in which bodily involvement takes place in the absence of direct environmental stimuli

Whole and part bodily motion in which bodily involvement takes place in the absence of direct environmental stimuli

Observing and influencing others/objects’ motion in which bodily involvement takes place in the presence of direct environmental stimuli

Whole and part bodily motion in which bodily involvement takes place in the presence of direct environmental stimuli
Immediate

Own motion

Whole bodily motion

Part bodily motion

Others/objects’ motion

Influencing others/objects’ motion

Looking at or observing others/objects’ motion

Non-immediate

Own motion

Whole bodily motion

Part bodily motion

Others/objects’ motion

Influencing others/objects’ motion

Looking at or observing others/objects’ motion

Class I

Class II

Class III

Class IV

(\(n=26\))

(\(n=8\))

(\(n=4\))

(\(n=8\))

(\(n=3\))

(\(n=1\))

(\(n=6\))

(\(n=6\))
Mediating factors within embodied learning environments

- Real-world context *Referring to experiences of students with or in the real world*
- Multimodality *Referring to intertwining modalities*
- Linking motion to graph *Referring to linking motion to the graphical representation*
- Multiple representations *Referring to multiple representations of a particular motion*
- Semiotics *Referring to the use of meaning-supported sign systems*
- Student control *Referring to students being in control of the learning activity*
- Attention capturing *Referring to aspects in the learning environment that captures students attention*
- Cognitive conflict *Referring to conflicting conceptions*
Immediate Own Motion (n=34)

Immediate Others/Objects’ Motion (n=12)

Non-immediate Own Motion (n=4)

Non-immediate Others/Objects’ Motion (n=12)
Question 1

Embodied learning environment supporting students’ understanding of graphing motion, what is reported on:

1. Embodied configuration?
2. Presumed factors mediating learning?
3. Relationship between configuration and the factors mediating learning?
4. Efficacy of embodied learning environments.
Room for discussion I

How can we define the role of conceptual metaphors (or the building of conceptual metaphor) across embodied learning environments (differing on their embodied configuration)? Think for example about:

A. Learning environments providing a direct physical link with relevant source-domain (embodied) experiences versus learning environments that provide an indirect physical link with relevant source-domain (embodied) experiences.

B. Conceptual metaphors that are explicit and active versus conceptual metaphors that are implicit and internal
In the review we did not distinguish between observing the movement of a human model, and observing the movement of an object, even though some studies have suggested this is a relevant aspect to consider.

A. Mirroring hypothesis
B. Action repertoire (human versus object)
Part II-A

The potential of an embodied learning environment – consisting of a six-lesson teaching sequence – to support students’ HOT, as their reasoning about graphing motion
Understanding graphs describing dynamic situations (e.g., distance changing over time) is a core goal of mathematics education.

Developing a “graph sense”

“looking at the entire graph (or parts of it) and gaining meaning about the relationship between the two variables and, in particular, their pattern of co-variation”

(Leinhardt et al., 1990)

Some empirical evidence that students are able to reason with, and construct (graphical) representations of dynamic data:

- Inventing graphical representations (of motion) (DiSessa et al., 1991)
- Reasoning about graphical representations generated by motion sensors (Nemirovsky & Tierney, 1998)
- Reasoning about discrete and continuous change (deBeer et al., 2015)

(Duijzer et al., 2018)
The current study

Teaching sequence
• Goal: Understanding of graphs describing motion (i.e., time-distance graphs)
• 6 lessons
• Embodied learning environment

Students move in front of a motion sensor, graphing their movement

Participants:
• 70 fifth-grade students (about 10/11 years)

(Duijzer et al., 2018)
Motion sensor technology

- Activities in which students create **time-distance graphs describing their own movements**
- Coupling **action, perception** and **reasoning**

Alignment with embodied cognition theory

(Duijzer et al., 2018)
Example: Bas (7 years old)

‘This is an example of a graph, can you try to walk this graph?’
Example: Timon (11 years old)

‘This is an example of a graph, can you try to walk this graph?’
Question 2

How does students’ reasoning about graphing motion develop over a six-lesson teaching sequence within an embodied learning environment?

Micro-development over the teaching sequence

Focus:
A. Changes over time in students’ level of reasoning when interpreting and constructing graphs.
B. Relation between students’ perceptual motor experiences and their reasoning about the graphs.
Teaching sequence

LESSON 1
Motion: reflecting and representing

LESSON 2
From discrete to continuous representations of change

LESSON 3
Continuous graphs of ‘distance to’ (1)

LESSON 4
Continuous graphs of ‘distance to’ (2)

LESSON 5
Scaling on the graphs’ axes

LESSON 6
Multiple movements and their graphical representation

Lesson-specific graph interpretation tasks

Task 1
Task 2

Task 2
Task 3

Task 3
Task 4

Task 4
Task 5

Task 5
Task 6

Task 6

(Duijzer et al., 2018)
Lesson-specific test items (micro)

Assessing the students’ ability to (1) **represent**, (2) **interpret** and (3) **reason about** graphical representations

**Graph construction**

**Graph interpretation**

**Example:**

Assessing the students’ ability to (1) **represent**, (2) **interpret** and (3) **reason about** graphical representations

**Graph construction**

**Graph interpretation**
Assessing the students’ ability to (1) represent, (2) interpret and (3) reason about graphical representations
Example 1: reasoning on the tasks

[NO] “The boat goes straight over the water, and does not go up sidewards.”

[YES] “The hot air balloon goes up sidewards, just as the line”

[YES] “The ferris wheel makes a turn, and the line is slanted”
Example 1: reasoning on the tasks

Lesson 3

[YES] “The distance becomes bigger, just as with the hot air balloon, and that is also represented in the graphs.”

[YES] “He goes straight up, so the distance becomes larger, which is also shown in the graph.”

[NO] “The ferris wheel makes a turn, so the distance has to become bigger and smaller, small bumps [in the graph]”
<table>
<thead>
<tr>
<th>Reasoning Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>R0 Unrelated reasoning</td>
<td>“Because you cannot make turns”</td>
</tr>
<tr>
<td>R1 Iconic reasoning</td>
<td>“In the graph it goes up and here the airplane also goes up”</td>
</tr>
<tr>
<td>R2 Single variable reasoning</td>
<td>“The boat moves forwards and the graph as well, if the graph goes up it means you go forward.”</td>
</tr>
<tr>
<td>R3 Dual variable reasoning</td>
<td>“Yes, because he travelled a ‘certain amount of distance’ within a certain amount of time”</td>
</tr>
</tbody>
</table>
RQ 1: Development over time

Students’ level of reasoning ($N_{Lesson 1}=67$, $N_{Lesson 2}=63$, $N_{Lesson 3}=67$, $N_{Lesson 4}=69$, $N_{Lesson 5}=65$, $N_{Lesson 6}=64$)

(Duijzer et al., 2018)
Perceptual motor experiences and students’ reasoning about graphs

Celine: “He is making them bigger.”
Celine: “They have to be closer together”
Teacher: “How could we make the graphs more similar?”
Celine: “A little faster…a little faster and a slightly shorter distance?”

(Duijzer et al., 2018) Math seminar – Nov 2020
How does students’ reasoning about graphing motion develop over a six-lesson teaching sequence within an embodied learning environment?

• Reasoning became more sophisticated (i.e., iconic reasoning $\rightarrow$ multiple variable reasoning)

• Students’ reasoning about the graphical representation of the motion was stimulated and strengthened by:
  - Perceptual-motor experiences (provided by the motion sensor)
  - Working together in groups and looking at each others’ movements
Part II-B

The potential of an embodied learning environment – consisting of a six-lesson teaching sequence – to support students’ HOT, as their reasoning about graphing motion
Question 3

To what extent does embodied support in a six-lesson teaching sequence on graphing motion affect the development of students’ graphical reasoning?

Macro-development over the school year
Condition 1
Indirect embodied support
(on paper – digital blackboard)

Condition 2
Direct embodied support
(physical – digital blackboard)
Conditions: Differences & similarities

Experimental set-up:
- Two parallel conditions: same content and tasks
- Different tools

Embodied condition

1. Embodied tasks (lesson 1)
2. Worksheet model:

Non-embodied condition

1. Disembodied tasks (lesson 1)
2. Worksheet model:
Embodiment setting:

(Duijzer et al., 2020) Math seminar – Nov 2020
Taking into account students’ level of reasoning

Four categories:

R0 No reasoning
R1 Iconic reasoning
R2 Single-variable reasoning
R3 Multiple-variable reasoning
Between which points does the car travels fastest?

<table>
<thead>
<tr>
<th>Geen redeneren [R0]</th>
<th>Iconisch redeneren [R1]</th>
<th>Redeneren met 1 variabele [R2]</th>
<th>Redeneren met meerdere variabelen [R3]</th>
</tr>
</thead>
<tbody>
<tr>
<td>It was a guess</td>
<td>[F] Because this is the highest point</td>
<td>[B, C] Between point B and C, there the line goes straight up, and he travels 8 kilometer</td>
<td>[B,C] The car travels 8 kilometer in 5 minutes and that is the longest distance in the shortest time period</td>
</tr>
</tbody>
</table>
A train ride.
A train travels **twice as fast** between 10:00 and 11:00 o’clock than between 11:00 and 12:00 o’clock. The train stands still from 12:00 to 13:00 o’clock.

3a. Draw a graph that fits the description above.
3b. How do you know?

*Score: correct (1), incorrect (0)*
Indirect embodied support

(Duijzer et al., 2020)
Direct embodied support

(Duijzer et al., 2020)

Math seminar – Nov 2020
Indirect embodied support

Direct embodied support

(Duijzer et al., 2020)
A train ride.
A train travels twice as fast between 10:00 and 11:00 o’clock than between 11:00 and 12:00 o’clock. The train stands still from 12:00 to 13:00 o’clock.

3a. Draw a graph that fits the description above.
3b. How do you know?

Score: correct (1), incorrect (0)

(Duijzer et al., 2020)
Question 3

To what extent does embodied support in a six-lesson teaching sequence on graphing motion affect the development of students’ graphical reasoning?

• Strong effect of the intervention (both direct and indirect embodied support)
• Condition was found to be a predictor of the intervention effect. Thus student’s’ receiving direct embodied support during the teaching sequence displayed higher levels of reasoning after the intervention than students that received indirect embodied support.
Room for discussion III

How does the embodied cognitive mechanism (metaphor?), that mediates bodily experiences directly, bring forth new abstract forms of meaning (HOT)?
Room for discussion IV

What are **opportunities** and **challenges** to measure embodied learning. Which tools could be used; how do we obtain a deeper understanding of how perception-action processes in embodied learning environments activate, change, combine, and blend elementary embodied cognitions to ground abstract mathematical concepts? (What are **opportunities** and **challenges** when these types of activities (embodied learning tasks) are implemented in the primary school (mathematics) classrooms?)
MOVING TOWARDS UNDERSTANDING: REASONING ABOUT GRAPHS IN PRIMARY MATHEMATICS EDUCATION
CAROLIEN DUIJZER
Publications related to this thesis (further reading)


Thank you for your attention!

c.duijzer@hsmarnix.nl