Fourierreeksen: daar zit muziek in!

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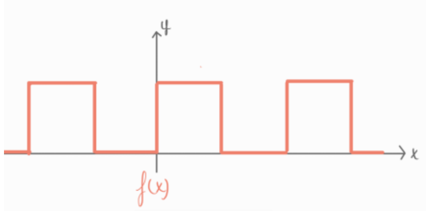
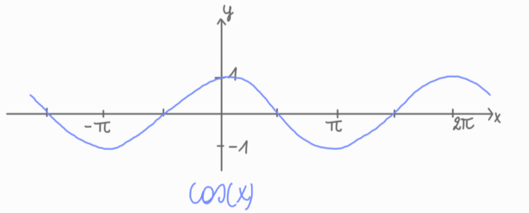
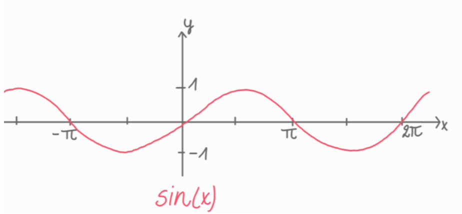
Fourierreeks

= oneindige *reeks* die een periodieke functie weergeeft in termen van cosinus en sinus







$$f(x) = \frac{1}{2} + \frac{2}{\pi}\sin(x) + \frac{2}{3\pi}\sin(3x) + \frac{2}{5\pi}\sin(5x) + \dots$$

Taylor- en Maclaurinreeks

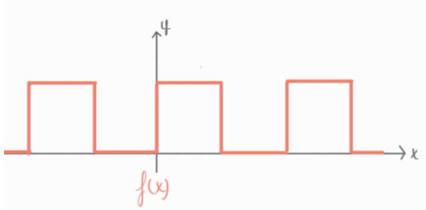
$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \dots = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

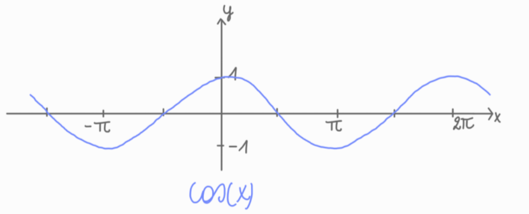
$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

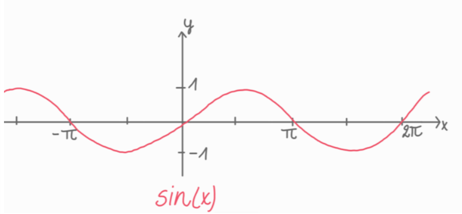
Jean-Baptiste Joseph Fourier
Franse wiskundige en natuurkundige
1768-1830

Fourierreeks

= oneindige *reeks* die een periodieke functie weergeeft in termen van cosinus en sinus







$$f(x) = \frac{1}{2} + \frac{2}{\pi}\sin(x) + \frac{2}{3\pi}\sin(3x) + \frac{2}{5\pi}\sin(5x) + \ldots$$

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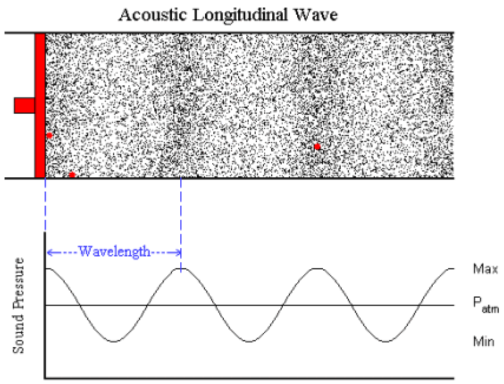


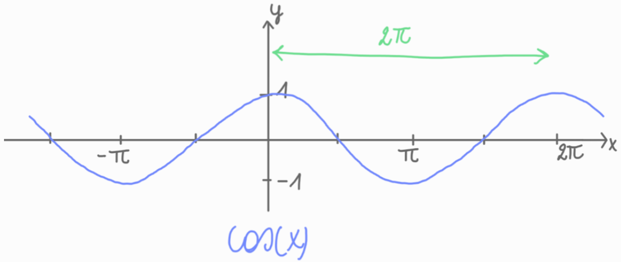
1

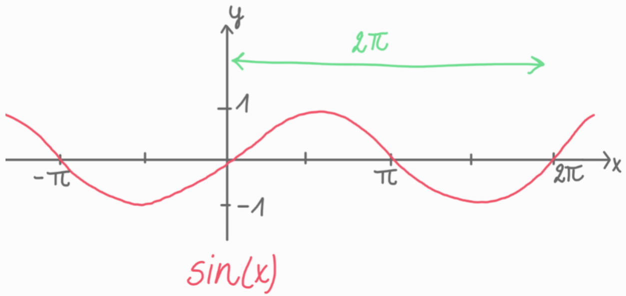
Periodieke functies

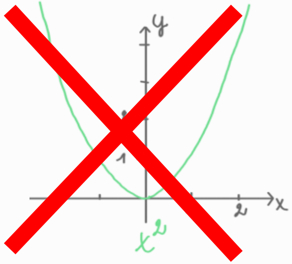
Periodieke functies

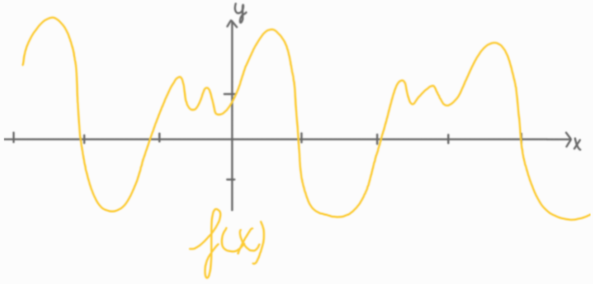
$f(x)$ is een periodieke functie als

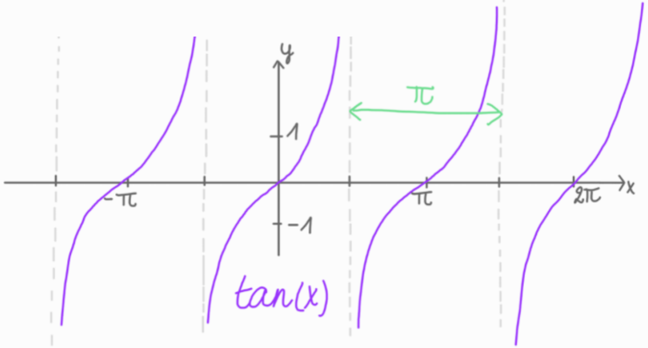
1. $f(x)$ is gedefinieerd voor alle reële x (eventueel met uitzondering van bepaalde x)
2. Er is een positief getal p, een periode van $f(x)$, zodat $f(x+p) = f(x)$ voor alle x

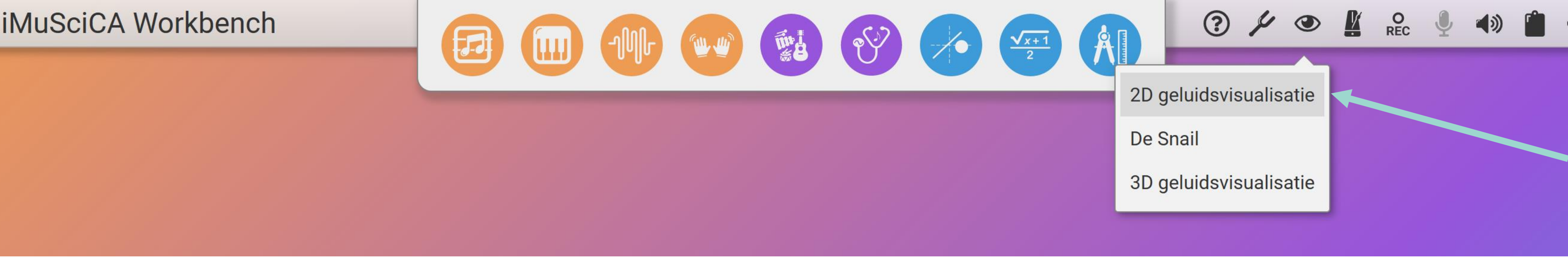












Periodieke functies in de muziek!

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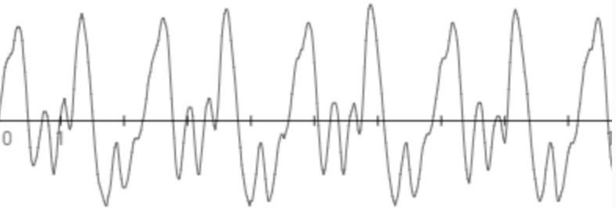

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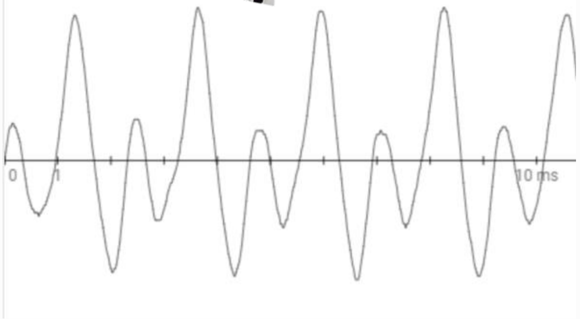

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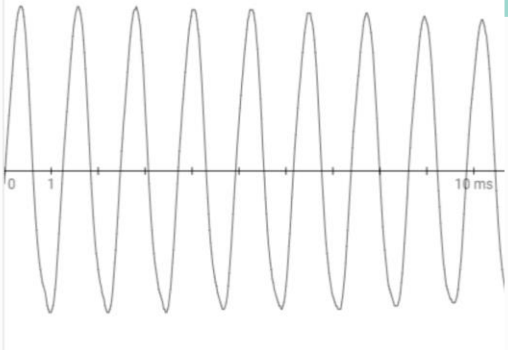

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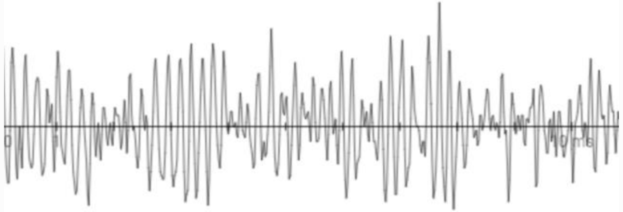
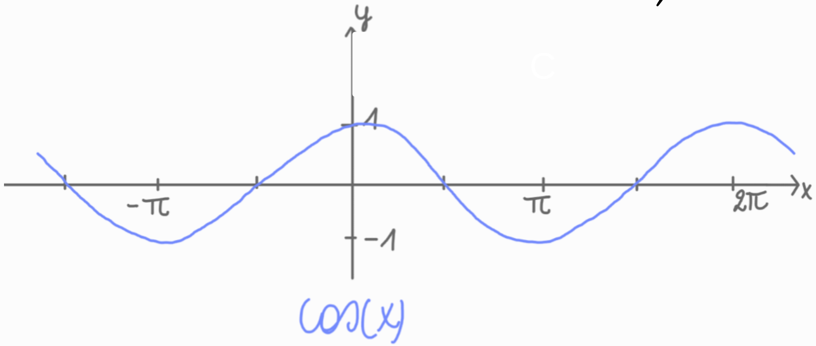
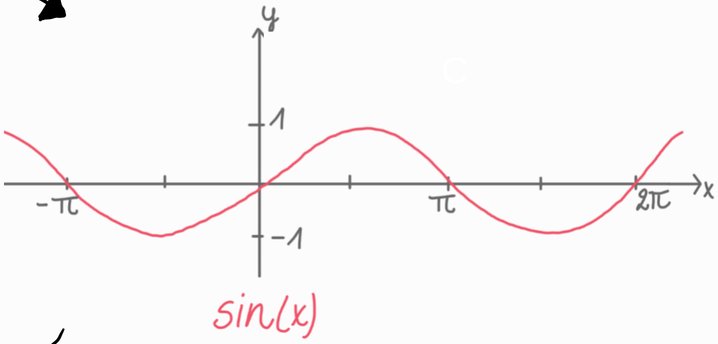


$$y = a \cdot sin(b(x-c)) + d$$

$$cos\left(x - \frac{\pi}{2}\right) = \sin(x)$$

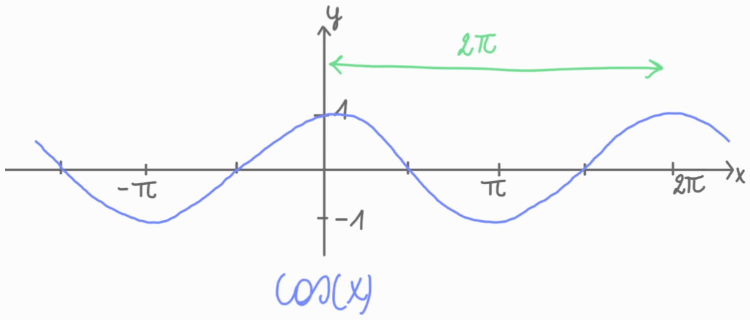


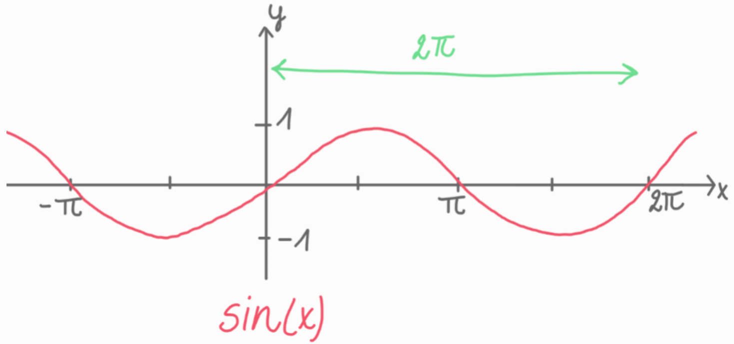


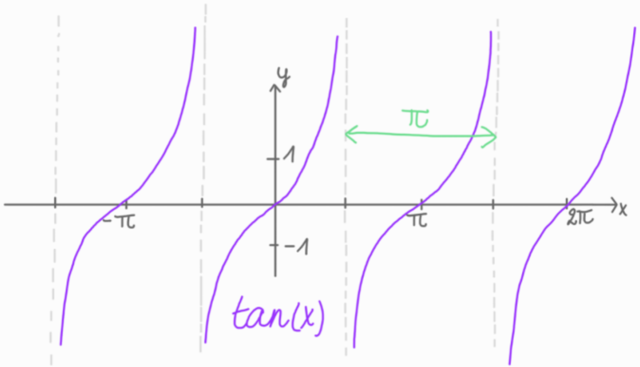
$$cos(x) = \sin\left(x + \frac{\pi}{2}\right)$$

Fundamentele periode

de kleinste positieve periode







Fundamentele periode

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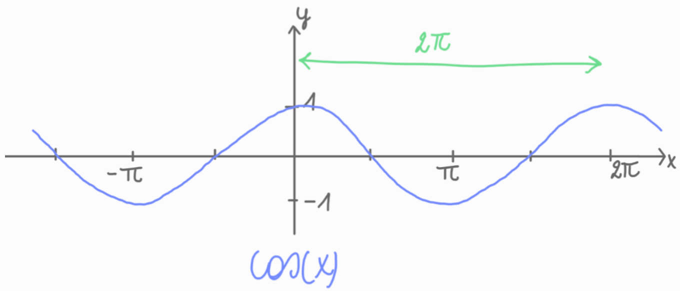
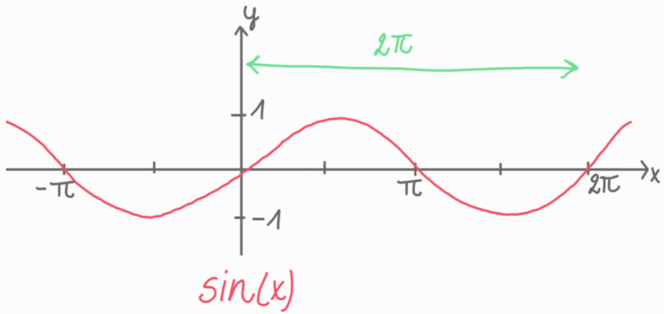
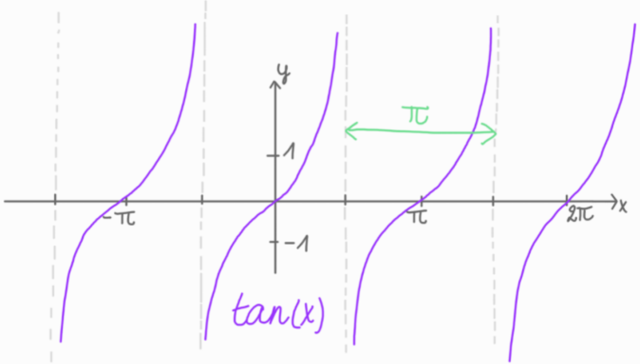


fundamentele periode = 2π



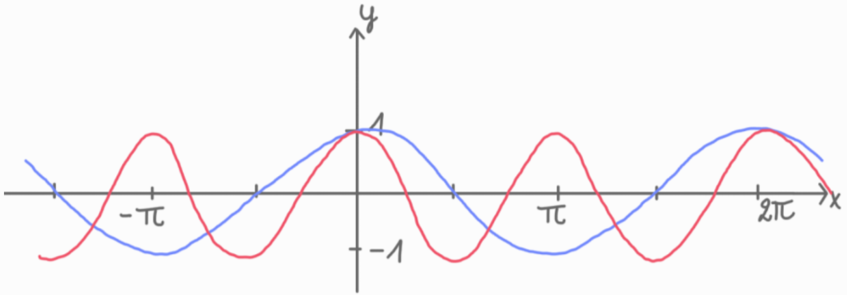
fundamentele periode = 2π



fundamentele periode = π

Fundamentele periode

= de kleinste positieve periode $\frac{2\pi}{b}$



cos(x) fundamentele periode = 2π

cos(2x) fundamentele periode = π = $\frac{2\pi}{2}$

Enkele andere voorbeelden

fundamentele periode van $\sin(3x)$ = $\frac{2\pi}{3}$

fundamentele periode van $\cos(4x)$ = $\frac{2\pi}{4} = \frac{\pi}{2}$

vermenigvuldiging t.o.v. y –as met factor $\frac{1}{b}$

$$y = a \cdot sin(b(x - c)) + d$$

$\cos(x)$, $\sin(x)$, $\cos(2x)$, $\sin(2x)$, $\cos(3x)$, $\sin(3x)$ … $\cos(bx)$, $\sin(bx)$ hebben allemaal **een periode 2π**

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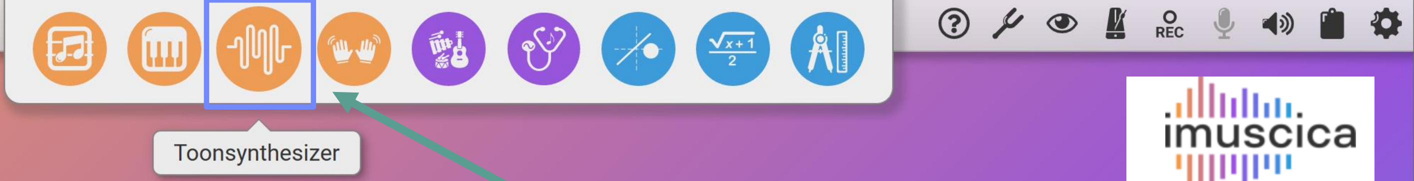

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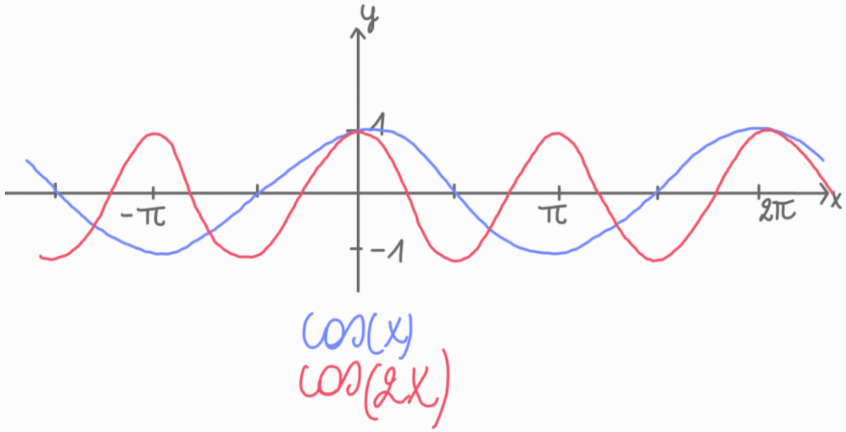
Op zoek naar de fundamentele periode van functies

Luisteren naar functies waarvan de periodes een gemeenschappelijk veelvoud zijn!



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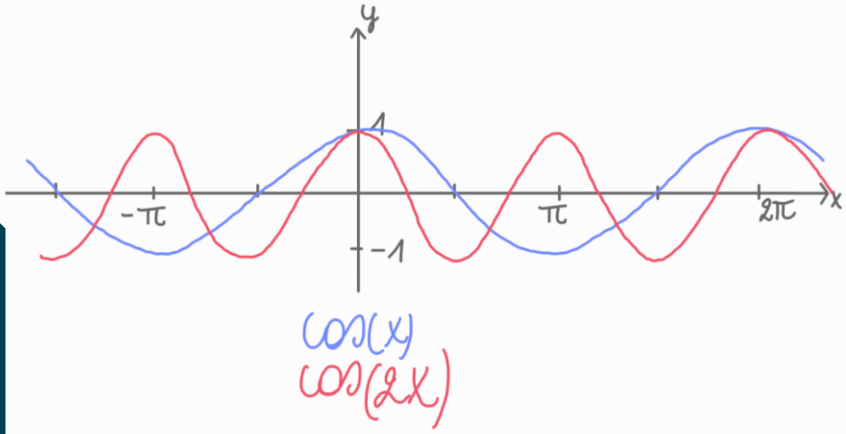
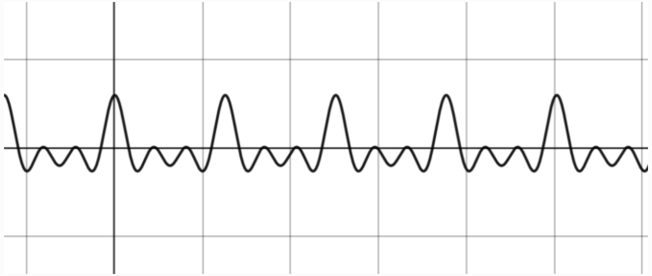
Op zoek naar de fundamentele periode van functies

Luisteren naar de *som* van functies met een periode die een gemeenschappelijk veelvoud hebben!





$f(x) = cos(x) + cos(2x)$

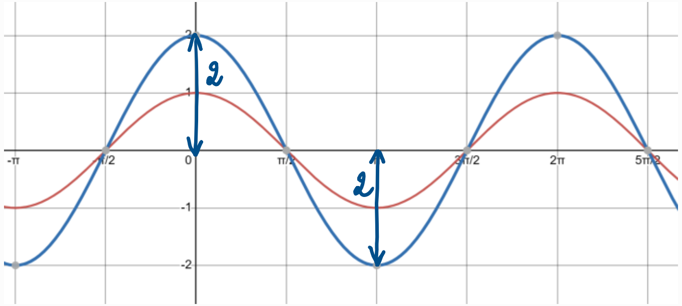
Coëfficiënt voor de functie

Wat gebeurt er als we een coëfficiënt voor de sinus- of cosinusfunctie plaatsen?

vermenigvuldiging t.o.v. x –as met factor

$$y = a \cdot sin(b(x - c)) + d$$

$f(x) = cos(x)$



$f(x) = 2\ cos(x)$

Laten we luisteren!

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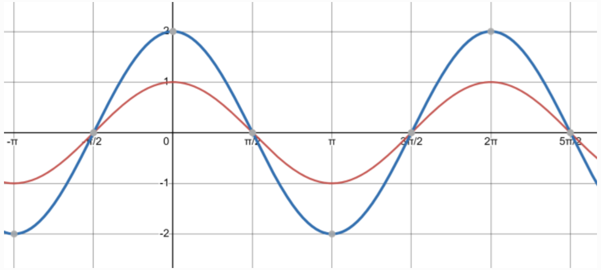
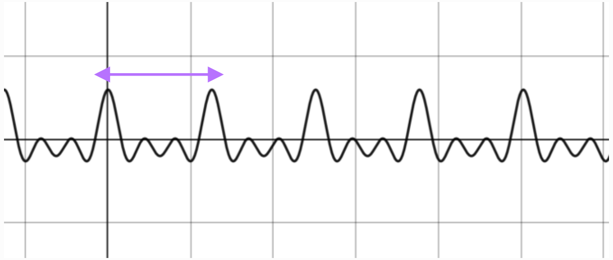
Luisteren naar de *som* van functies

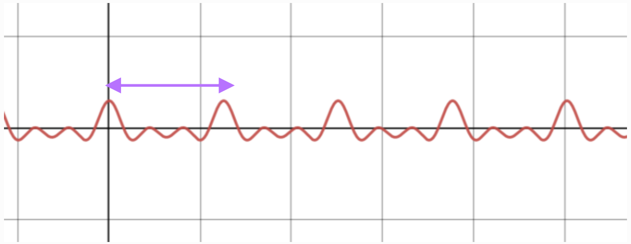


$f(x) = cos(x)$

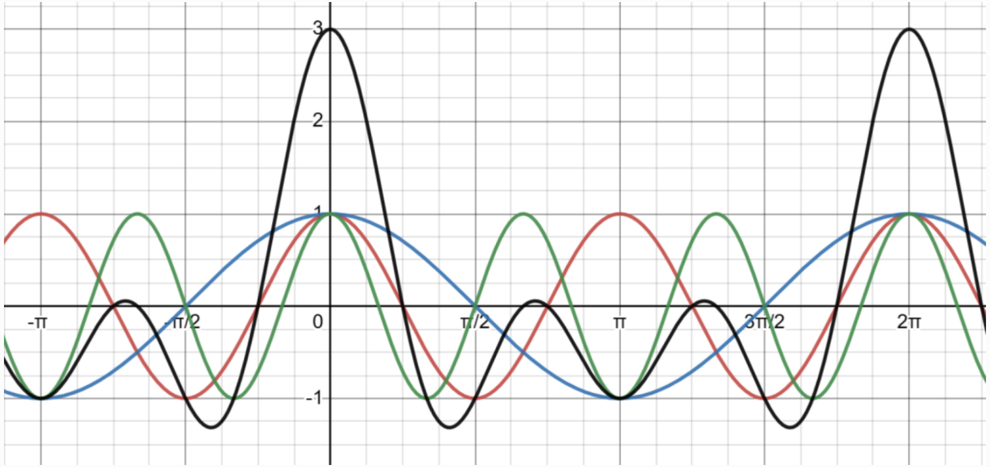
$f(x) = 2\,cos(x)$



$f(x) = cos(x) + cos(2x)$



$f(x) = cos(x) + \frac{1}{2} cos(2x)$

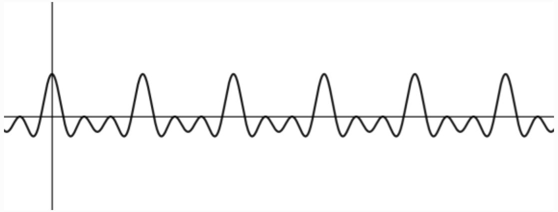


$\cos(x)$ 1e harmoniek = grondtoon $p_1 = 2\pi$

$\cos(2x)$ 2e harmoniek $p_2 = \frac{p_1}{2} = \pi$

$\cos(3x)$ 3e harmoniek $p_3 = \frac{p_1}{3} = \frac{2\pi}{3}$

$\cos(x) + \cos(2x) + \cos(3x)$



Wiskundetaal omzetten naar muziektaal

Maximale uitwijking t.o.v. y –as (amplitude)
Vermenigvuldiging t.o.v. de x –as met factor a

Volume/geluidsterkte

Horizontale verschuiving
$c < 0$: verschuiving naar links
$c > 0$: verschuiving naar rechts

Doorgaans geen invloed

$$y = a \cdot sin(b(x - c)) + d$$

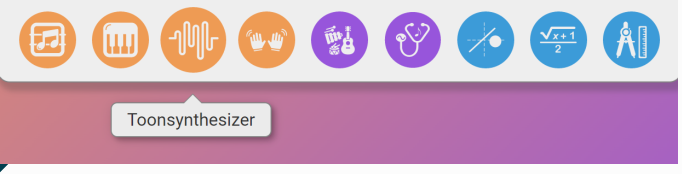
Vermenigvuldiging t.o.v. de y –as met factor $\frac{1}{b}$ verandert de periode, $p = \frac{2\pi}{b}$

toonhoogte

Verticale verschuiving
$d < 0$: verschuiving naar onder
$d > 0$: verschuiving naar boven

Geen invloed

Periode en fundamentele periode in de muziek



Onze eigen synthesizersound!

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frequentie $f = \frac{1}{p}$

aantal trillingen per tijdseenheid

$$f(x) = a \cdot \sin(b(x-c))$$

$$\text{periode } p = \frac{2\pi}{b} \Rightarrow b = \frac{2\pi}{p} = 2\pi \cdot \frac{1}{p} = 2\pi f$$

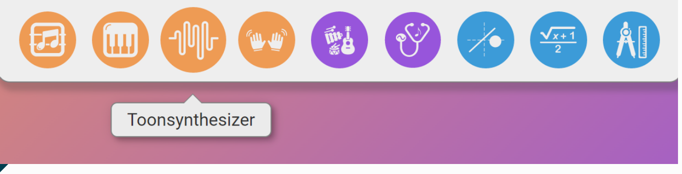
$$f(t) = a \cdot \sin(2\pi f(t-c))$$

$$= a \cdot \sin(2\pi f t - 2\pi f c)$$

$$= a \cdot \sin(2\pi f t + q)$$

$$= a \sin(2\pi \cdot f \cdot t + q)$$

waardes die je kan invullen



Onze eigen synthesizersound!

$$f(x) = a \cdot \sin(b(x-c))$$

$$f(t) = a \sin(2\pi \cdot f \cdot t + q)$$

$q = -2\pi f c$

frequentie $f = \frac{1}{p}$
aantal trillingen per tijdseenheid

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Opdracht 1: Combineer een hoge en een lage toon!
Zet één toon stiller dan de andere en een creëer een interessante sound!

Opdracht 2: Maak je eigen sound met functies die op een veelvoud na dezelfde periode hebben.
Experimenteer met verschillende veelvouden en verschillende geluidssterktes.

*De luidsprekers van een gemiddelde laptop produceren frequenties tussen **200** en 20 000 trillingen per seconde.*

*Het menselijk oor van een twintiger hoort frequenties tussen 25 en **17 000**.*

Let er dus op dat je frequentiewaarden ingeeft tussen *200* en *17 000*.

2

Fourierreeksen van functies met periode 2π

Fourierreeks van een functie met periode 2π

functie $f(x)$ met periode 2π uitdrukken in termen van cosinussen en sinussen

$$f(x) = a_0 + a_1\cos(x) + b_1\sin(x) + a_2\cos(2x) + b_2\sin(2x) + a_3\cos(3x) + b_3\sin(3x) + \cdots$$

Elke term heeft periode 2π

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n\cos(nx) + b_n\sin(nx))$$

= Fourierreeks van $f(x)$

a_0, a_n en b_n zijn de fouriercoëfficiënten

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\cos(nx)\,dx \qquad n = 1,2,3\ldots$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\sin(nx)\,dx \qquad n = 1,2,3\ldots$$

Bewijs van de fouriercoëfficiënten

Te bewijzen: $a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, dx$

Bewijs: $$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n\cos(nx) + b_n\sin(nx))$$

$$\int_{-\pi}^{\pi} f(x)\, dx = \int_{-\pi}^{\pi}\left(a_0 + \sum_{n=1}^{\infty}(a_n\cos(nx) + b_n\sin(nx))\right) dx$$

$$\int_{-\pi}^{\pi} f(x)\, dx = \boxed{\int_{-\pi}^{\pi} a_0\, dx} + \sum_{n=1}^{\infty}\int_{-\pi}^{\pi}(a_n\cos(nx) + b_n\sin(nx))\, dx$$

$$a_0\,[x]_{-\pi}^{\pi} = a_0\,(\pi + \pi) = 2\pi a_0$$

$$\int_{-\pi}^{\pi} f(x)\, dx = 2\pi a_0 + \sum_{n=1}^{\infty}\left(a_n \int_{-\pi}^{\pi}\cos(nx)\, dx + b_n \int_{-\pi}^{\pi}\sin(nx)\, dx\right)$$

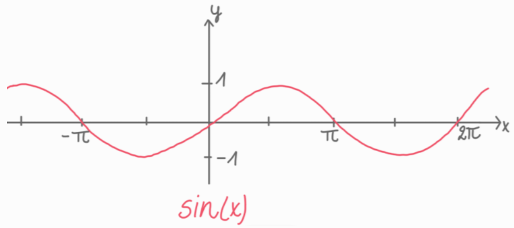
Te bewijzen: $a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$$\int_{-\pi}^{\pi} f(x)\,dx = 2\pi a_0 + \sum_{n=1}^{\infty}\left(a_n \int_{-\pi}^{\pi}\cos(nx)\,dx + b_n \int_{-\pi}^{\pi}\sin(nx)\,dx\right)$$

$$a_n \int_{-\pi}^{\pi}\cos(nx)\,dx = a_n \left[\sin(nx)\cdot\frac{1}{n}\right]_{-\pi}^{\pi} = a_n\left(\sin(\pi n)\frac{1}{n} - \sin(-\pi n)\frac{1}{n}\right) = a_n(0-0) = 0$$

$$b_n \int_{-\pi}^{\pi}\sin(nx)\,dx = b_n \left[-\cos(nx)\cdot\frac{1}{n}\right]_{-\pi}^{\pi} = b_n\left(-\cos(\pi n)\frac{1}{n} + \cos(-\pi n)\frac{1}{n}\right) = b_n\left(-\cos(\pi n)\frac{1}{n} + \cos(\pi n)\frac{1}{n}\right) = 0$$

$$\int_{-\pi}^{\pi} f(x)\,dx = 2\pi a_0 + \sum_{n=1}^{\infty}(0+0)$$



$$\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = a_0 \blacksquare$$

Fourierreeks van een functie met periode 2π

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

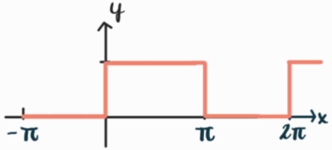
$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos(nx)\, dx \qquad n = 1,2,3 \ldots$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \sin(nx)\, dx \qquad n = 1,2,3 \ldots$$

Voorbeeld

Geef de Fourierreeks van de functie $f(x)$.

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$$



Fourierreeks

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(nx) + b_n \sin(nx) \right)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos(nx)\, dx$$

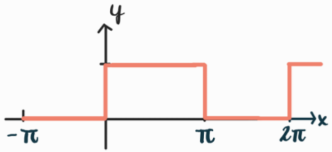
$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \sin(nx)\, dx$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(nx) + b_n \sin(nx)\right)$$

Voorbeeld

Geef de Fourierreeks van de functie $f(x)$.

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$$



$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\left(\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi} 1\,dx\right) = \frac{1}{2\pi}\left[x\right]_0^{\pi}$$

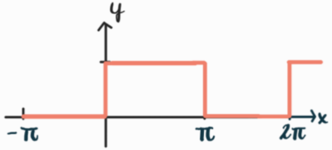
$$= \frac{1}{2\pi}(\pi - 0) = \frac{\pi}{2\pi} = \frac{1}{2}$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(nx) + b_n \sin(nx)\right)$$

Voorbeeld

Geef de Fourierreeks van de functie $f(x)$.

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$$



$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\cos(nx)\,dx = \frac{1}{\pi}\left(\int_{-\pi}^{0} 0\cdot\cos(nx)\,dx + \int_{0}^{\pi} 1\cdot\cos(nx)\,dx\right)$$

$$= \frac{1}{\pi}\left[\sin(nx)\cdot\frac{1}{n}\right]_0^{\pi} = \frac{1}{\pi}\left(\sin(n\pi)\cdot\frac{1}{n} - \sin(0)\cdot\frac{1}{n}\right) = \frac{1}{\pi}\cdot 0 = 0$$

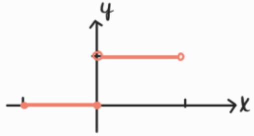
$\sin(n\pi) = 0$

$\sin(0) = 0$

Voorbeeld

Geef de Fourierreeks van de functie $f(x)$.

$$f(x) = \begin{cases} 0 & -\pi \leqslant x \leqslant 0 \\ 1 & 0 \leqslant x \leqslant \pi \end{cases}$$



$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(nx) + b_n \sin(nx) \right)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \sin(nx)\, dx = \frac{1}{\pi} \left(\int_{-\pi}^{0} 0 \cdot \sin(nx)\, dx + \int_{0}^{\pi} 1 \cdot \sin(nx)\, dx \right)$$

$$= \frac{1}{\pi} \left[-\cos(nx) \cdot \frac{1}{n} \right]_0^{\pi} = \frac{1}{\pi} \left(-\cos(n\pi) \cdot \frac{1}{n} + \cos(0) \cdot \frac{1}{n} \right) = \frac{1}{\pi} \left(-\cos(n\pi) \cdot \frac{1}{n} + \frac{1}{n} \right)$$

$\cos(0) = 1$

$$= \frac{1}{n\pi} \left(-\cos(n\pi) + 1 \right) = \frac{1}{n\pi} \left(1 - \cos(n\pi) \right)$$

$$b_n = \frac{1}{n\pi} \left(1 - (-1) \right) = \frac{2}{n\pi} \quad \text{als } n = 1, 3, 5 \ldots$$

$$b_n = \frac{1}{n\pi} \left(1 - 1 \right) = 0 \quad \text{als } n = 2, 4, 6 \ldots$$

Voorbeeld

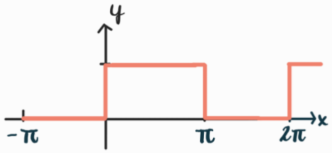
Geef de Fourierreeks van de functie $f(x)$.

$a_0 = \frac{1}{2}$ $\quad a_n = 0$

$b_n = \frac{1}{n\pi}\left(1-(-1)\right) = \frac{2}{n\pi}$ als $n = 1,3,5\ldots$

$b_n = \frac{1}{n\pi}\left(1-1\right) = 0$ als $n = 2,4,6\ldots$

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$$



Fourierreeks

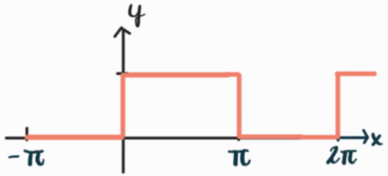
$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos(nx) + b_n\sin(nx)\right)$$

$$= a_0 + b_1\sin(x) + b_2\sin(2x) + b_3\sin(3x) + \ldots$$

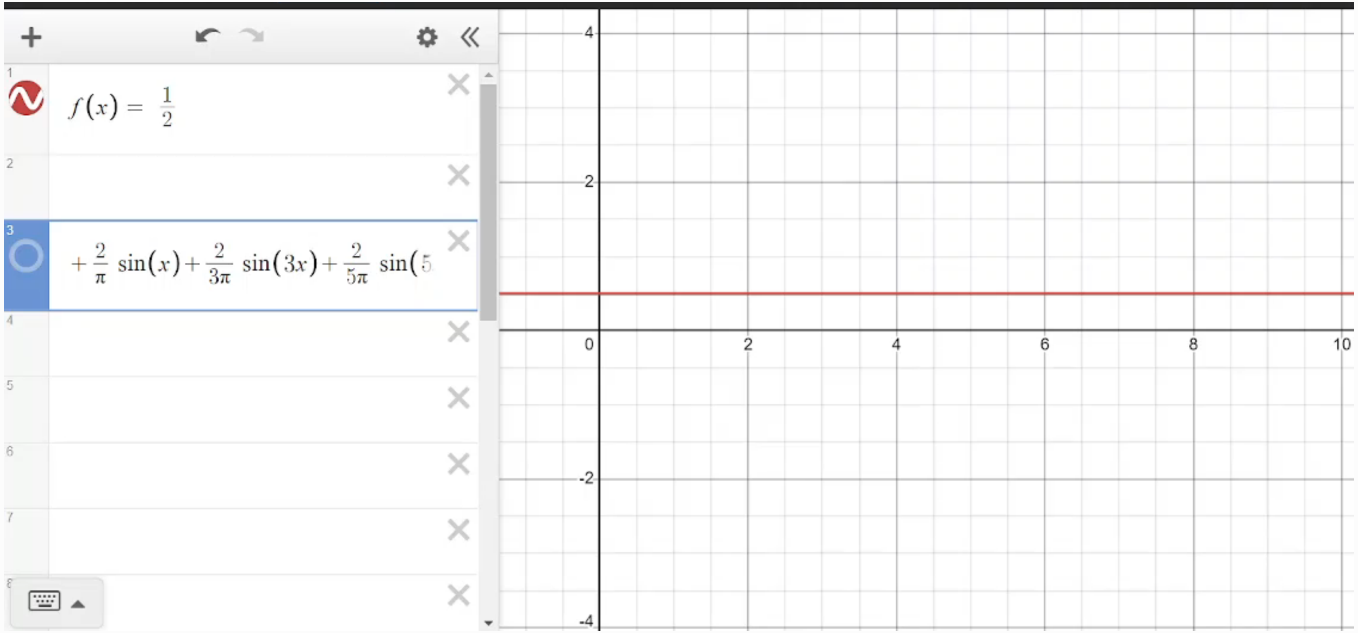
$$= \frac{1}{2} + \frac{2}{\pi}\sin(x) + \frac{2}{3\pi}\sin(3x) + \frac{2}{5\pi}\sin(5x) + \ldots$$

$$= \frac{1}{2} + \frac{2}{\pi}\left(\sin(x) + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \ldots\right)$$

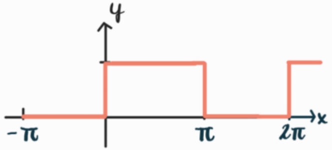
Even kijken of dat klopt!



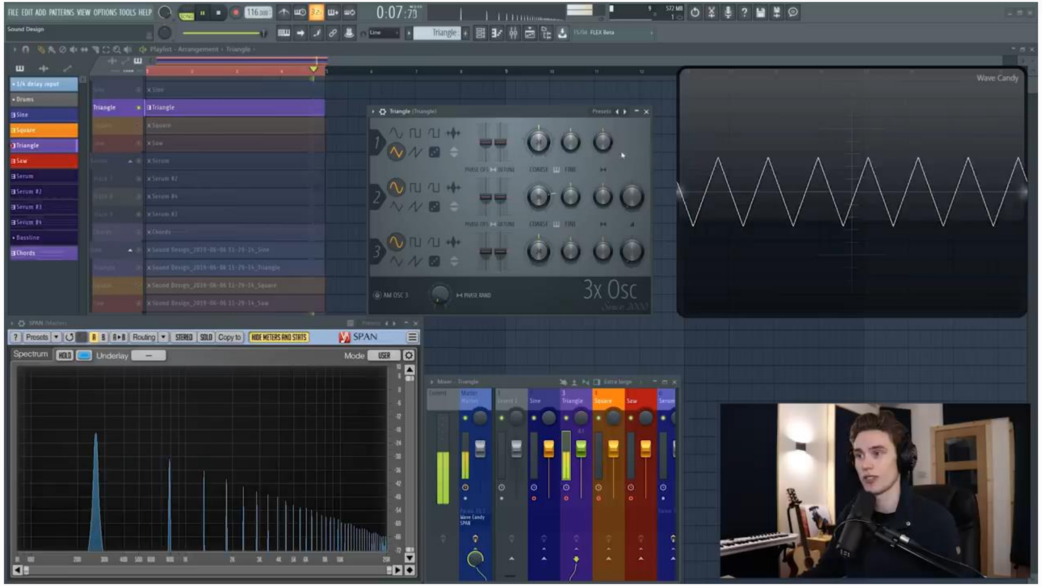
$$f(x) = \frac{1}{2} + \frac{2}{\pi}\left(\sin(x) + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \ldots\right)$$



Even horen hoe dit klinkt!



$$f(x) = \frac{1}{2} + \frac{2}{\pi}\left(\sin(x) + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \dots\right)$$



Visuele weergave van de termen in de fourierreeks

Fourierreeks van $g(u)$ met periode 2π

$$g(u) = a_0 + \sum_{n=1}^{\infty}(a_n cos(nu) + b_n sin(nu))$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} g(u)\, du$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} g(u) \cdot \cos(nu)\, du$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} g(u) \cdot \sin(nu)\, du$$

$$u = kx$$

$$2\pi = k \cdot 2L$$

$$\pi = k \cdot L$$

$$\frac{\pi}{L} = k$$

$$u = \frac{\pi}{L}x$$

$$du = \frac{\pi}{L}dx$$

Fourierreeks van $f(x)$ met periode $2L$

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \sin\left(\frac{n\pi}{L}x\right)\right)$$

$$a_0 = \frac{1}{2\pi}\int_{-L}^{L} f(x) \cdot \frac{\pi}{L}dx = \frac{1}{2L}\int_{-L}^{L} f(x)\, dx$$

$$a_n = \frac{1}{\pi}\int_{-L}^{L} f(x) \cdot \cos\left(\frac{n\pi}{L}x\right)\frac{\pi}{L}dx = \frac{1}{L}\int_{-L}^{L} f(x) \cdot \cos\left(\frac{n\pi}{L}x\right) dx$$

$$b_n = \frac{1}{\pi}\int_{-L}^{L} f(x) \cdot \sin\left(\frac{n\pi}{L}x\right)\frac{\pi}{L}dx = \frac{1}{L}\int_{-L}^{L} f(x) \cdot \sin\left(\frac{n\pi}{L}x\right) dx$$

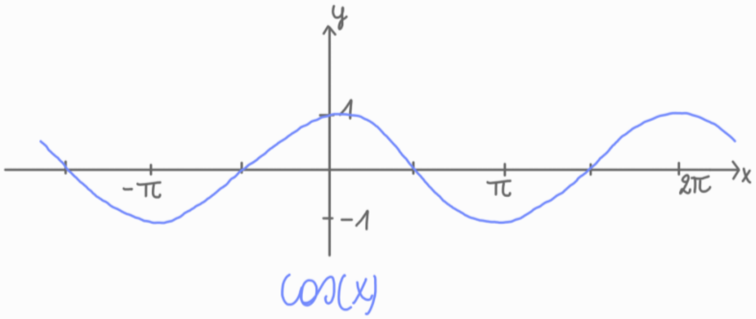
Schaalverandering

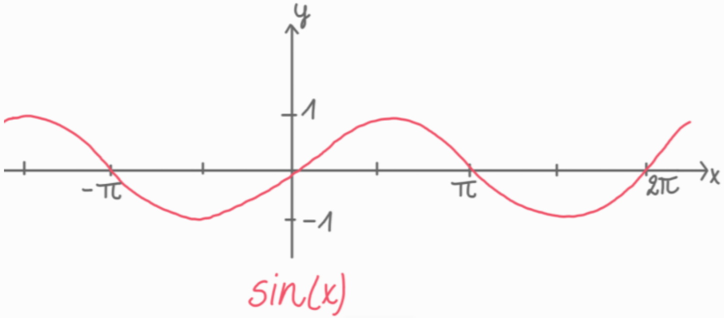
Neem $u = kx$ zodat de oude periode $u = 2\pi$ voor de nieuwe variabele x de nieuwe periode $x = 2L$ geeft

4

Even en Oneven Functies - Halfbereik Uitbreidingen

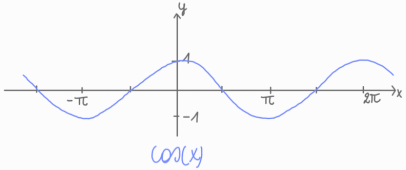
Even functies en Oneven functies





$$f(x) = f(-x)$$

$$f(x) = -f(-x)$$



$f(x) = f(-x)$

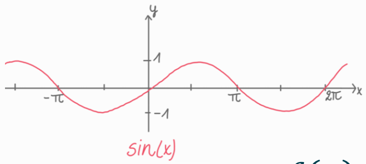
Fourierreeks van een even functie met periode $2L$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{L}x\right)$$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx = \frac{1}{L}\int_{0}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cdot\cos\left(\frac{n\pi}{L}x\right)dx = \frac{2}{L}\int_{0}^{L} f(x)\cdot\cos\left(\frac{n\pi}{L}x\right)dx$$

Fourier Cosinusreeks



$f(x) = -f(-x)$

Fourierreeks van een oneven functie met periode $2L$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{L}x\right)$$

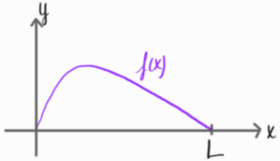
$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\cdot\sin\left(\frac{n\pi}{L}x\right)dx$$

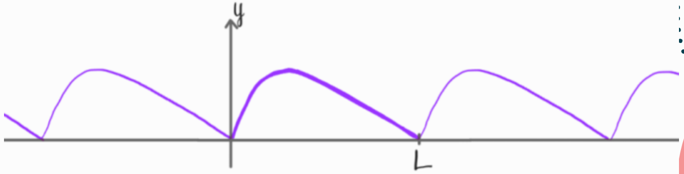
$$= \frac{2}{L}\int_{0}^{L} f(x)\cdot\sin\left(\frac{n\pi}{L}x\right)dx$$

Fourier Sinusreeks

Half Bereikuitbreiding

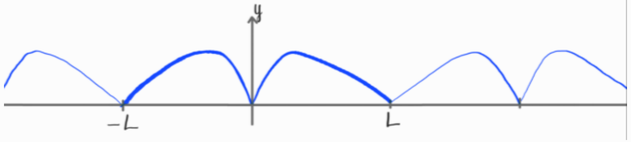
$f(x)$ is gegeven op een half bereik (= de helft van het periodieke interval met lengte 2L)





even periodieke uitbreiding

Fourier Cosinusreeks

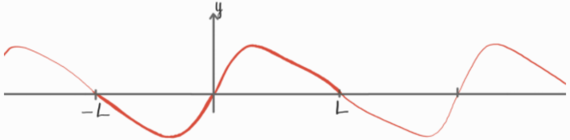


$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{L}x\right)$$

$$a_0 = \frac{1}{L}\int_0^L f(x)\,dx \qquad a_n = \frac{2}{L}\int_0^L f(x)\cdot\cos\left(\frac{n\pi}{L}x\right)dx$$

oneven periodieke uitbreiding

Fourier Sinusreeks



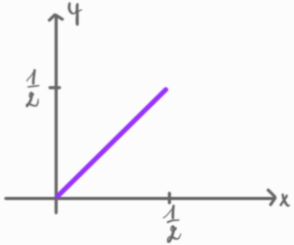
$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{L}x\right)$$

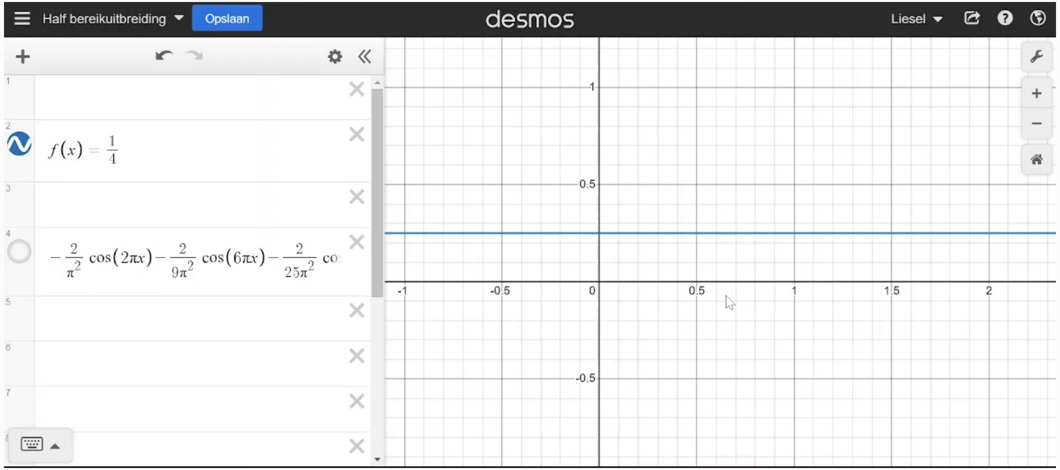
$$b_n = \frac{2}{L}\int_0^L f(x)\cdot\sin\left(\frac{n\pi}{L}x\right)dx$$

Half Bereikuitbreiding

$f(x)$ is gegeven op een half bereik (= de helft van het periodieke interval met lengte 2L)

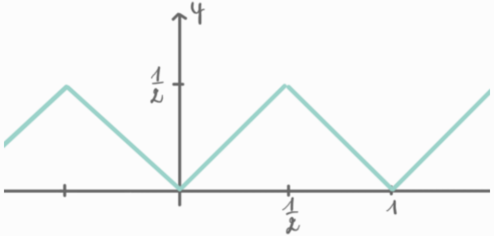
$$f(x) = x \qquad 0 < x < \frac{1}{2} \qquad \text{met periode} = \frac{1}{2}$$





even periodieke uitbreiding

Fourier Cosinusreeks

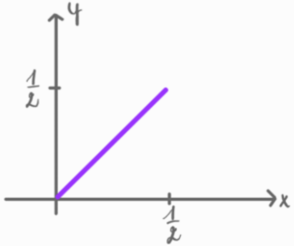


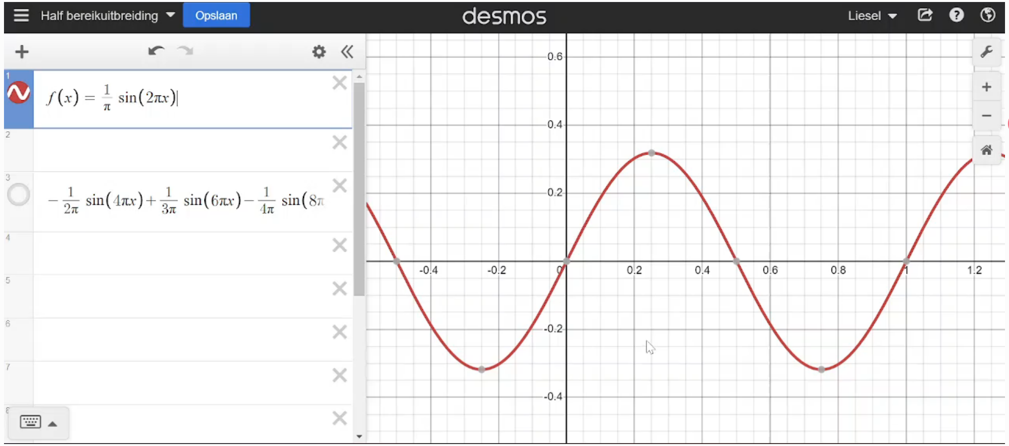
$$f(x) = \frac{1}{4} - \frac{2}{\pi^2}\cos(2\pi x) - \frac{2}{9\pi^2}\cos(6\pi x) - \frac{2}{25\pi^2}\cos(10\pi x) - \frac{2}{49\pi^2}\cos(14\pi x) - \frac{2}{81\pi^2}\cos(18\pi x) - \frac{2}{121\pi^2}\cos(22\pi x) - \ldots$$

Half Bereikuitbreiding

$f(x)$ is gegeven op een half bereik (= de helft van het periodieke interval met lengte 2L)

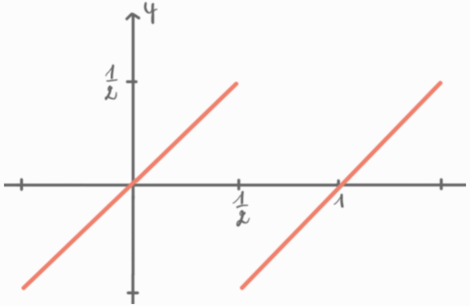
$$f(x) = x \qquad 0 < x < \frac{1}{2} \quad \text{met periode} = \frac{1}{2}$$



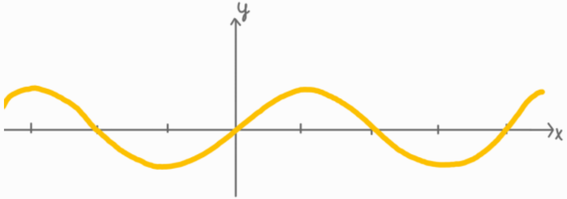


oneven periodieke uitbreiding

Fourier Sinusreeks

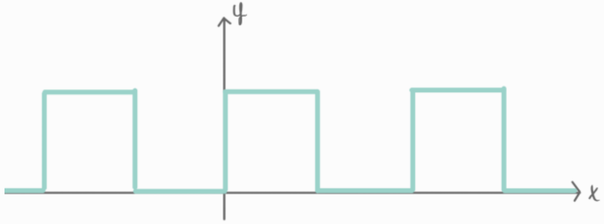


$$f(x) = \frac{1}{\pi}\sin(2\pi x) - \frac{1}{2\pi}\sin(4\pi x) + \frac{1}{3\pi}\sin(6\pi x) - \frac{1}{4\pi}\sin(8\pi x) + \frac{1}{5\pi}\sin(10\pi x) - \frac{1}{6\pi}\sin(12\pi x) + \frac{1}{7\pi}\sin(14\pi x) - \ldots$$

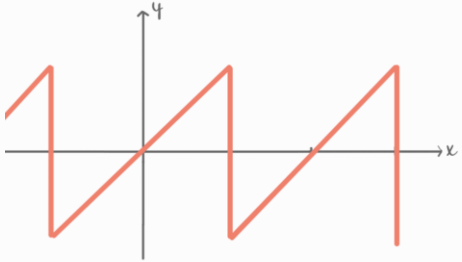


sinus

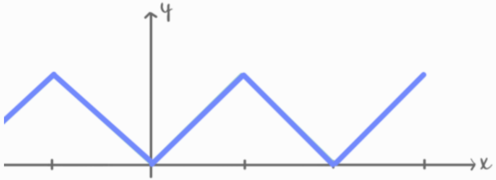
$$\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$$



vierkant



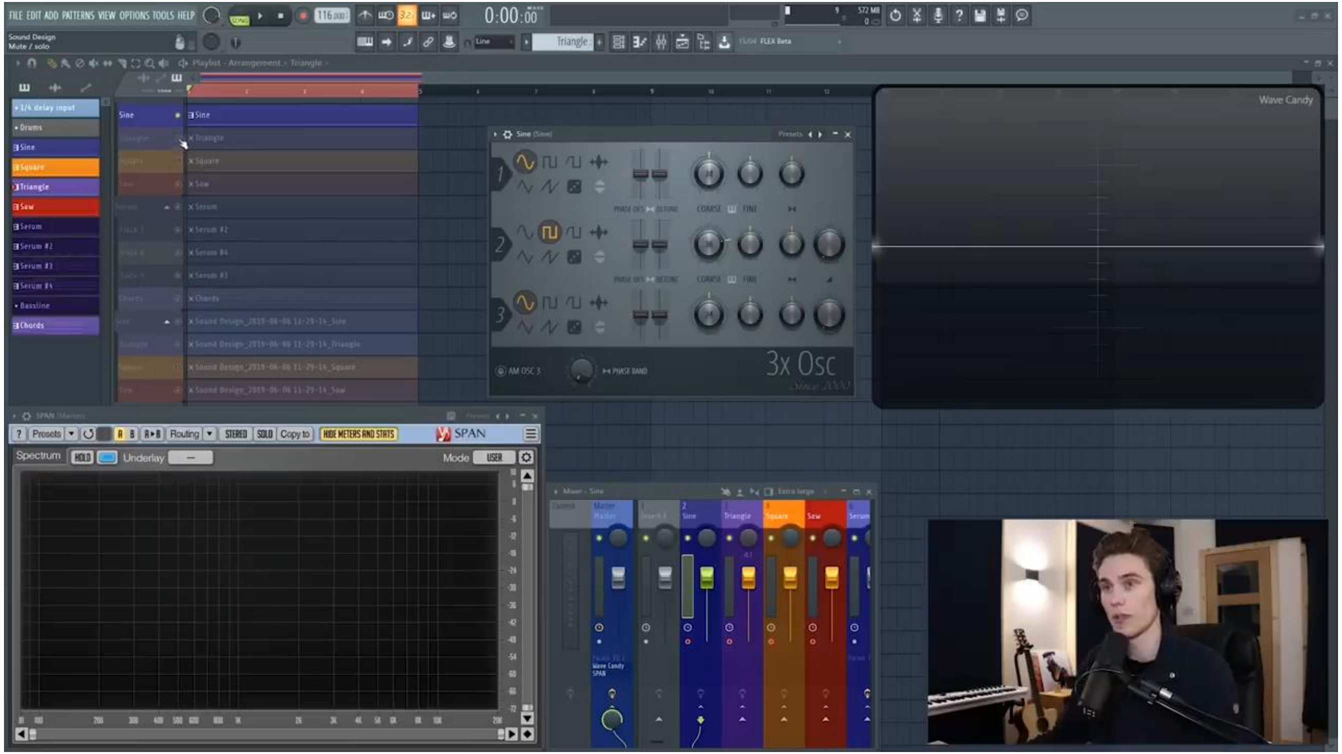
zaagtand



driehoek

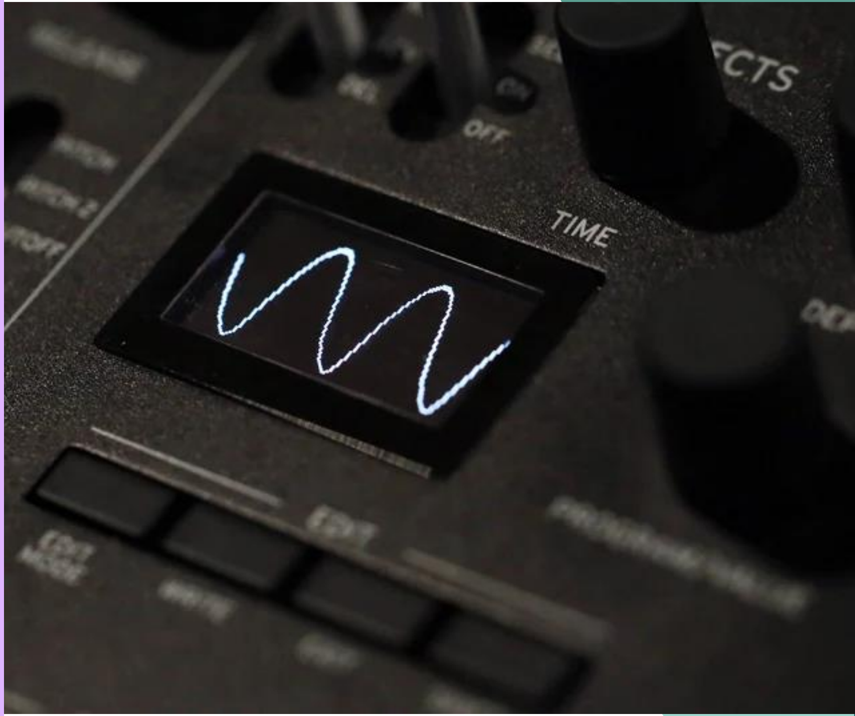
$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \sin\left(\frac{n\pi}{L}x\right)\right)$$

Luisteren naar fourierreeksen

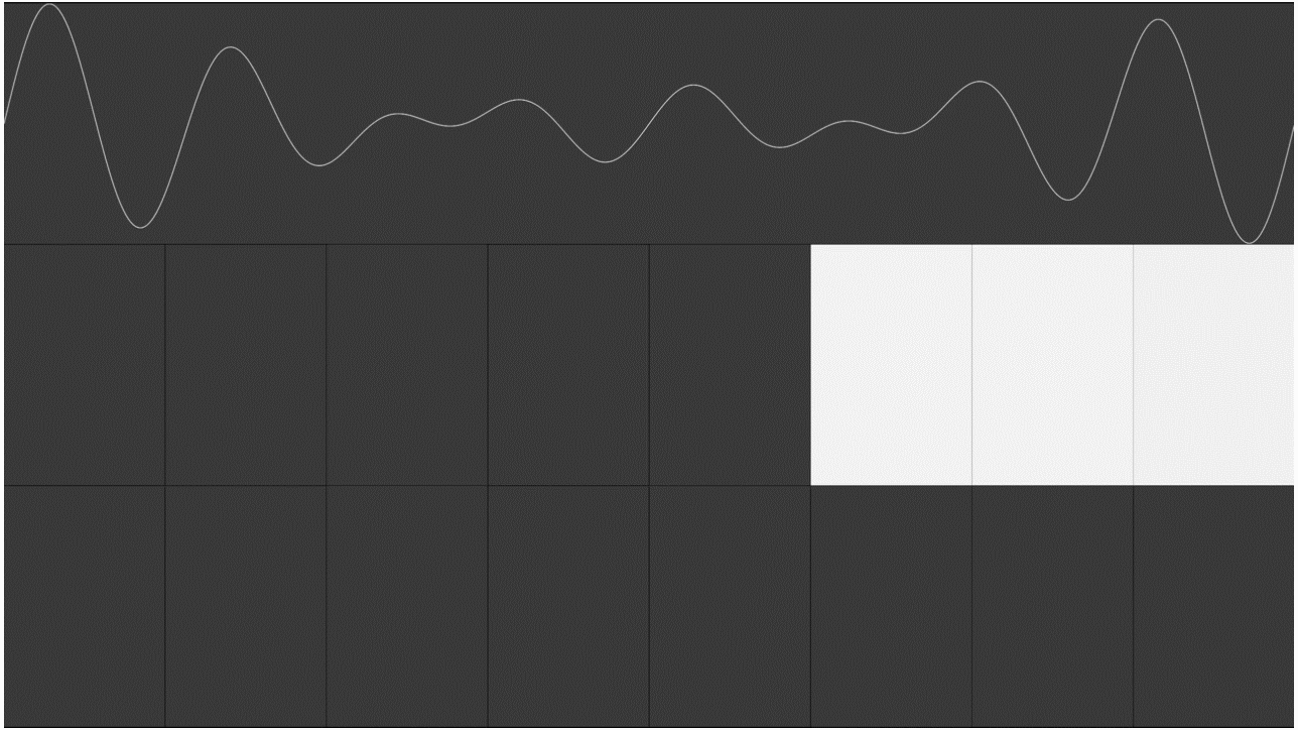


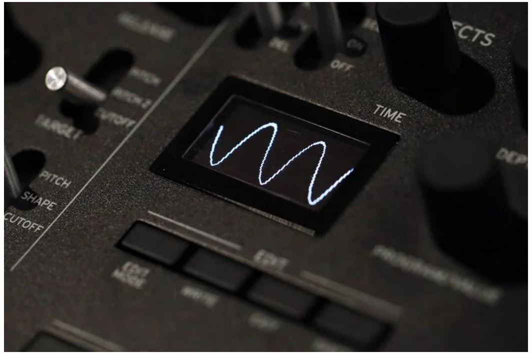
5

Maak je eigen synthesizersound!

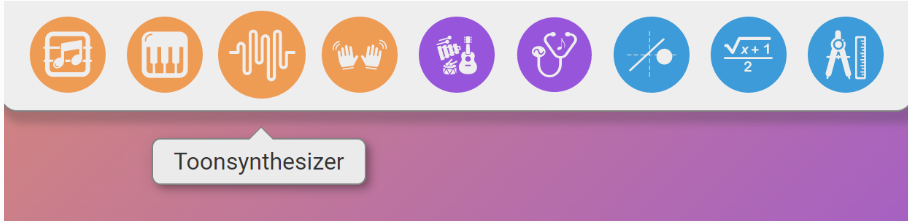


De synthesizer maakt een synthese van verschillende sinusfuncties.





Onze eigen synthesizersound!

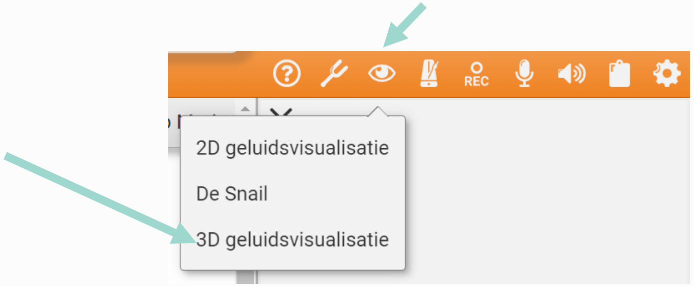


Opdracht 3: Maak een bestaande toon na (vb.fluiten, zingen, een meldingsgeluid op je telefoon, viool…)
Meet hiervoor eerst de verschillende frequenties van jouw klank op!
(Speel jouw klank en druk op pauze om de waardes af te lezen.)

iMuSciCA Online Workbench
www.workbench.imuscica.eu

De luidsprekers van een gemiddelde laptop produceren frequenties tussen ***200*** *en 20 000 trillingen per seconde.*

Het menselijk oor van een twintiger hoort frequenties tussen 25 en ***17 000****.*



Hopelijk zitten we nu wat meer op dezelfde golflengte!

(of op een veelvoud daarvan)

Bedankt voor jullie aandacht!