Interactions between amphoteric surfaces with strongly overlapping double layers

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The entropic repulsion between strongly overlapping electrical double-layers from two parallel amphoteric plates is described via the Donnan equilibrium in the limit of zero electric field. The plates feature charge-regulation and the inter-plate solution is in equilibrium with a reservoir of a monovalent electrolyte solution. A finite electric potential and disjoining pressure is found at contact between the plates, due to a complete discharging of the plates. For low potentials, the decay of potential and pressure is fully governed by a characteristic length scale and the contact potential. Additionally, for large separations we find a universal inverse square decay of disjoining pressure, irrespective of the contact potential. The results of the Donnan theory show quantitative agreement with self-consistent field computations that solve the full Poisson equation.

1 Introduction

When two charged surfaces immersed in an electrolyte solution move sufficiently far apart such that mid-plane electrical potentials are small, double-layer repulsions between the surfaces can be evaluated from the Debye–Hückel solution of the linearized Poisson–Boltzmann (PB) equation. Upon approach of the two plates, mid-plane potentials increase such that the approximation of weak double-layer overlap breaks down.

It has been pointed out that when two identical charged surfaces come sufficiently near, another simplifying limit is approached: for inter-plate distances below the Debye length, the electric field between the surfaces gradually vanishes throughout. Then a regime is entered where ions diffuse in a spatially constant electrical potential (in contrast, for larger separations, the electric field is only zero at the mid-plane between the plates). For the case of ideal ions, the calculation of this spatially constant potential, also known as the Donnan potential, is straightforward. It leads—without any further assumptions—to analytical expressions for the disjoining pressure, i.e., the external pressure that is needed to fix the repulsive plates at a certain distance. It should be stressed that this inverse square decay differs in origin from the inverse-square decay that is observed in the salt-free, counterion-only limit.

These Donnan disjoining pressures agree quite well with numerical solutions of the PB equation up to inter-plate distances as large as about the Debye length, in marked contrast to the exponential decay that results from the weak-potential Debye–Hückel approximation. Furthermore, recently it was shown by Landman et al. that the algebraic Donnan disjoining pressure could explain peculiar scaling of interbilayer separation with concentration in structures of self-assembled microtubes. Their observations represent, as far as we know, the first experimental support for the zero-field Donnan model. The Donnan model, it should be noted, was also employed by Per Linse and co-workers in their profound computational study of polyelectrolyte gels in equilibrium with a salt reservoir solution.

The zero-field Donnan treatment for constant-charge plates, was recently extended to surfaces with a regulated charge density. There, surface charge was due to a single association–dissociation equilibrium, which excludes the possibility of a net-charge sign reversal induced by a pH change: in other words, those surfaces had no iso-electric point (IEP). Surfaces of common inorganic colloids, composed of materials such as silica, aluminium (hydr)oxides and iron (hydr)oxides, of course do have an IEP owing to pH-dependent protonation and deprotonation of surface hydroxyl groups. The aim of the present work is therefore to generalize the Donnan treatment to amphoteric surfaces, i.e., surfaces grafted with a single type of molecules that can accept as well as release cations, yielding positive or negative charge densities depending on pH.

This paper is structured as follows. Derivations of electrical potentials, dissociation degrees, and disjoining pressures for amphoteric surfaces via Donnan zero-field theory are given in...
Section 2. Self-consistent field (SCF) computations, detailed in Section 3, are done to assess the validity of the zero-field calculations done for ideal ions. The SFC lattice computations, which solve the full PB equation for ions and solvent molecules of finite size, are compared in Section 4 to predictions from Donnan theory. The paper is finalized with general conclusions in Section 5.

2 Zero-field Donnan theory

In this section we will describe the repulsion between two amphoteric, charge regulated surfaces using zero-field Donnan theory. We will first give a description of the system at hand, followed by derivations of the Donnan potential, degree of dissociation, and disjoining pressure, with a focus on the limiting behavior at low potentials.

2.1 General description of the system

Consider two parallel surfaces separated by a distance \( h \), immersed in an electrolyte solution. The electrolyte is a simple strong monovalent acid such as HCl or HNO3. The inter-plate region is in equilibrium with a reservoir of constant salt concentration \( r_s \), as sketched in Fig. 1.

The surface contains amphoteric groups \(-AH\), that can both accept and release ions. The following equilibria are accounted for:

\[
\begin{align*}
-H_2A^+ &\rightleftharpoons -HA + H^+ , \\
-HA &\rightleftharpoons -A^- + H^+ ,
\end{align*}
\]

with the following equilibrium constants:

\[
\begin{align*}
K_1 = &\frac{\sigma_{HA} \rho_+}{\sigma_{H^+}} , \\
K_2 = &\frac{\sigma_A \rho_-}{\sigma_{HA}} .
\end{align*}
\]

Here \( \rho_+ \) denotes the number density of cations, in this case \( H^+ \), at the surface of the plates. The surface densities are denoted by \( \sigma_i \); the total surface density of the amphoteric groups is given by

\[
\sigma_{tot} = \sigma_{H^+} + \sigma_{HA} + \sigma_{A^-}
\]

and is assumed to be constant. It should be mentioned that, through eqn (1) and (2), we implicitly also incorporate the following surface equilibrium

\[
-H_2A^+ + A^- \rightleftharpoons 2-HA,
\]

with equilibrium constant

\[
K_3 = \frac{\sigma_{HA}^2}{\sigma_{H^+} \sigma_{A^-}} = \frac{K_1}{K_2} .
\]

We will follow the convention of referring to the numerical values of the equilibrium constants through their pK’s, defined as \( pK_i = -\log K_i \).

From now on, we consider the limit of strongly overlapping electric double layers, i.e., it is assumed that all inter-plate separations are smaller than approximately the Debye length, defined as

\[
\lambda_D \equiv \frac{1}{\sqrt{8 \pi \epsilon_0 \kappa_b I}}
\]

where \( \lambda_B \equiv e^2/(4 \pi \epsilon_0 \epsilon_r k_B T) \) is the Bjerrum length of the medium (with \( e \) the elementary charge, \( \epsilon_0 \) the vacuum permittivity, \( \epsilon_r \) the relative permittivity, \( k_B \) the Boltzmann constant, and \( T \) the absolute temperature) and \( I \) is the ionic strength. Under these conditions, the electric field between the plates is negligible and the ions are distributed homogeneously: \( \rho_+ \) is a spatial constant.

By solving eqn (1), (2) and (5) for \( \sigma_{A^-} \) and \( \sigma_{H^+} \) simultaneously, we find that the degree of dissociation into positively and negatively charged sites is given by

\[
\alpha_+ \equiv \frac{\sigma_{H^+}}{\sigma_{tot}}
\]

\[
= \left( \frac{\rho_+}{\rho_s} \right)^2 + k_1 \frac{\rho_+}{\rho_s} + k_2 ,
\]

and

\[
\alpha_- \equiv \frac{\sigma_{A^-}}{\sigma_{tot}}
\]

\[
= k_1 k_2 \left( \frac{\rho_-}{\rho_s} \right)^2 + k_1 \frac{\rho_-}{\rho_s} + k_1 k_2 ,
\]

where the dimensionless equilibrium constants \( k_1 \) and \( k_2 \) have been introduced.

Fig. 1. Schematic overview of the system under consideration. Two plates, grafted with amphoteric groups \(-AH\), are in equilibrium with a salt reservoir containing a simple strong acid (HCl) at fixed concentration \( r_s \). The amphoteric groups, with density \( \sigma_{tot} \), can acquire or release protons with equilibrium constants \( K_1 \) and \( K_2 \) respectively, such that the surfaces become positively or negatively charged. An external pressure \( \Delta P_s \) is required to fix the distance \( h \) between the plates.
When the plates are infinitely far apart, the concentration of ions in the inter-plate region becomes equal to that of the reservoir. Hence we find for the maximum degree of dissociation:

\[ \alpha^\text{max} = \frac{1}{1 + k_1 + k_1k_2}, \]  

(14)

\[ \alpha^\text{-max} = \frac{k_1k_2}{1 + k_1 + k_1k_2}. \]  

(15)

For comparison with SCF computations, it is convenient to vary the pH of systems while keeping the Debye length \( \lambda_D \) constant through addition of inert background salt. This scenario can be described by choosing effective values for pH, \( pK_1 \), and \( pK_2 \). For instance, SCF computations at \( I = 10^{-8} \text{ M} \), pH = 6.5, \( pK_1 = 1 \), and \( pK_2 = 13 \) may be mapped onto a system with pH = 6.0, \( pK_1 = 0.5 \), and \( pK_2 = 12.5 \).

2.2 Donnan potential

The concentration of cations and anions in the inter-plate region is assumed to be linked to the salt concentration in the reservoir through a Boltzmann distribution,

\[ \rho_\pm = \rho_s \exp(\mp u), \]  

(16)

where \( u = e\psi/(k_BT) \) is the dimensionless Donnan potential difference of the inter-plate region w.r.t. the reservoir, with \( \psi \) the potential difference. It is emphasized that \( u \) is assumed to be a spatial constant in our zero-field Donnan model. It should be compared to the mid-plane potential resulting from more elaborate approaches (such as the full Poisson–Boltzmann theory incorporated in our SCF computations), because halfway between the two plates the local electric field is zero.

An expression for the potential \( u \) may be derived based on the boundary condition that the inter-plate region is electrically neutral. Electroneutrality entails that

\[ \rho_+h + 2\sigma_{tot}x_+ = \rho_-h + 2\sigma_{tot}x_-, \]  

(17)

which can be rewritten as

\[ \sinh u = \frac{\sigma_{tot}(x_+ - x_-)}{\rho_s h}. \]  

(18)

Using eqn (10), (12) and (16) and \((\rho_+ - \rho_-)/(2\rho_s) = -\sinh u\). It turns out that it is convenient to rewrite this as

\[ \sinh u = \frac{\lambda}{\hbar} \frac{x_+ - x_-}{\alpha^\text{max} - \alpha^\text{-max}}, \]  

(19a)

\[ = \frac{\lambda}{\hbar} \frac{1 + k_1 + k_1k_2}{1 - k_1k_2} \frac{\exp(-2u) - k_1k_2}{\exp(-2u) + k_1 \exp(-u) + k_1k_2}, \]  

(19b)

where

\[ \lambda \equiv \frac{\sigma_{tot}(\alpha^\text{max} - \alpha^\text{-max})}{\rho_s} \]  

(20a)

\[ = \frac{\sigma_{tot}}{\rho_s} \frac{1 - k_1k_2}{1 + k_1 + k_1k_2}. \]  

(20b)

The parameter \( \lambda \) has the unit of length and its absolute magnitude sets the characteristic length scale for repulsions between strongly overlapping double layers. Note that the sign of \( \lambda \) in this definition follows the sign of the surface charge: it is positive for a net cationic surface and negative for a net anionic surface, because \( k_1k_2 < 1 \) for the former and \( k_1k_2 > 1 \) for the latter.

The length scale \( \lambda \) can be related to the Gouy-Chapman length,\(^{12} \) which is inversely proportional to the number of charges \( \sigma \) per unit area of a surface:

\[ \lambda_{GC} \equiv (2\pi \alpha_s |\sigma|)^{-1}. \]  

(21)

In our case, \( \sigma \) (and consequently \( \lambda_{GC} \)) continuously varies with separation distance between the plates, but as a reference we may take the Gouy-Chapman length at infinite separation:

\[ \lambda_{GC}^\text{max} \equiv \left(2\pi \alpha_s^\text{max} |\alpha^\text{max} - \alpha^\text{-max}|\right)^{-1}, \]  

(22)

from which \( \lambda \) follows as

\[ |\lambda| = 4\lambda_D^2/\lambda_{GC}^\text{max}. \]  

(23)

Eqn (19) has a rather involved (though analytic) solution since it is cubic in \( \exp u \). for brevity, we do not show it here. A straightforward solution for the Donnan potential at contact is available however. It follows from the fact that, for \( h \rightarrow 0 \), the last fraction in eqn (19b) must become zero:

\[ \frac{\exp(-2u) - k_1k_2}{\exp(-2u) + k_1 \exp(-u) + k_1k_2} = 0 \]  

(24)

Therefore, the contact electric potential \( u_0 \) is given by

\[ u_0 = -\ln \sqrt{k_1k_2} \]  

(25)

which is valid for potentials of arbitrary magnitude. This result entails that the contact potentials for amphoeteric surfaces remain finite, whereas for surfaces with a single dissociation equilibrium \((k_1 \rightarrow \infty \text{ or } k_2 \rightarrow 0)\) or with a fixed charge \((k_1 \rightarrow \infty \text{ and } k_2 \rightarrow 0)\) the contact electric potential diverges.\(^{13,14,19} \) It should also be noted that the contact potential is independent of the surface density \( \sigma_{tot} \) of the amphoteric groups.

In the limit of small potentials, more straightforward limiting solutions can be obtained. When retaining only the first order terms of eqn (19), it can be shown that

\[ u \simeq \frac{1}{\alpha_0^\text{eff} + h} \]  

(26)

for \( |u| \ll 1 \), where we denote \( \alpha_0^\text{eff} \) as the effective Donnan potential at contact \((h = 0)\) in the low-potential limit, defined as

\[ \alpha_0^\text{eff} \equiv \frac{-(1 - k_1k_2)(1 + k_1 + k_1k_2)}{k_1(1 + 4k_2 + k_1k_2)}. \]  

(27)

It is noted that \( u_0 \) and \( \alpha_0^\text{eff} \) have the same value for small contact potentials. This does not hold for large contact potentials (e.g., far from the IEP), however the definition of eqn (27) ensures that whenever the Donnan potential \( u \) becomes small upon increasing separation, eqn (26) will still predict the correct Donnan potential, even if \( u_0 \) and/or \( \alpha_0^\text{eff} \) are large. This would not be the case if one would naively use \( u_0 \) (eqn (25)) in place of \( \alpha_0^\text{eff} \) in eqn (26). This is the reason that eqn (27) can be viewed as an effective or apparent contact potential that one would probe at intermediate separations.
From eqn (26) it is evident that the decay of the Donnan potential with increasing separation \( h \) is fully set by the effective contact potential \( u_0^{\text{eff}} \) and length scale \( \lambda \) alone. We can make a series expansion of eqn (26) for small inter-plate separations \( h \) to find that
\[
 u \simeq u_0^{\text{eff}} \left( 1 - u_0^{\text{eff}} \frac{h}{\lambda} + \left( u_0^{\text{eff}} \frac{h}{\lambda} \right)^2 + \cdots \right) \tag{28}
\]
and
\[
 z_- \simeq \frac{k_1 k_2}{(1 - 2u) + k_1 (1 - u) + k_1 k_2} \tag{38}
\]
where eqn (26) was employed. Likewise, the corresponding limits of small and large separations may be obtained using eqn (29) and (31), but for brevity we omit them here.

2.4 Disjoining pressure

The expressions for \( u \) also allow to calculate the disjoining pressure of the charge-regulated plates. The disjoining pressure, scaled to the osmotic pressure of the salt reservoir, is defined as
\[
 \Delta \tilde{\Pi} \equiv \frac{\Delta \Pi}{2 \rho_a k_B T} = \frac{\rho_+ + \rho_- - 2 \rho_a}{2 \rho_a}. \tag{41}
\]

Inserting the Boltzmann distribution of eqn (16) yields
\[
 \Delta \tilde{\Pi} = \cosh u - 1. \tag{42}
\]

Eqn (42) is also valid when there is only weak overlap of the double layers, as long as the electric field in the mid-plane is zero and the ions in the mid-plane behave ideally.

For small potentials, \( \cosh u - 1 \simeq \frac{1}{2 u^2} \). Using eqn (26), (29) and (31), we find:
\[
 \Delta \tilde{\Pi} \simeq \frac{1}{2} \left( \frac{1}{u_0^{\text{eff}}} + \frac{h}{\lambda} \right)^{-2} \quad \text{for } \left| u \right| \ll 1, \tag{43}
\]
\[
 \Delta \tilde{\Pi} \simeq \frac{1}{2} \left( \frac{1}{u_0^{\text{eff}}} \right)^2 \left( 1 - 2 u \frac{h}{\lambda} \right)^{-2} \quad \text{for } \left| u \right| \ll 1 \text{ and } h \to 0, \tag{44}
\]
\[
 \Delta \tilde{\Pi} \simeq \frac{1}{2} \left( \frac{h}{\lambda} \right)^2 \quad \text{for } \left| u \right| \ll 1 \text{ and } h \to \infty. \tag{45}
\]

It follows that the inverse square decay, first predicted for constant-charge surfaces\(^{13}\) and charge-regulated surfaces with a single equilibrium,\(^{14}\) also holds for amphoteric surfaces, as long as \( u_0^{\text{eff}} \frac{h}{\lambda} \gg 1 \). The prefactor \( u_0^{\text{eff}} \) in this limit entails, perhaps somewhat counter-intuitively, that for small contact potentials, large separations are necessary to achieve this condition. For constant-charge and single-equilibrium surfaces, the contact potential is infinite, thus this subtlety is not present for those surfaces. For infinite contact potentials, eqn (43) directly reduces to eqn (45).

3 SCF computations

In order to assess the validity of our theoretical approximations in the previous section, we performed self-consistent field (SCF) calculations to compute the thermodynamic properties of
two infinite parallel amphoteric plates in an electrolyte solution. In contrast to the zero-field approximation, the SCF computations solve the full Poisson equation (on a mean-field level) and assume a finite size for ions and solvent. SCF theory has been extensively discussed in the literature before, see ref. 20–27 for details. Here we will mostly focus on a description of the parameters used in the computations.

We perform our calculations on the basis of the numerical lattice approximation of Scheutjens and Fleer.20,23 In the Scheutjens–Fleer method, space is represented by a set of lattice sites. For the application of SCF theory in this work, the focus is on a flat geometry in which \( T \) lattice layers are considered. The lattice layers are numbered \( z = 0, 1, 2, \ldots, T, T + 1 \), where the first plate is located at \( z = 0 \) and the second plate at \( z = T + 1 \). The inter-plate distance is given by \( h = bT \), where \( b \) is the size of a lattice site. The solution between these plates is in equilibrium with a reservoir of constant chemical potential. The size of a lattice site is set to \( b = 0.3 \) nm and all ions, solvent molecules, and surface groups in the system occupy a single lattice site. We assume ideal behavior, so all Flory–Huggins molecules, and surface groups in the system occupy a single lattice site. However, the system is still in equilibrium with a reservoir of constant chemical potential.

Our calculations are performed with the following parameters. The number of lattice sites is \( \mathcal{N} = \mathcal{N}_x = \mathcal{N}_z = 7 \) at a pH of 7 and higher. For two amphoteric plates at pH 6 (1 pH unit from the IEP), we observe large potentials for small separations. Consequently, the low-potential approximations (dashed blue curves) for the electric potential and pressure are not valid in such cases. However, upon increasing the separation, the electric potential gradually decreases and the low-potential approximation becomes valid.

### 4 Results and discussion

In this section, we will first discuss the analytical results, followed by a comparison with SCF computations.

#### 4.1 Analytical results

When two charged amphoteric plates in an electrolyte solution are brought to inter-plate separations \( h \leq \lambda_D \), the electric field between the plates should be sufficiently small for the zero-field treatment of the preceding sections to apply. In the limit of two touching plates, we find a finite contact Donnan potential due to a complete discharging of the plates. This behavior can be clearly seen in Fig. 2: there is no divergence of the potential \( \psi = u_0k_BT/\epsilon \) for small \( h/\lambda \). Since the disjoining pressure \( \Delta \Pi_D \) is directly determined by the Donnan potential, the disjoining pressure at contact also remains finite. Remarkably, the contact potential \( u_0 \) (eqn (23)) is fully determined by the equilibrium constants \( pK_1 \) and \( pK_2 \) and the salt concentration; it is therefore independent of the density \( \sigma_{tot} \) of amphoteric surface groups.

Fig. 2 also shows various approximations to the zero-field theory for the parameters listed in Table 1 (first section, \( pK_1 = 1 \)). For two amphoteric plates at pH 6 (1 pH unit from the IEP), we observe large potentials for small separations. Consequently, the low-potential approximations (dashed blue curves) for the electric potential and pressure are not valid in such cases. However, upon increasing the separation, the electric potential gradually decreases and the low-potential approximation becomes valid.

\[
\Delta \Pi_D = \frac{\phi_+(T/2) + \phi_-(T/2) - 2\phi_s}{2\phi_s} \tag{46}
\]

where \( \phi_+ \) and \( \phi_- \) are the total volume fractions of positive and negative ions (including background salt), respectively, and where \( \phi_s \) is the volume fraction of inert salt in the reservoir.
Table 1  Overview of relevant parameters for zero-field theory and SCF computations shown in Fig. 2–5. The contact potential and effective contact potential are displayed here as $\psi_0 = u\delta_0^\text{eff}/e$ and $\psi_0^\text{eff} = u\delta_1^\text{eff}/e$, respectively. Note that the presence of additional inert background salt in the zero-field theory is implicitly accounted for through a shift in the pK’s and pH; the reported values for $k_1$ and $k_2$ include this shift. For all systems, $\sigma_{\text{tot}} = 1 \text{ nm}^{-2}$. Identical values are not repeated.

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even though the contact potential is large. For pH 6.9 and 6.99, closer to the IEP, the low potential approximation is valid over the entire range.

Within the low potential approximation, the contact potential $u_0^\text{eff}$ dictates the decay of the electric potential and disjoining pressure with the scaled distance $h/\lambda$. The initial decay is linear, with a slope set by $u_0^\text{eff}$, see eqn (29) and (44) (red dotted curve). This contrasts surfaces with a constant charge or a single dissociation equilibrium, which feature a divergence of the disjoining pressure with $h^{-1}$ and $h^{-1}\lambda^2$, respectively, in the limit of small separations. This is the so-called Van’t Hoff regime, where the repulsion is dominated by an ‘ideal gas’ of counterions. The full discharging of amphoteric surfaces at contact (eqn (35)) prevents this divergence.

However, at large separations the decay becomes independent of $u_0^\text{eff}$, see eqn (31) and (45) (green dotted curve). When scaling the disjoining pressure as $[h/\lambda]^2\Delta\Pi$, this universal inverse square decay displays as a horizontal plateau. It is evident however that, closer to the IEP an increasingly large separation $h/\lambda$ is required to reach this limit. This is connected to the small contact potential $u_0^\text{eff}$, see Table 1, because the large-separation limit only holds for $h \gg \lambda u_0^\text{eff}$.

This brings us to the question: under which conditions can the inverse square decay occur in practical amphoterics systems? First of all, we note that the zero-field approximation is only valid within the unscreened regime, i.e., for strongly overlapping double layers or $h \lesssim \lambda_0$. Second, the electric potential must be small, which is generally true for $h \gg \lambda$ even in case of large contact Donnan potentials. (Naturally, for small contact potentials, this condition is met regardless of separation.) Finally, to access the inverse square decay of eqn (45), the condition $h \gg \lambda u_0^\text{eff}$ must be met. This suggests that, depending on the value of $\lambda_0$, it may become impossible to access the inverse square decay for increasingly small contact potentials, since the Debye screening may come into effect first. However, the values in Table 1 do suggest that at least under certain combinations of parameters the inverse square decay could be observable.

4.2 Comparison to SCF

To investigate the validity and accuracy of our zero-field theory for two amphoterics surfaces, we will now compare the results to self-consistent field (SCF) computations that solve the full Poisson–Boltzmann equation and in which ions have a finite size.

First we devote our attention to the situation at large Debye length, when the ionic strength is low ($I = 10^{-6}$ M, $\lambda_0 = 0.3$ nm), see Fig. 3. For these systems, we set a large difference between the two equilibrium constants ($pK_1 = 1, pK_2 = 13$) and the pH is rather close to the IEP, entailing very small degrees of dissociation. A full overview of all parameters is given in the first section of Table 1 ($pK_1 = 1$).

From Fig. 3 it is evident that the zero-field Donnan treatment gives a quantitative prediction of the electric potential, the net degree of dissociation, and the disjoining pressure for the parameters in question. We can identify three distinct regimes in Fig. 3:

1. Relatively far from the IEP (e.g., a pH of 6), the electric potential is large for small $h$. In this regime, the decay of $\delta$ and $\Delta\Pi$ is governed by $\lambda$.

2. For intermediate separations, $h \gtrsim \lambda$, the system enters the low-potential limit, and the length scale of the decay is set by $\lambda u_0^\text{eff}$ following eqn (26) and (43).

3. Finally, for large separations, the decay is once again set by $\lambda$, however the decay is now universal: the absolute value of potential and pressure becomes independent of the contact potential, following eqn (31) and (45).

Close to the IEP (pH 6.9 and 6.99), the contact potentials are small: regime (1) is not present and regime (2) holds until the plates touch. Additionally, all curves appear to be similar, except for a different pre-factor. This is because close to the IEP, where $k_1 k_2 \approx 1$, the factor $\lambda u_0^\text{eff}$ becomes constant:

$$\frac{\lambda}{u_0^\text{eff}} \approx \frac{\sigma_{\text{tot}}}{\rho_s} \frac{2}{2 + k_1} \text{ for } k_1 k_2 \approx 1.$$  (47)

This can also be verified numerically from the data in Table 1. Thus, in the vicinity of the IEP, a particular system will have a universal behavior regardless of the precise distance to the IEP, differing only by a pre-factor set by $u_0^\text{eff}$.

We now turn our attention to the situation an ionic strength of $I = 10^{-4}$ M ($\lambda_0 = 30$ nm), see Fig. 4. The systems shown here have a fixed pH of 7 and $pK_1 = 3$. The second equilibrium
constant $pK_2$ is varied, but in general $pK_1$ and $pK_2$ are significantly closer together than in the previous example. A detailed overview of all parameters is shown in the second section of Table 1 ($pK_1 = 3$).

From Fig. 4 it is immediately apparent that beyond separations comparable to the Debye length, the zero-field assumption starts to break down: screening comes into effect and the predicted potential, degree of dissociation, and disjoining pressure as predicted by the Donnan model are all higher than the numerical SCF results. At small separations $h \leq 15$ nm the zero-field approach is still valid. However, because Debye screening occurs at larger separations, the low potential limit cannot be realized in these systems.
There are also qualitative differences between the systems currently at hand and those previously discussed in Fig. 3. The systems in Fig. 4 feature a very long-ranged decay of potential and disjoining pressure with distance. The reason is that the length scale \( \lambda \) for these systems is very large, of the order of 1 \( \mu \)m. As can be seen from eqn (20a), this is because there is a large net degree of dissociation, as one of the \( K_1 \)'s is rather close to the pH.

Additionally, for \( pK_2 = 10 \) there is a characteristic (rather sharp) upturn of \( \psi = \kappa_b T e \) and \( \Delta \Pi_d \) at small \( h \), which precedes the slow decay of potential and pressure. This upturn is related to the fact that surfaces with a single dissociation equilibrium have an infinite contact potential. Here, the second dissociation equilibrium is very weak compared to the first equilibrium, such that it is practically absent. The divergence of the contact electric potential and contact disjoining pressure thus slowly reappears when one of the equilibria becomes negligible.

Finally, we turn our attention to the inverse square decay of pressure with separation. Since the focus of the current work is on amphoteric surfaces, we will restrict ourselves to systems close to the IEP, such that both equilibria play a role. As discussed in the previous section, the observation of this behavior is most likely when simultaneously the Debye length is relatively large and both \( \lambda \) and \( \Delta b_0^\text{eff} \) are small. The latter can be judged from eqn (47). For systems close to the IEP, it turns out that \( \Delta b_0^\text{eff} \) is small when \( k_1 \) is large. Because close to the IEP \( k_1 k_2 \approx 1 \), this means that \( k_2 \) must be small. In other words, there should be a large difference between \( pK_1 \) and \( pK_2 \).

This requirement is illustrated in Fig. 5, which shows two systems with (top) \( \Delta pK = pK_2 - pK_1 = 12 \) (same as in Fig. 3, pH 6.5) and (bottom) \( \Delta pK = 16 \) (see also Table 1, third section, pH \( pK_1 = -1 \)). The pressure is scaled as \( (h/\lambda)^2 \Delta \Pi_d \), and the effective inter-plate distance

\[
h_{\text{eff}} = h - 2\sigma_{\text{tot}} b^3
\]

(48)
takes into account the finite volume of the surface groups.\(^{14}\) It is evident that in the first case the plateau value \( (h/\lambda)^2 \Delta \Pi_d = 0.5 \), characteristic for the inverse square decay, is not fully developed due to the occurrence of Debye screening at \( h \approx 10^2 \) nm. Increasing \( \Delta pK \) to 16 decreases \( \lambda \) hundredfold, while keeping \( u_0^\text{eff} \) constant, see Table 1. This indeed makes the inverse square decay clearly apparent.

It should be stressed that our SCF computations incorporate a treatment of Poisson–Boltzmann theory on a mean-field level. Therefore, deviations from Poisson–Boltzmann theory (such as due to overcharging\(^{29}\) or under confinement\(^{30}\)) are not accounted for. We leave further discussion on the general validity of Poisson–Boltzmann theory for other work; we note here that our results show that the zero-field Donnan model represents an insightful treatment of repulsions between amphoteric plates with strongly overlapping double layers on the level of Poisson–Boltzmann theory.

### 5 Conclusions

The entropic repulsion between two amphoteric surfaces has been analyzed in the limit of zero electric field, where ideal ions are homogeneously distributed in the inter-plate solution and are in equilibrium with a reservoir with constant salt concentration. We show that there is an inherent length scale \( \lambda \) that governs the decay of electric potential and disjoining pressure with inter-plate distance, which serves as the unscreened counterpart of the Debye length. In the limit of low potentials, we show that the decay is set by an effective contact potential \( u_0^\text{eff} \) and the length scale \( \lambda \). Further, the zero-field disjoining pressure between amphoteric surfaces has an inverse-square decay for low potentials and sufficient inter-plate distances, just as for constant-charge surfaces and surfaces with a single dissociation equilibrium, but with a smaller amplitude due to the charge regulation. This is remarkable, because in general scaling depends on the precise boundary conditions (i.e., constant charge, constant potential, or charge regulation). Numerical self-consistent field lattice computations quantitatively confirm the predictions of the zero-field model, including the inverse square decay for large separations and small potentials. Additionally, the zero-field model remains valid at large electric potentials, showing that Donnan theory is a tractable way to describe repulsions between amphoteric charged plates separated by distances smaller than the Debye length. The results described here have direct practical relevance, for instance to understand electrostatic interactions in self-assembled multi-layered structures, such as microtubes or membranes. A pertinent continuation for future work would be an extension of the present model towards non-flat geometries, such as concentrated dispersions of amphoteric colloids with average particle separations of the order of the Debye length.
Conflicts of interest
There are no conflicts to declare.

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