Generativity in design research: the case of developing a genre of action-based mathematics learning activities

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Educational research is often of limited use to designers, as it typically offers descriptions of or explanations for how education was or currently is. In this paper we argue that generalizability is not enough; what deserves more attention is generativity. *Designers and educators need actionable knowledge on how education could be. To substantiate the criterion of generativity, we here develop the construct of a design genre—a theory-driven class of designs targeting a particular form of learning. As an example we discuss the evolution of an action-based genre of embodied design through iterative theory-design clarifications. Initially instantiated in the form of activities for promoting the understanding of proportional reasoning, the genre has been elaborated through the process of applying it to additional notions such as parabolas and trigonometry. We argue that this genre was generative for design, theory development, and methodological innovation.*

Keywords: Mathematics curriculum, generalization, sensory experience, instructional design, learning theory.

Generativity

Educational research mostly investigates practice "as it was" or "as it is." Generating new desirable types of education—"as it could be"—has largely been left to practitioners and designers. With its focus on generalizability, educational theories typically provide little help to designers or educators in making design decisions, for example on how to use new tools, develop a new teaching approach, or achieve new learning goals (Klaassen & Kortland, 2013). It has been argued that educational research is a linking science (Dewey, 1900) or a design science (Glaser, 1976), more similar to engineering, medicine, and management than to physics, biology, or psychology. What linking or design sciences are looking for is "actionable knowledge" (Argyris, 1996). Design research originates in the wish to produce such actionable, advisory knowledge (Bakker, 2018).

Discussing the role of theory in design research, diSessa and Cobb (2004) distinguished several kinds of theory such as grand theories, orienting frameworks, frameworks for action, and domain-specific theories. The authors observe that the more general such theories are, the less informative these typically are to the designer or educator. In their view, "theory must do real design work in generating, selecting, and validating design alternatives at the level at which they are consequential for learning" (p. 77). An example of what they consider good innovation is the general construct of socio mathematical norms, yet they admit this notion is not very prescriptive for design.

In line with the aforementioned need to produce actionable knowledge, we propose that educational researchers should not just care about generalizability (deriving general ideas from empirical specificities) but also about *generativity* (deriving ideas for specific design solutions from general principles). The aim of this paper is to argue that so-called *design genres* (Abrahamson, 2014), as theory-driven types of designs targeting a particular form of learning, offer useful knowledge to educational researchers, design researchers, and designers alike. We also submit that design genres may fulfill the criteria of generalizability and generativity in the sense if they are research-based and productive in generating new actionable knowledge, but also provide concrete advice on how to design tasks in this particular genre (cf. Watson & Othani, 2015). Thus they would fulfill the role theories should play in design research according to diSessa and Cobb (2004).

To illustrate the notion of generativity, we explore how the formulation of such a genre facilitated the process of creating similar designs outside of the source laboratory. We reflect on the evolution of an *action-based genre of embodied design* (see Figures 1 and 2). This design genre originated in Abrahamson's Embodied Design Research Laboratory (Berkeley), and was extended by Shvarts in Moscow and Bakker, Alberto, and colleagues in Utrecht to other mathematical domains. In terms of design, we reflect on a set of general and generative principles that slowly developed through iterative cycles of collaborative design research.

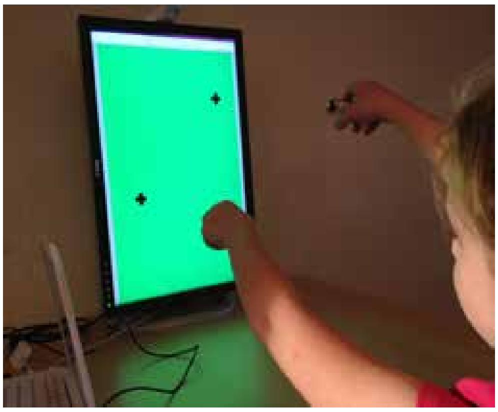


Figure 1: Solving a movement riddle in the Mathematics Imagery Trainer, a child learns to move physically in a new way that is then articulated formally as governed by a proportional function

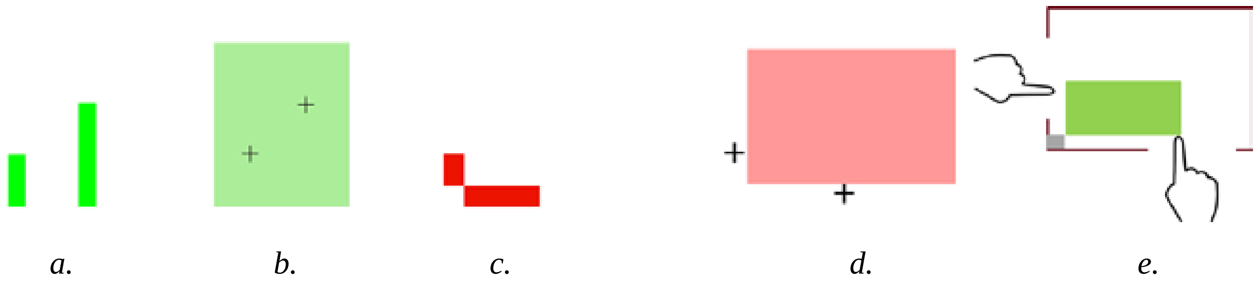


Figure 2: Five versions of the MIT-proportion applications: (a) parallel bars; (b) parallel with full screen feedback; (c) orthogonal bars; (d) orthogonal with full screen feedback; (e) rectangle feedback

A prototypical action-based design for proportion

Our design research is inspired by a range of philosophical positions and cognitive-science research arguing for the embodied origin and nature of cognition (Dreyfus & Dreyfus, 1999)—none of which yielded specific ideas for design. Accordingly, we sought to develop and evaluate action-based learning activities that could potentially support students' sensorimotor grounding of mathematical content. As part of our research program, we deliberately focus on mathematical domains beyond simple arithmetic to explore whether theories of embodiment hold promise for more advanced topics too.

Figure 2 presents a set of schematic screen images featuring computer-based materials we have been exploring (there are also versions with lines or grids, and with numbers). These activities build on, yet extend, earlier research by Abrahamson and colleagues on the Mathematics Imagery Trainer for Proportion (MIT-proportion). Clinical research on the MIT-proportion has demonstrated the emergence of students' sensorimotor schemes, as they attempt to satisfy the task specifications: First, manipulate two virtual objects on the screen so that the bars (2a) or entire screen (2b) becomes green; and then move both objects simultaneously keeping the green feedback. Unknown to the students, the favorable feedback of green appears only if the objects they manipulate make for a particular proportion between key magnitudes relative to a spatial frame of reference, for example a 1:2 ratio between the heights of the two bars in Figure 2a or between the *y*- and *x*-axis, linear extensions in Figure 2c.

We found patterns in students' solution process on these tasks that could be summarized well in Piagetian terms of reflective abstraction (Abrahamson, Shayan, Bakker, & Van der Schaaf, 2016). Eye-tracking technology allowed us to study the emergence of sensorimotor action schemes in more detail than hitherto possible. For example, the gaze data showed that students focus their attention at particular locations on the screen so as to coordinate their bimanual movement. We came to understand these sensory behaviors as evidence that students were developing *attentional anchors*, spontaneous perceptual constructions that facilitate the enactment of challenging movements by focalizing the coordination of motor actions. That is, we were witnessing the emergence of new goal-oriented sensorimotor schemes. For example, Figure 3a shows a typical scan path when students keep the bars green (see Duijzer, Shayan, Bakker, Van der Schaaf, & Abrahamson, 2017, for an automated analysis of patterns in how students looked in particular areas of interest). Students look halfway up along the tall bar before they express the insight that it is twice as tall as the shorter bar. Arriving at this insight took between 10 seconds and 9 minutes.

Figure 3: (a) typical scan path when trying to keep the bars green while moving them up. (b–e) attentional anchors when keeping green in the orthogonal task with full screen color feedback

Expanding a genre of action-based embodied designs

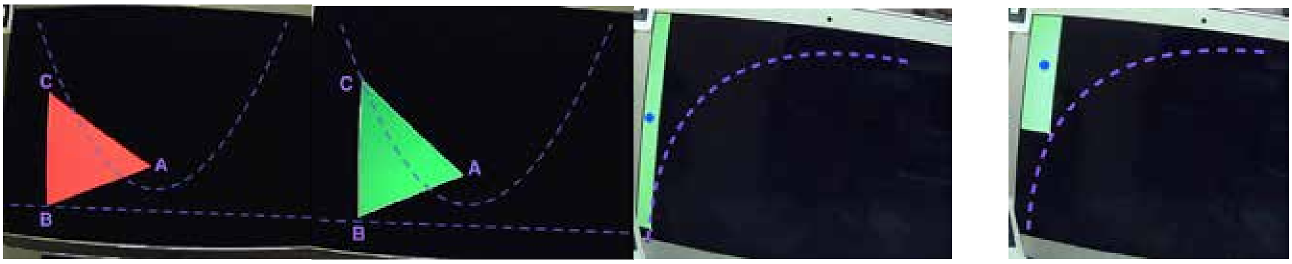
Intending to expand this genre of educational designs, we varied on the initial interactive activity along several dimensions, such as mathematical domain, the form of interactive feedback, or characteristics of input movements (bimanual/unimanual, in the air in front of the screen / on a touchscreen or a tablet / via touchpad). These new designs were somewhat intuitive, unstructured, or loosely meditated. While we were aiming to keep the core of the design, stating just what that core might be was still work in progress. We found ourselves in a situation not unlike a child participating in a cognitive developmental psychology experiment investigating the emergence of categorical reasoning (Vygotsky, 1962): Tasked to select from a pool of colored geometrical shapes a set of some yet-to-be-defined class, the child groups a red and a yellow triangle, since they both are triangles, but then adds a yellow circle, since it too is yellow, and then, perhaps, a trapezoid, since it is similar in another way. In this process of iteratively varying on different aspects of the design, we came to recognize a need to articulate the limits of variation that would keep intact some constant qualities of this genre's essential features, whatever these would turn out to be.

Orthogonal design for proportional reasoning

Urged by a request from a high school mathematics teacher, Lee, Hung, Negrete and Abrahamson (2013) envisioned a first variant on the original MIT-proportion activity, in which the hands would move not in parallel but orthogonally. This variant, which expands the original activity from 1D to 2D, was implemented at Utrecht University (Figures 2c–e; 3b). The new task demanded of students to solve the challenging movement problem of coordinating simultaneous motor action along orthogonal axes. Figures 3b–e include a student's eye fixations (the orange blobs overlain on screenshots from the video-recording). The student told us he is imagining a diagonal line connecting his left- and right-hand index fingertips and that he is moving *this diagonal line* sideways to the right, keeping constant the line's angular orientation to the base line. As such, the diagonal line constituted an attentional anchor for this student and others who arrived at similar solutions. We speculate that this sensorimotor scheme—a bimanual coordination oriented on a perceptual construction—could serve as the foundation for further developing various mathematical concepts, such as function, covariation, or slope. This task elicited rich geometrical language as students explained their strategy, including reference to angles, lines, rectangles, triangles et cetera.

Conic sections

It has been shown that visualization of the parabola as a vase-like curve on a surface may lead to a wrong conceptualization (Aspinwall, Shaw, & Presmeg, 1996). As an alternative Shvarts designed opportunities for individual students to experience a parabola as a locus of equidistant points (MIT-parabola, see Figure 4a–b). The solution is to move Vertex C on the screen such that it is equidistant from a straight horizontal line (Vertex B directly below Vertex C, running along the parabola's directrix) and a separate point (Vertex A, fixed on the parabola's focus). Consistent with the embodied-design principle that semiotic symbols should be absent in the initial activity (Abrahamson, 2014), only a triangle is featured on the screen during the first stage of the task (the dashed lines in Figure 4 are for illustration only and are never shown to the students). Once students had accomplished the embodied task and were ready to mathematize their solution in the form of the canonical parabola formula, the axes were introduced. Dual eye-tracking of this activity (i.e., of both the tutor and the student) revealed the traces of attentional anchors along the triangle's median. These traces appeared in the eye movements both of the students and the tutors (Shvarts & Abrahamson, 2018).



a. *b.* *c.* *d.*

Figure 4: (a, b) MIT-parabola; and (c–d) MIT-area. The dashed lines were not shown to the students

Yet another task (MIT-area, see Figures 4c–d) implies another conic section, the hyperbole: The top left vertex of a rectangle on the screen was fixed, while a participant was moving the opposite vertex to change the size and form of the rectangle. The rectangle turned green if it had a particular area. The eye-movement data (blue) demonstrate a simple attentional anchor of fixating in the center of the rectangle (Figures 4c-d) while grasping the area by extrafoveal vision.

Trigonometry

Alberto, Bakker, Van Aalst, Boon and Drijvers (submitted) designed a new task in the same genre, now for trigonometry (MIT-trig). The student slides the left-hand fingertip on the perimeter of a unit circle and the right-hand fingertip on a sine graph (Figure 5). Whenever the radian value on the circle corresponds to the *x*-value in the sine graph, the frame around these two objects becomes green. The student needs to keep the frame green while moving both hands. Analysis of a case-study participant indicates that she used an attentional anchor on a segment connecting the two fingers, which seemed to help her keep the two fingers at the same height. Mathematized, this came to mean that the left- and right- fingertip positions are equally "high" or "low" on the grid, thus sharing the same *y*-value, *sin(x)*. This awareness appeared to support her bimanual coordination process of keeping green while moving.

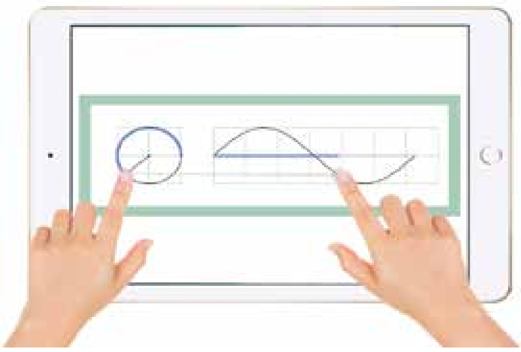


Figure 5: The frame is green if the radian (unit circle, left) and *x*-value (sine graph, right) correspond

A reflection on theoretically driven and empirically grounded principles

We have referred to the set of embodied tasks discussed above as exemplars of a design genre, an action-based genre of embodied design (Abrahamson, 2014). In so doing, we conceptualize a design genre as capturing both the "why" and "how" of generating learning activities in the form of a set of principles that instantiate the theoretical presumptions concerning the process of learning (the "why") and that guide a designer through the design process (the "how"). The embodied design framework builds on enactivist and dynamic systems theory to stipulate the creation of a goal-oriented task that calls for a sensorimotor investigation of the problem field. The design *core objective* is that students' new knowledge will emerge in a form of sensorimotor coordination constituted as a solution for a motor problem (Bernstein, 1967), when they encounter unexpected constraints in the course of attempting to accomplish the activity task. The student's solution to the enactment problem is a new sensorimotor coordination applied to a spontaneous construction of an attentional anchor in the perceptual field; this new perceptuomotor scheme constitutes an embodied form of mathematical knowledge underlying the task design. The motor coordination undergoes automation that frees space for mathematical reflection and cultural referencing.

As design genres evolve, theoretically driven design objectives settle as definitive and essential principles guide the process of designing new activities. At the same time, the introduction of new activities as plausible exemplars of the genre may challenge both conscious and unconscious assumptions, causing a team to rethink the principles that had been taken as definitive and essential. Wherever a resolution cannot be reached, new research questions may emerge. For example, the design dimension of *feedback*, which had not been on the fore of our agenda, drew our interest only once we realized the variable effect of feedback on attention: An abrupt change of color or contour within the visual field may attract overt attention and thus disrupt the vulnerable construction of new eye-movement patterns governing manual control. The solution in MIT-trig was to introduce a rectangular frame around the central interactive arena (Figure 5). This rectangle creates a uniform stimulus amenable to peripheral vision, thus enabling the student to retain foveal vision on the virtual manipulatives. That said, we recognize that there is much more for us and our colleagues to learn and experiment so as to optimize for design decisions along the dimension of feedback.

Another design dimension in need of further research is that of *uni*- vs. *bimanuality*. Is bimanuality an essential attribute of the genre? Our current designs entail both bi- and unimanual tasks. A simplistic assumption would be that two hands are necessary to provoke the need for new coordination. Moreover, much of mathematics is relational in nature. However it could be that coordinations are formed between manual and visual tracking of object on the screen. We are currently devising a study to investigate this problem. Thus the theory–design dialectics determining the research team's iterative efforts contributes to the theoretical rather than empirical generalization of the activities into a design genre (Davydov, 1990).

Conclusions

In this paper we proposed that design genres do not only allow for generalizability but also generativity. We have argued that the emerging design genre illustrated here was generative: It helped in making concrete design decisions and thus in speeding up the design process outside of the source laboratory. Note that this was also possible due to our attempts to generalize while designing: By framing any design decision as a case of a more general theoretical idea (cf. Abrahamson, 2018), we were able to identify features of the design that were important in achieving particular goals (e.g., development of sensorimotor action schemes that were relevant grounding for mathematical reasoning). Based on these features, new designs were developed for promoting reasoning about other mathematical topics.

As a bonus we can also highlight the notion of *generativity* in a more research-oriented sense: The innovative designs led to phenomena of learning that could hitherto not be studied, and thus led to many new research questions. Given the multimodal nature of students' observed sensorimotor behaviors, we were urged to draw on multidisciplinary bodies of literature, including sports and kinesiology, such as the construct of an attentional anchor developed by Hutto and Sánchez–García (2015). Throughout, we have debated what counts as mathematical activity. If students coordinate two hands in a way that we consider mathematically relevant, in what sense does this capacity constitute a precursor to conceptual understanding? Would the sensorimotor experience enable them to evoke the meaning of particular mathematical signs and reason through relevant problems? What design features help to promote reflection and argumentation? Is there any evidence that students re-use the sensorimotor schemes that have proved useful in earlier tasks?

The design-research approach to develop theory by design has generated methodological contributions, too, for example dual eye-tracking, by which we can now investigate student–student or student–tutor joint attention. Such research settings, where people collaboratively develop new sensorimotor action schemes, open up new areas of multidisciplinary research that accounts for intersubjective multimodal coordination processes (Shvarts & Abrahamson, 2018). Contemporary educational theory appears to lack powerful conceptual and methodological frameworks for studying these processes. It is hence also at this level of motivating and developing new research programs that, we believe, design genres can be generative.

Aforementioned types of generativity may also impact educational practice. We speculate that the learning of many mathematical topics across the curriculum (function, covariation, etc.) can be supported by activities in the same design genre. When students recognize a new task as related to a previous task, the instrumentation phase could be more efficient. Finally, we expect that evidence of design genre's generativity for design, theory, and methods will make design research more appealing to researchers, educators, and policymakers.

Acknowledgment

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References

Abrahamson, D. (2014). Building educational activities for understanding: An elaboration on the embodied-design framework and its epistemic grounds. *International Journal of Child-Computer Interaction, 2*(1), 1–16. doi:10.1016/j.ijcci.2014.07.002

Abrahamson, D. (2018). Teaching design research as a case of cultivating a community of professional practice. In A. Bakker (Ed.), *Design research in education: A practical guide for early career researchers* (pp. 153–171). London, UK: Routledge.

Abrahamson, D., Shayan, S, Bakker, A. & Van der Schaaf, M. F. (2016). Eye-tracking Piaget: Capturing the emergence of attentional anchors in the coordination of proportional motor action. *Human Development, 58*(4–5), 218–224.

Alberto, R. A., Bakker, A., Van Aalst, O., Boon, P., & Drijvers, P. (submitted to CERME11, 2019). Embodied Instrumentation: An eye-tracking case study in trigonometry.

Argyris, C. (1996). Actionable knowledge: Design causality in the service of consequential theory. *The Journal of Applied Behavioral Science, 32*(4), 390–406.

Aspinwall, L., Shaw, K. L., & Presmeg, N. C. (1996). Uncontrollable mental imagery: Graphical connections between a function and its derivative. *Educational Studies in Mathematics, 33*(3), 301–317.

Bakker, A. (2018). *Design research in education: A practical guide for early career researchers*. London, UK: Routledge.

Bernstein, A. N. (1967). *The co-ordination and regulation of movements.* Oxford, UK: Pergamon.

Davydov, V. V. (1990). *Types of generalization in instruction: Logical and psychological problems in the structuring of school curricula*. (J. Teller, Ed.). Reston, VA: National Council of Teachers of Mathematics.

diSessa, A. A., & Cobb, P. (2004). Ontological innovation and the role of theory in design experiments. *The Journal of the Learning Sciences, 13*(1), 77–103.

Dreyfus, H. L., & Dreyfus, S. E. (1999). The challenge of Merleau-Ponty's phenomenology of embodiment for cognitive science. In G. Weiss & H. F. Haber (Eds.), *Perspectives on embodiment: The intersections of nature and culture* (pp. 103–120). New York: Routledge.

Duijzer, C. A., Shayan, S., Bakker, A., Van der Schaaf, M., & Abrahamson, D. (2017). Touchscreen tablets: Coordinating action and perception for mathematical cognition. *Frontiers in Psychology, 8(*144).

Hutto, D. D., & Sánchez-García, R. (2015). Choking RECtified: Embodied expertise beyond Dreyfus. *Phenomenology and the Cognitive Sciences, 14*(2), 309–331.

Klaassen, K., & Kortland, K. (2013). *The role and nature of theory in design research*. Paper presented at ESERA, Cyprus, September 2–7.

Lee, R. G., Hung, M., Negrete, A. G., & Abrahamson, D. (2013). *Rationale for a ratio-based conceptualization of slope: Results from a design-oriented embodied-cognition domain analysis.* Paper presented at AERA, San Francisco, CA, April 27–May 1.

Shvarts, A., & Abrahamson, D. (2018). *Towards a complex systems model of enculturation: A dual eye-tracking study.* Paper presented at the annual conference of the American Educational Research Association (Special Interest Group: Learning Sciences). New York, NY, April 13–17.

Vygotsky, L. S. (1962). *Thought and language*. Cambridge, MA: MIT Press.

Watson, A., & Ohtani, M. (Eds.) (2015). *Task design in mathematics education.* New York: Springer.