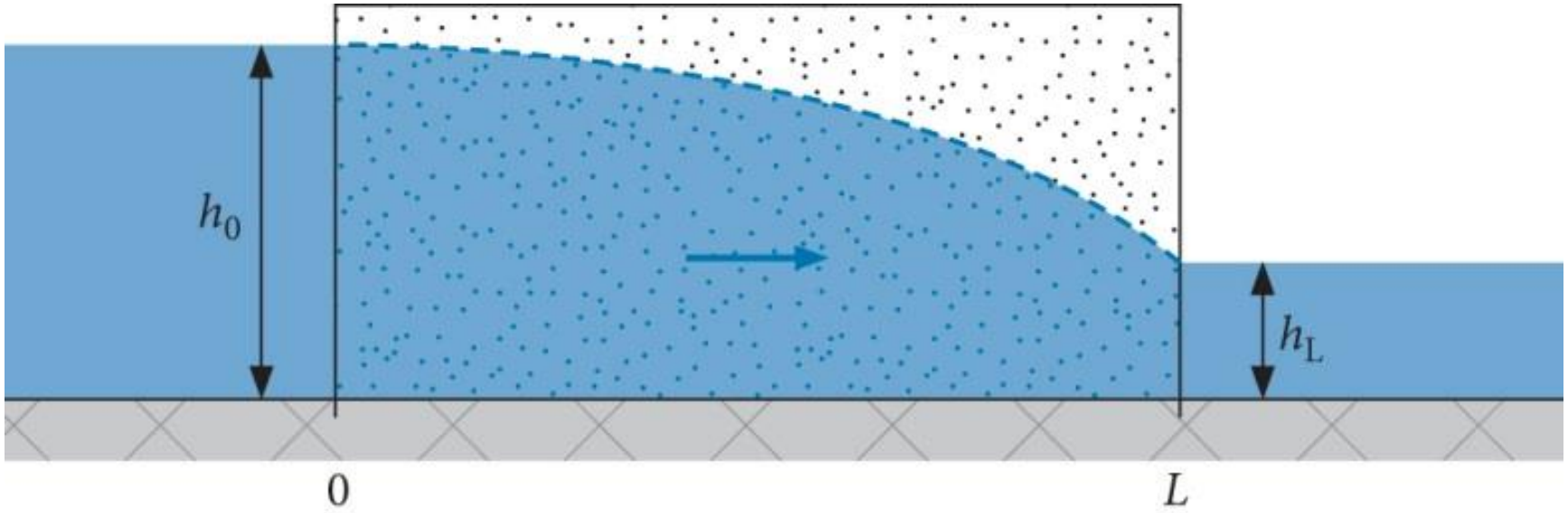
Unconfined aquifer

https://www.youtube.com/user/MartinRHendriks/videos



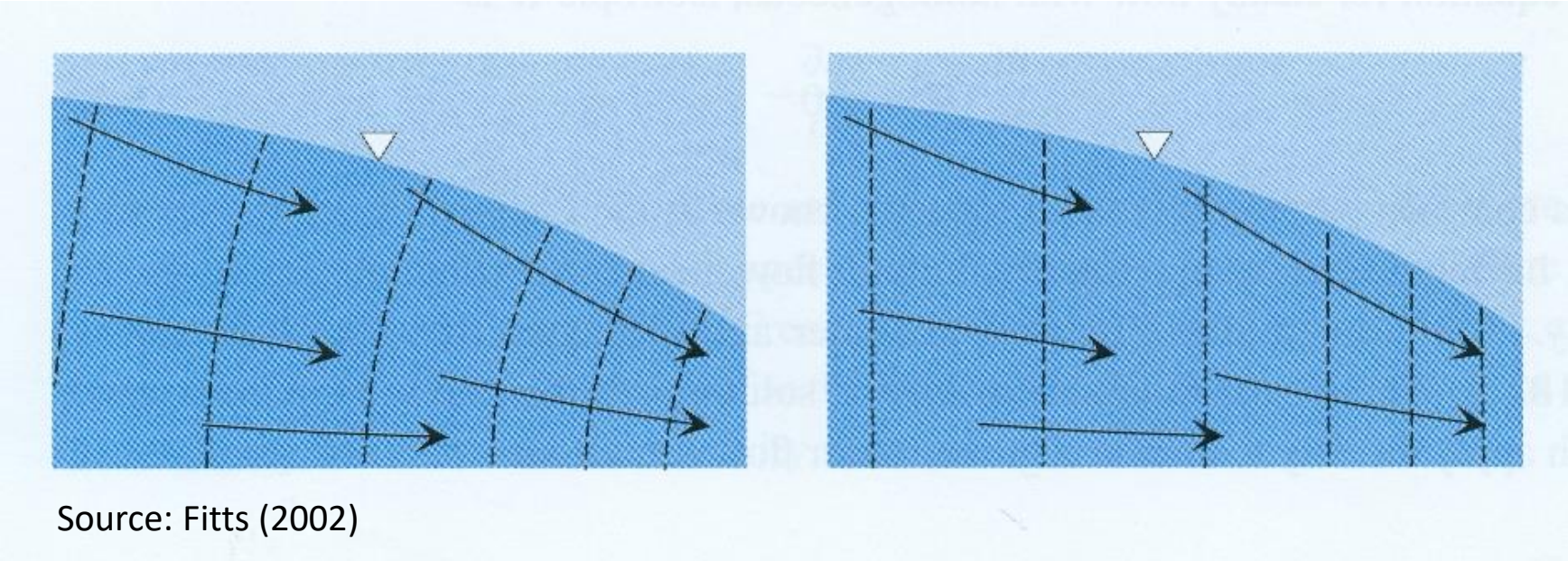
Q' = constant

$$Q' = -Kh\frac{dh}{dx}$$

$$Q' = \text{constant} \Rightarrow -Kh\frac{dh}{dx} = \text{constant}$$

Unconfined aquifer

https://www.youtube.com/user/MartinRHendriks/videos



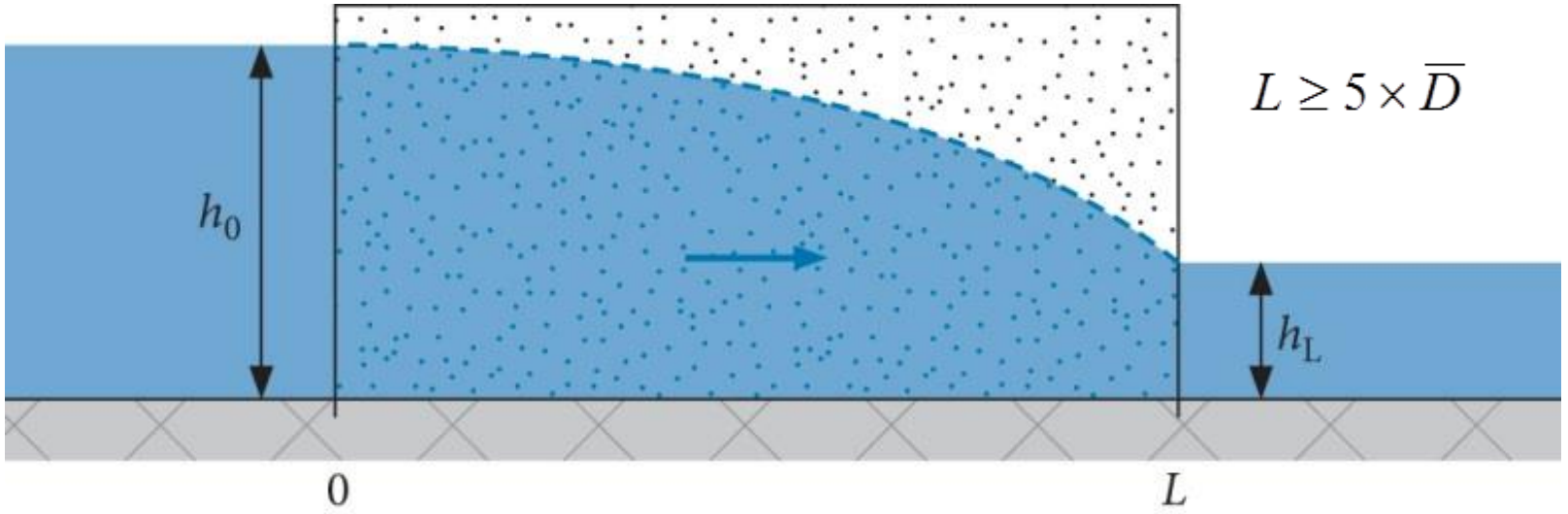
Source: Fitts (2002)

$L \geq 5 \times \overline{D}$: Dupuit-Forchheimer approximation

$$Q' = -K\overline{D}i$$

Unconfined aquifer

https://www.youtube.com/user/MartinRHendriks/videos



$$Q' = -K\overline{D}i \qquad \overline{D} = \frac{h_0 + h_L}{2} \qquad i = \frac{h_L - h_0}{L}$$

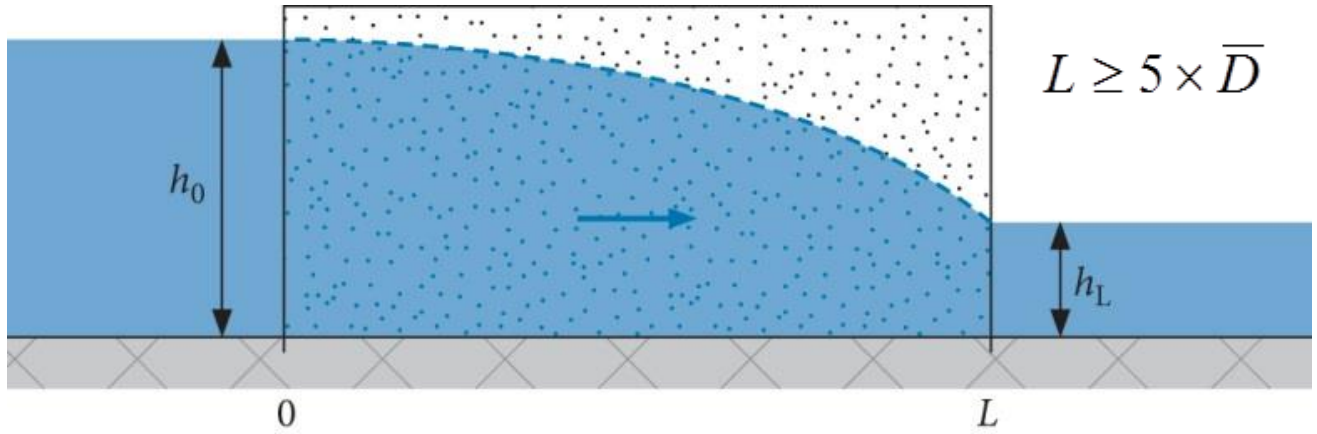
$$Q' = -K\frac{h_L + h_0}{2}\frac{h_L - h_0}{L} = -K\frac{h_L^2 - h_0^2}{2L}$$

Dupuit-Forchheimer equation

Unconfined aquifer

https://www.youtube.com/user/MartinRHendriks/videos

$$Q' = -Kh\frac{dh}{dx}$$



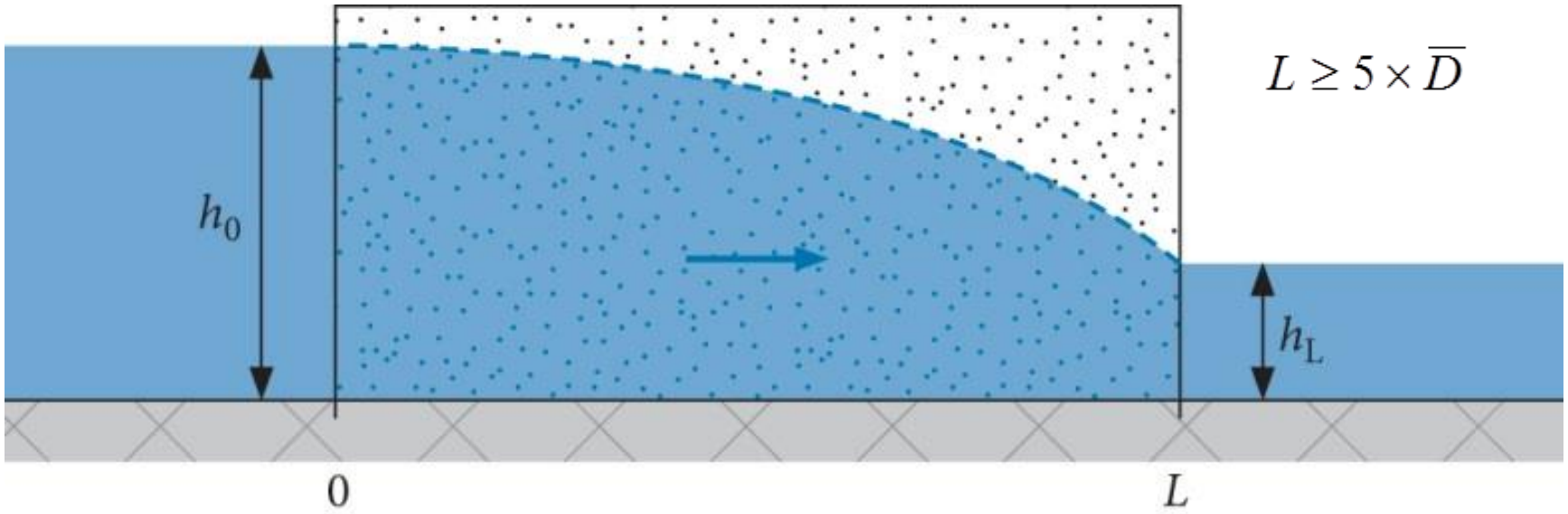
$$Q' = \text{constant} \Rightarrow -Kh\frac{dh}{dx} = \text{constant}$$

$$\text{chain rule } \frac{dy}{dx} = \frac{dy}{dh}\frac{dh}{dx} \text{ with } y = h^2 \text{ gives } \frac{dh^2}{dx} = \frac{dh^2}{dh}\frac{dh}{dx} = 2h\frac{dh}{dx}$$

$$h\frac{dh}{dx} = \text{constant} \Rightarrow \tfrac{1}{2}\frac{dh^2}{dx} = \text{constant} \Rightarrow \frac{dh^2}{dx} = C_1 \Rightarrow h^2 = C_1 x + C_2$$

Unconfined aquifer

https://www.youtube.com/user/MartinRHendriks/videos

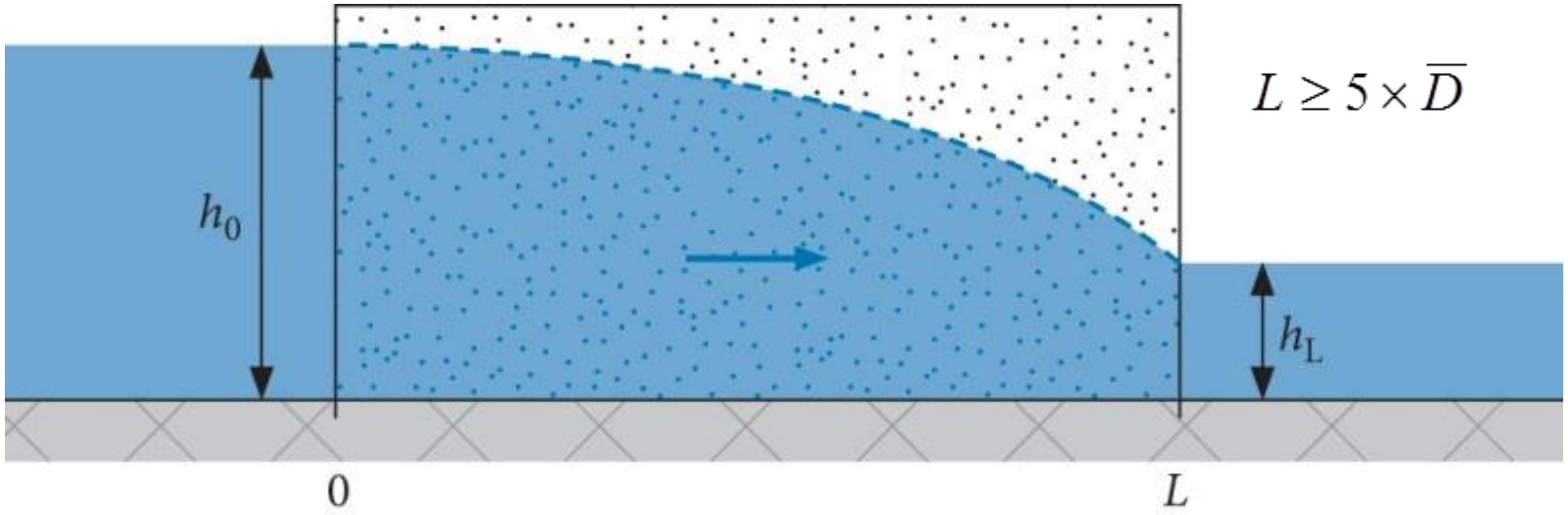


$$h^2 = C_1 x + C_2$$

$$h^2 = \frac{h_L^2 - h_0^2}{L} x + h_0^2$$

Unconfined aquifer

https://www.youtube.com/user/MartinRHendriks/videos



$$h^2 = C_1 x + C_2 \quad \Rightarrow \quad h = \left(C_1 x + C_2\right)^{\frac{1}{2}} \quad \Rightarrow \quad \frac{dh}{dx} = \tfrac{1}{2}\left(C_1 x + C_2\right)^{-\frac{1}{2}} C_1$$

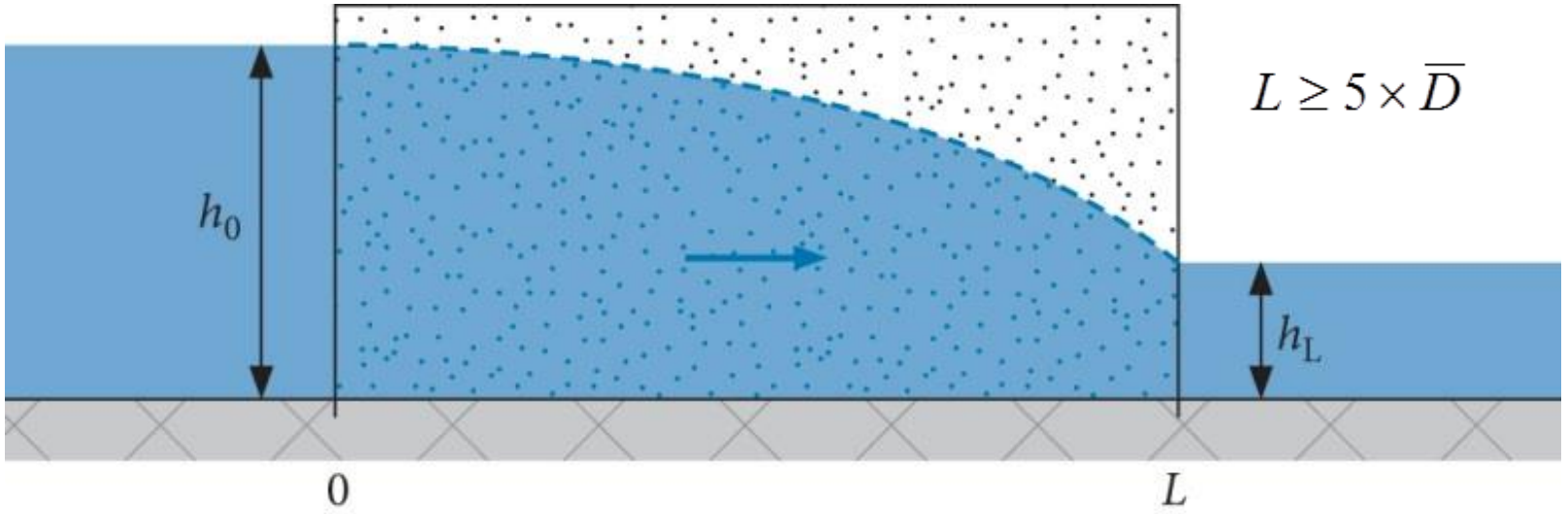
$$h\frac{dh}{dx} = (C_1 x + C_2)^{\frac{1}{2}}\, \tfrac{1}{2}\left(C_1 x + C_2\right)^{-\frac{1}{2}} C_1 = \frac{1}{2} C_1 \quad \Rightarrow \quad Q^{'} = -Kh\frac{dh}{dx} = -K\frac{1}{2}C_1$$

$$C_1 = \frac{h_L^2 - h_0^2}{L} \qquad Q^{'} = -\tfrac{1}{2} K \frac{h_L^2 - h_0^2}{L}$$

Dupuit-Forchheimer equation

Unconfined aquifer

https://www.youtube.com/user/MartinRHendriks/videos



$$h^2 = C_1 x + C_2 \quad \Rightarrow \quad \frac{dh^2}{dx} = C_1 \qquad \frac{dh^2}{dx} = 2h\frac{dh}{dx} \quad \Rightarrow \quad h\frac{dh}{dx} = \tfrac{1}{2}C_1$$

$$Q' = -Kh\frac{dh}{dx} \quad \Rightarrow \quad Q' = -\tfrac{1}{2}C_1 K \qquad C_1 = \frac{h_L^2 - h_0^2}{L}$$

$$Q' = -\tfrac{1}{2}K\frac{h_L^2 - h_0^2}{L}$$

Dupuit-Forchheimer equation

References

https://www.youtube.com/user/MartinRHendriks/videos

Fitts, C.R. (2002). Groundwater Science. Academic Press, Elsevier Science.

Hendriks, M.R. (2010). Introduction to Physical Hydrology. Oxford University Press.