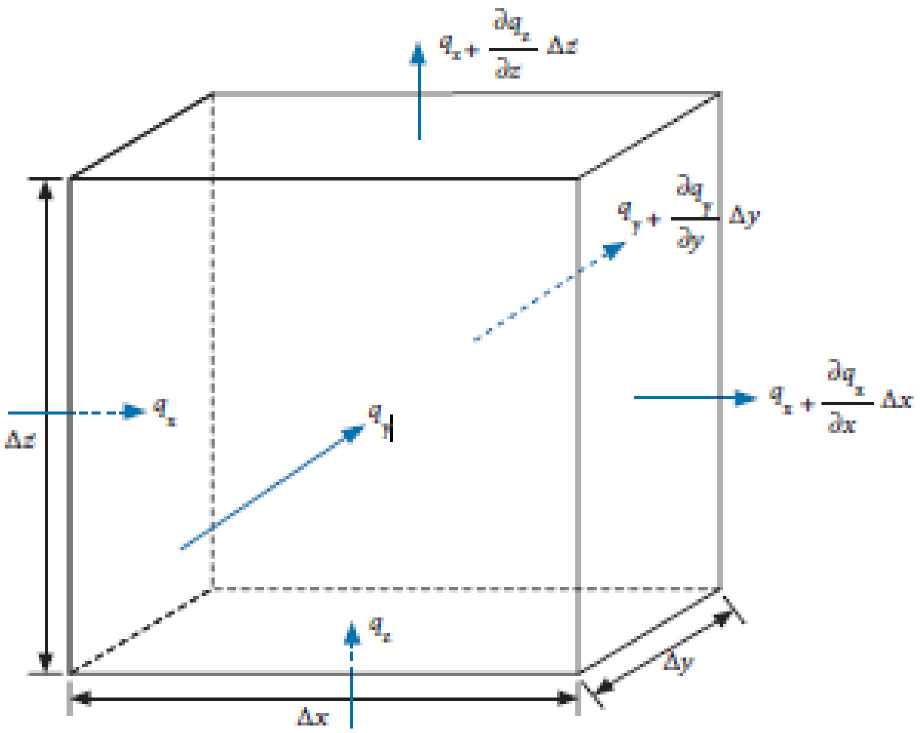
Elemental control volume

https://www.youtube.com/user/MartinRHendriks/videos

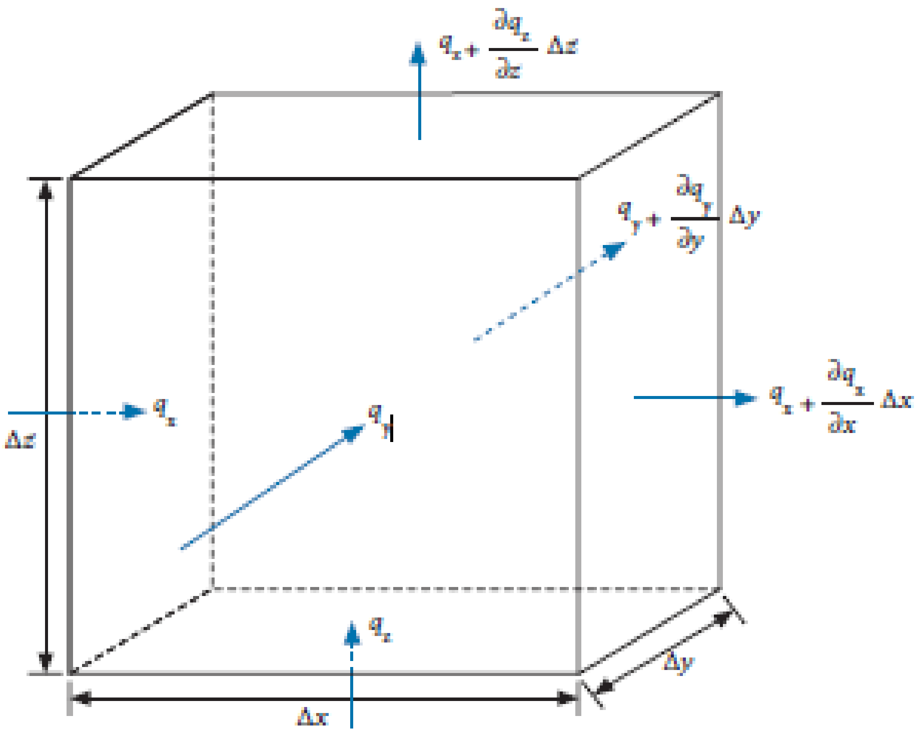


$\frac{\partial q_x}{\partial x}$ = the rate in which q_x changes with x

$\frac{\partial q_x}{\partial x}\Delta x$ = total change over the section Δx

Inflow - Outflow - Net inflow

https://www.youtube.com/user/MartinRHendriks/videos



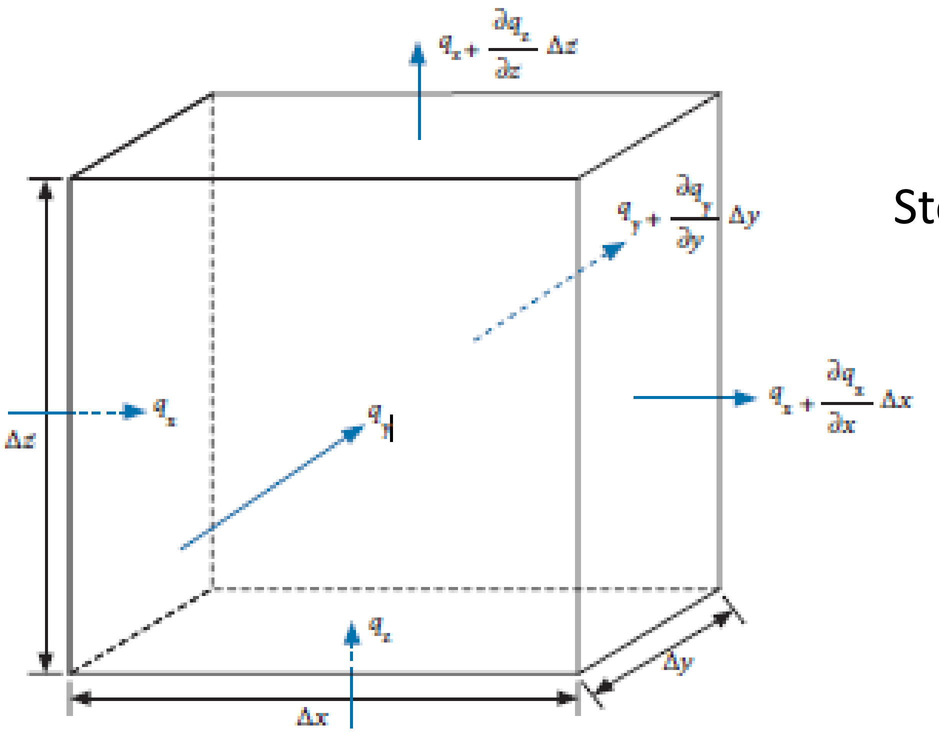
$$Q_{in} = q_x \Delta y \Delta z + q_y \Delta x \Delta z + q_z \Delta x \Delta y$$

$$Q_{out} = \left(q_x + \frac{\partial q_x}{\partial x}\Delta x\right)\Delta y \Delta z + \left(q_y + \frac{\partial q_y}{\partial y}\Delta y\right)\Delta x \Delta z + \left(q_z + \frac{\partial q_z}{\partial z}\Delta z\right)\Delta x \Delta y$$

Net inflow: $Q_{in} - Q_{out} = -\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right)\Delta x \Delta y \Delta z$

Continuity equation

https://www.youtube.com/user/MartinRHendriks/videos



Steady-state flow:

$\partial v/\partial t = 0$

Net inflow: $Q_{in} - Q_{out} = -\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right)\Delta x \Delta y \Delta z$

Continuity: net inflow = change of storage = 0

Continuity equation: $-\frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y} - \frac{\partial q_z}{\partial z} = 0$

Continuity and Darcy's law

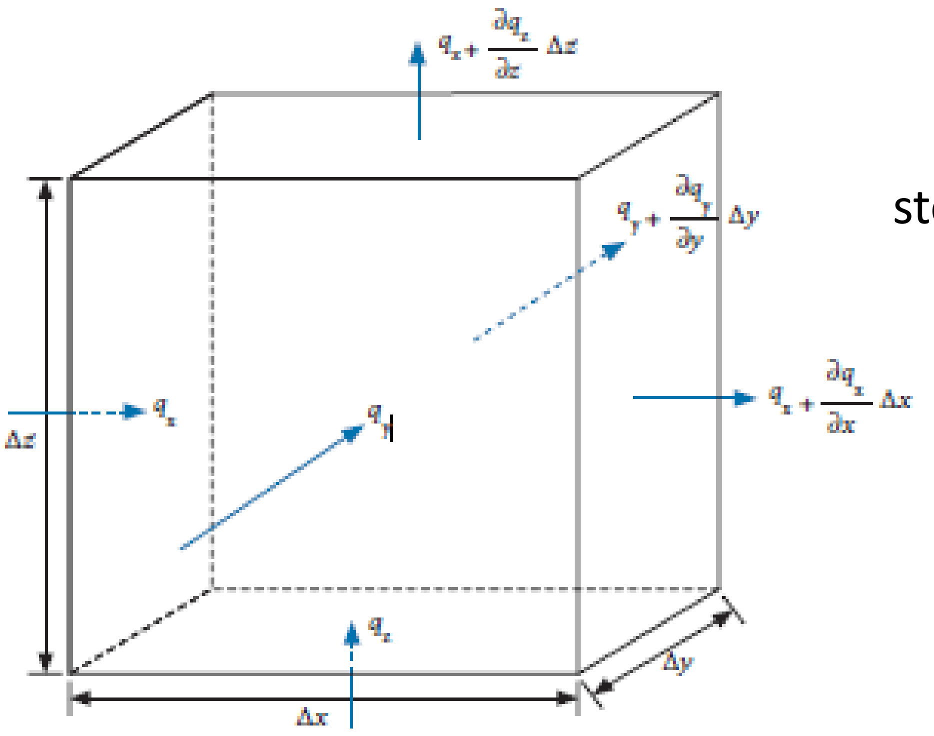
https://www.youtube.com/user/MartinRHendriks/videos

Darcy's law:

$$q_x = -K_x \frac{\partial h}{\partial x}$$

$$q_y = -K_y \frac{\partial h}{\partial y}$$

$$q_z = -K_z \frac{\partial h}{\partial z}$$



steady-state flow:

$$\partial v/\partial t = 0$$

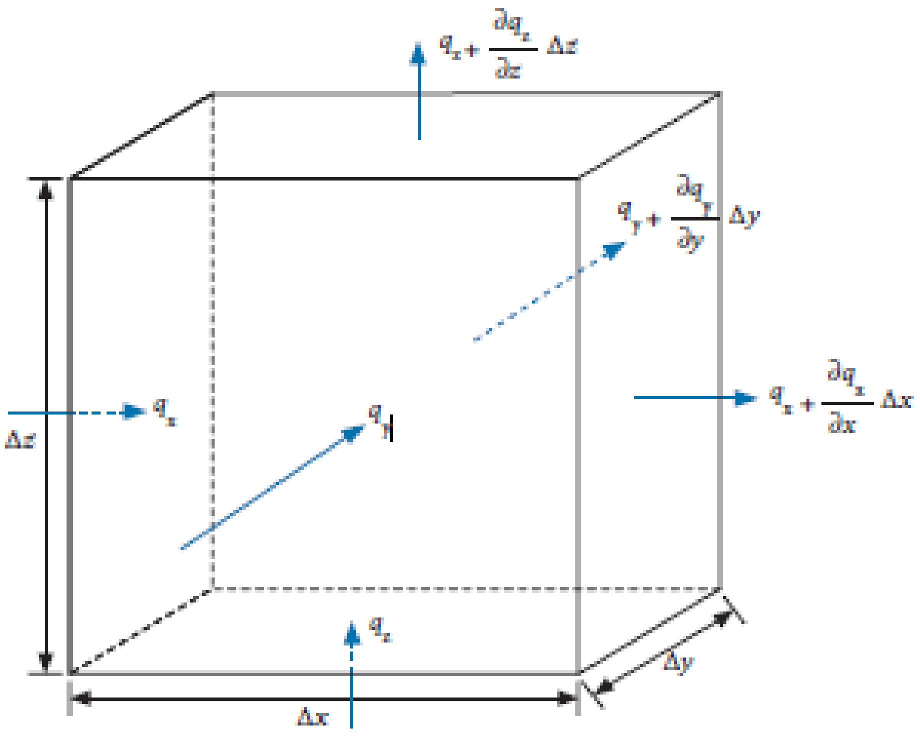
Continuity equation: $$-\frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y} - \frac{\partial q_z}{\partial z} = 0$$

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = 0$$

Laplace equation

homogeneous,
isotropic:

$$K_x = K_y = K_z$$



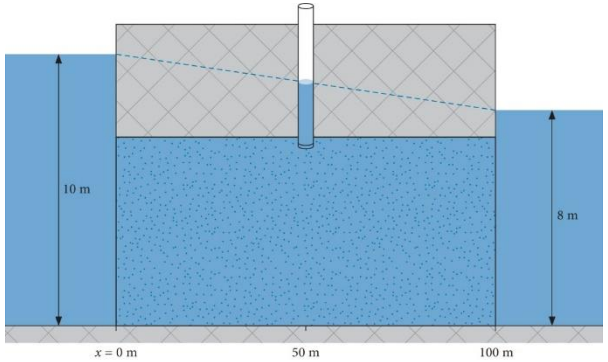
$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = 0$$

Laplace equation: $\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$

Laplacians

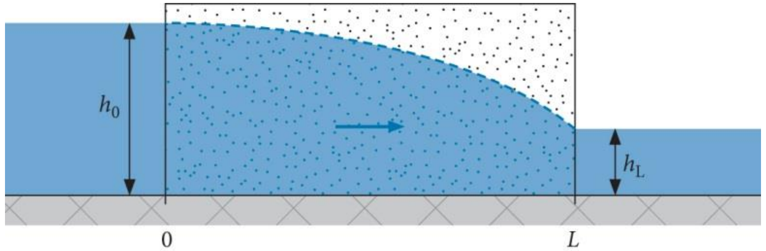
Laplace equation: $$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=0$$

confined:



$$\frac{d^2 h}{dx^2}=0 \quad \frac{dh}{dx}=C_1 \quad h=C_1x+C_2$$

unconfined:



$$\frac{d^2 h^2}{dx^2}=0 \quad \frac{dh^2}{dx}=C_1 \quad h^2=C_1x+C_2$$

Laplace and Boussinesq

steady-state flow:

$\partial v / \partial t = 0$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

transient flow:

$\partial v / \partial t \neq 0$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S_y}{K\bar{D}} \frac{\partial h}{\partial t}$$