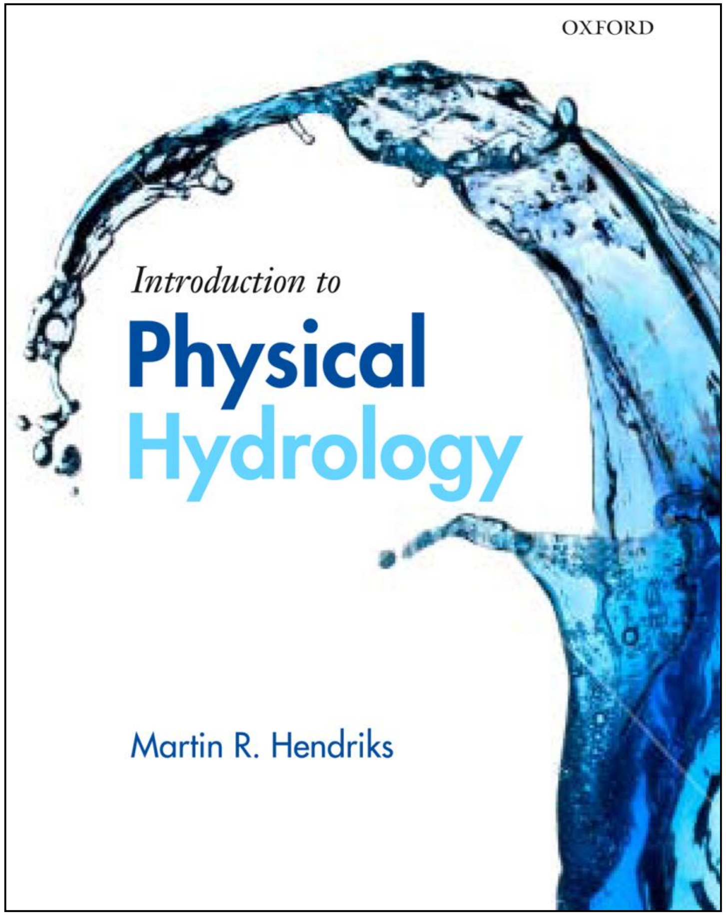
Groundwater hydraulics



Paperback | 351 pages
Follow the book's didactic concept!

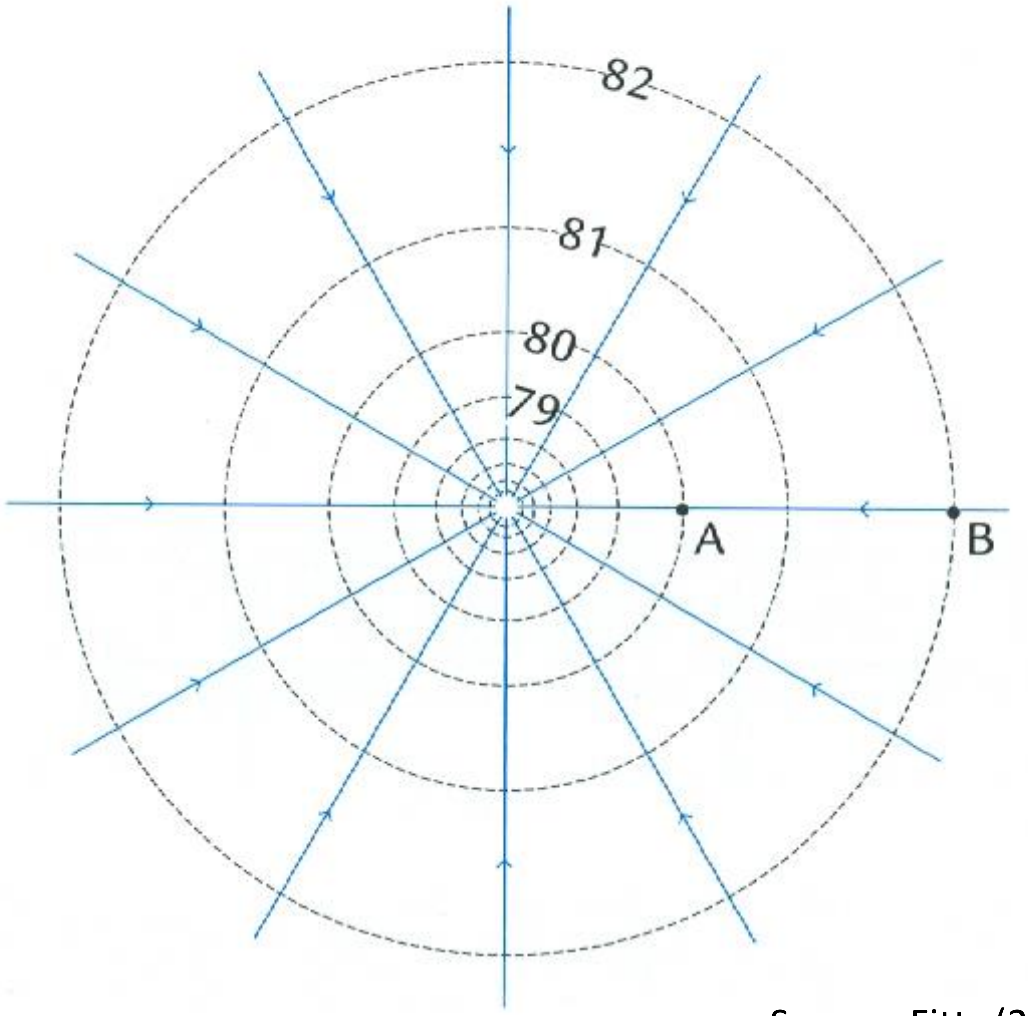
- Hydrological cycle
- Drainage basin
- Water balance

- Energy equation
- Flow equation
- Continuity equation

1. Introduction
2. Atmospheric water
3. Groundwater, including **Section 3.15**
4. Soil water
5. Surface water

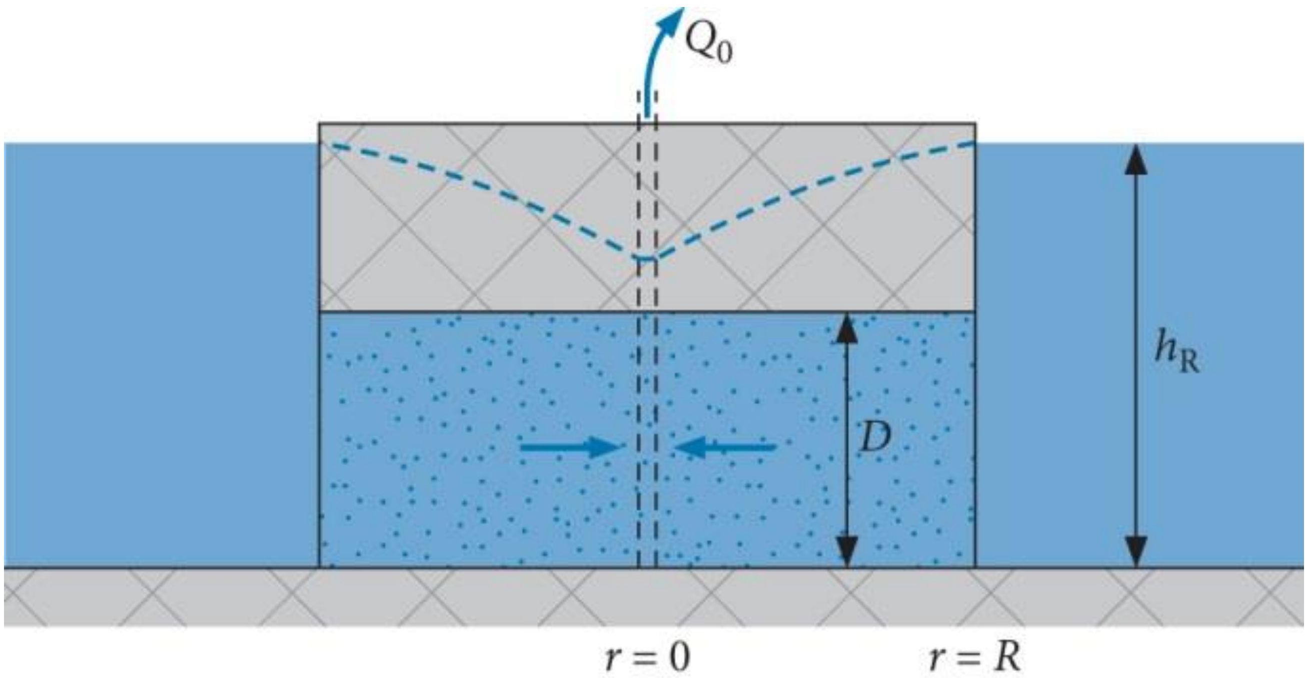
Exercises

Radial groundwater flow in plan view



Source: Fitts (2002)

Confined aquifer



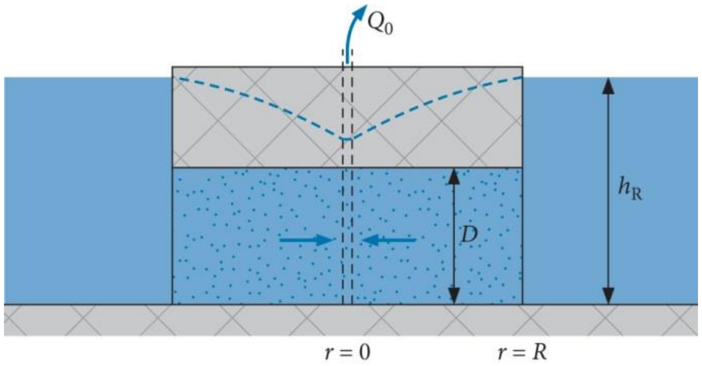
Dupuit equation

$$q_r = K\frac{dh}{dr} \qquad Q_0 = q_r 2\pi r D \qquad h = h_R + \frac{Q_0}{2\pi KD}\ln\frac{r}{R}$$

M1-5: hydraulic head *h* decreases with increasing *x* – minus sign in Darcy's law, linking *Q'* > 0 with *dh/dx* < 0
M6-7: hydraulic head *h* decreases with decreasing *r* – no minus sign in Darcy's law, linking *Q'* > 0 to pumping

Confined aquifer

$$q_r = K \frac{dh}{dr}$$



$$Q_0 = q_r 2\pi r D$$

$$Q_0 = K \frac{dh}{dr} 2\pi r D \Rightarrow \frac{dh}{dr} = \frac{Q_0}{2\pi KD} \frac{1}{r} \Rightarrow$$

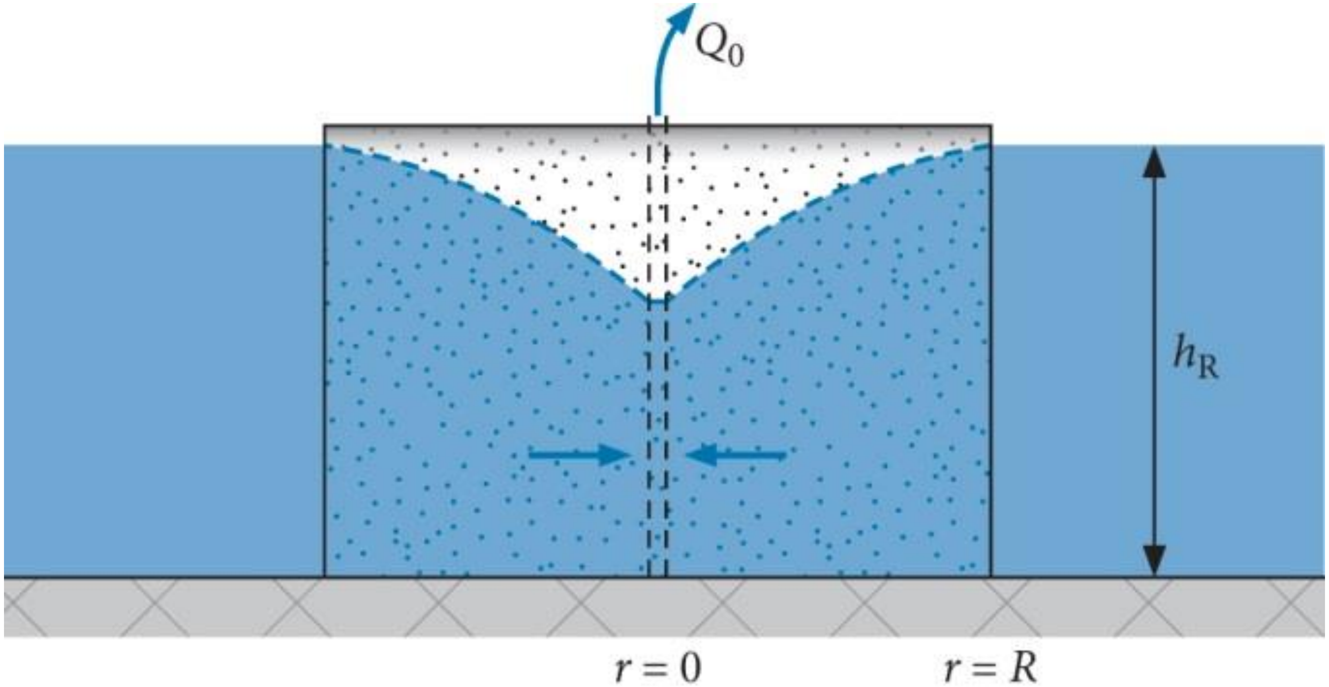
$$\int \frac{dh}{dr} dr = \int \frac{Q_0}{2\pi KD} \frac{1}{r} dr \Rightarrow h = \frac{Q_0}{2\pi KD} \ln r + C$$

$$r = R \Rightarrow h = h_R + \frac{Q_0}{2\pi KD} \ln \frac{r}{R}$$

Lowering of the hydraulic head: $h_R - h$

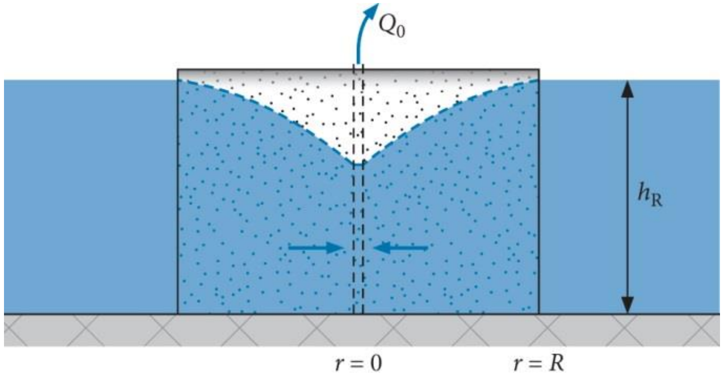
Drawdown: $h - h_R$

Unconfined aquifer



$$q_r = K\frac{dh}{dr} \qquad Q_0 = q_r 2\pi r h \qquad h^2 = h_R^2 + \frac{Q_0}{\pi K}\ln\frac{r}{R}$$

Unconfined aquifer



$$q_r = K\frac{dh}{dr}$$

$$Q_0 = q_r 2\pi r h$$

$$Q_0 = K\frac{dh}{dr}2\pi r h \Rightarrow \frac{dh}{dr} = \frac{Q_0}{2\pi K h r} \Rightarrow \frac{Q_0}{r}dr = 2\pi K h\, dh \Rightarrow$$

$$\int \frac{Q_0}{r}dr = \int 2\pi K h\, dh \Rightarrow Q_0 \ln r = \pi K h^2 + C \Rightarrow h^2 = \frac{Q_0}{\pi K}\ln r + C$$

$$r = R \Rightarrow h^2 - h_R^2 = \frac{Q_0}{\pi K}\ln\frac{r}{R}$$

$$h^2 - h_R^2 = (h + h_R)(h - h_R) = 2\overline{D}(h - h_R) \Rightarrow h - h_R = \frac{Q_0}{2\pi K\overline{D}}\ln\frac{r}{R}$$ Dupuit equation

Table 3.3 - Starting point of the exercises

One-dimensional steady groundwater flow

Confined	$h = C_1 x + C_2$
Unconfined	$h^2 = C_1 x + C_2$
Leaky	$h = h_a + C_1 e^{\frac{x}{\lambda}} + C_2 e^{\frac{-x}{\lambda}}$ with $\lambda = \sqrt{KDc}$
Recharge; equal water levels	$h^2 = -\frac{N}{K} x^2 + C$
Recharge; different water levels	$h^2 = -\frac{N}{K} x^2 + C_1 x + C_2$

Radial-symmetric steady groundwater flow

Confined	$h = h_R + \frac{Q_0}{2\pi KD} \ln \frac{r}{R}$ for $r_w \leq r \leq R$
Unconfined	$h^2 = h_R^2 + \frac{Q_0}{\pi K} \ln \frac{r}{R}$ for $r_w \leq r \leq R$

If you can't do the math

The beauty of section 3.15 is how easy the math is and how readily it applies to real hydrological settings!

Radial groundwater flow:

$$\int \frac{1}{r}\, dr = \ln r + C$$

$$\ln \frac{r}{R} = \ln r - \ln R$$

$$\frac{d \ln \frac{r}{R}}{dr} = \frac{d\left(\ln r - \ln R\right)}{dr} = \frac{d \ln r}{dr} = \frac{1}{r}$$

If you can't do the math

The beauty of section 3.15 is how easy the math is and how readily it applies to real hydrological settings!

Unconfined aquifer:

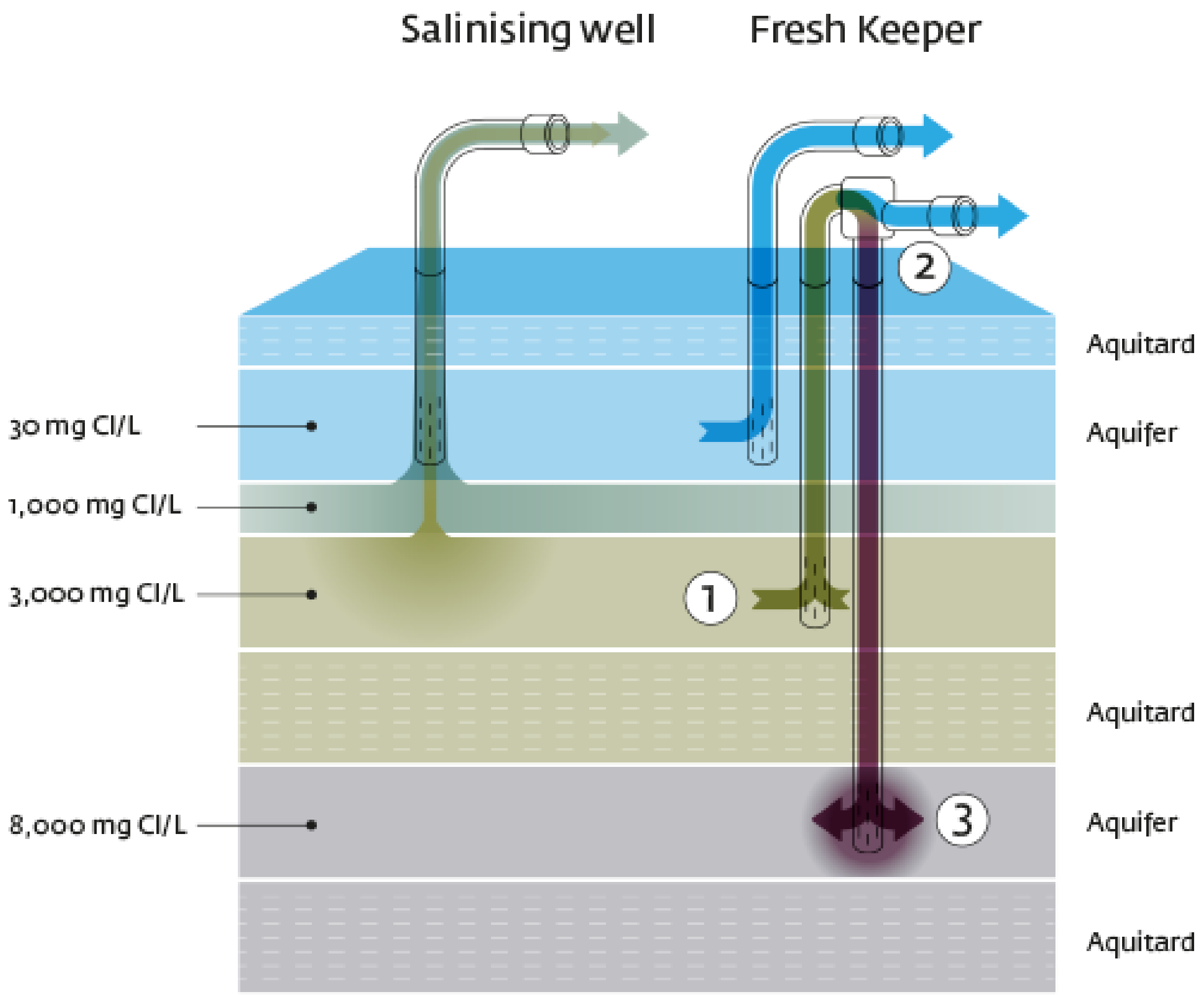
$$h_0^2 - h_L^2 = \left(h_0 + h_L\right)\left(h_0 - h_L\right)$$

Leaky aquifer:

$$y = e^{\frac{x}{\lambda}} \qquad \Rightarrow \qquad \frac{dy}{dx} = \frac{1}{\lambda} e^{\frac{x}{\lambda}}$$

$$y = e^{-\frac{x}{\lambda}} \qquad \Rightarrow \qquad \frac{dy}{dx} = -\frac{1}{\lambda} e^{-\frac{x}{\lambda}}$$

Freshkeeper concept



Source: Watershare

References

Fitts, C.R. (2002). Groundwater Science. Academic Press, Elsevier Science.

Hendriks, M.R. (2010). Introduction to Physical Hydrology. Oxford University Press.

Watershare. Freshkeeper: https://www.watershare.eu/projects/freshkeeper/