Geopotential, $\Phi$

\[ \Phi \equiv gz \]

\[ v_g = \frac{g}{f} \left( \frac{\partial z}{\partial x} \right)_p ; u_g = -\frac{g}{f} \left( \frac{\partial z}{\partial y} \right)_p \]

\[ f v_g = \left( \frac{\partial \Phi}{\partial x} \right)_p ; f u_g = -\left( \frac{\partial \Phi}{\partial y} \right)_p \]
Circumpolar flow at $p=250$ hPa

At which approximate height is $p=250$ hPa?
Geostrophic & hydrostatic balance in pressure coordinates

Geostrophic wind:
\[ v_g = \frac{g}{f} \left( \frac{\partial z}{\partial x} \right)_p ; u_g = -\frac{g}{f} \left( \frac{\partial z}{\partial y} \right)_p \]

Hydrostatic balance:
\[ \frac{\partial p}{\partial z} = -\rho g \quad \Rightarrow \quad \frac{\partial z}{\partial \ln p} = -\frac{RT}{g} \]

Thermal wind equation:
\[ \frac{\partial v_g}{\partial \ln p} = -R \frac{\partial T}{f \partial x} ; \frac{\partial u_g}{\partial \ln p} = R \frac{\partial T}{f \partial y} \]
Properties of the thermal wind

Thermal wind equation:

\[
\frac{\partial v_g}{\partial \ln p} = -\frac{R}{f} \frac{\partial T}{\partial x} \quad ; \quad \frac{\partial u_g}{\partial \ln p} = \frac{R}{f} \frac{\partial T}{\partial y}
\]

* Horizontal derivatives with pressure held constant

\[
\hat{k} \times \nabla \equiv \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
0 & 0 & 1 \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}
\end{vmatrix} = \left( -\frac{\partial}{\partial y} \right) \hat{i} + \left( \frac{\partial}{\partial x} \right) \hat{j}
\]

In vector notation:

\[
\frac{\partial \vec{V}_g}{\partial \ln p} = -\frac{R}{f} \hat{k} \times \nabla T
\]
Properties of the thermal wind

Thermal wind equation:

\[
\frac{\partial v_g}{\partial \ln p} = - \frac{R}{f} \frac{\partial T}{\partial x}, \quad \frac{\partial u_g}{\partial \ln p} = - \frac{R}{f} \frac{\partial T}{\partial y}
\]

*Horizontal derivatives with pressure held constant

In vector notation:

\[
\frac{\partial \vec{v}_g}{\partial \ln p} = - \frac{R}{f} \hat{k} \times \vec{\nabla}T
\]

The “thermal wind”:

\[
\vec{v}_T = \vec{v}_g(p_1) - \vec{v}_g(p_0) = - \frac{R}{f} \int_{p_0}^{p_1} (\hat{k} \times \vec{\nabla}T) d\ln p
\]

The “thermal wind” is parallel to the isotherms!
\[ \vec{v}_T = \vec{v}_g(p_1) - \vec{v}_g(p_0) = -\frac{R}{f} \int_{p_0}^{p_1} (\hat{k} \times \hat{\nabla} T) d\ln p \]

The thermal wind is parallel to the isotherms!

**Turning of the wind with height**

**Cold advection: backing with height**

**Warm advection: veering with height**
The thermal wind in a mid-latitude cyclone

NOAA, channel 4 (infra-red). 12 feb 1996, 1313 UTC
850 hPa
100 hPa
The thermal wind in a mid-latitude cyclone

NOAA, channel 4 (infra-red). 12 feb 1996, 1313 UTC
Mid-latitude cyclone: conceptual model
Exercise 1:

Identify areas of warm air advection and areas of cold air advection between 850 hPa and 500 hPa over western Europe on 3 March 1995, 00 UTC (see following two slides).
Thermal wind in a mid-latitude cyclone

NOAA image in channel 4 (IR) made on March 3, 1995, at 0157 UTC.
Exercise 2a

Vertical section across a cold front

Construct a vertical section (longitude pressure) at approximately 48.5°N on March 3, 1995, 00 UTC, across a cold front. Your analysis is based on the radiosonde measurements taken at, among other, Brest (nr. 07110; 48.45°N, -4.4°W), Paris (Trappes) (nr. 07145; 48.75°N, 2.0°E), Nancy (nr. 07180; 48.68°N, 6.21°E) and Munchen (nr. 10868; 48.25°N, 11.55°E). For radiosonde data see http://weather.uwyo.edu/. The cold front can easily be recognised in the satellite image, displayed in the next slide.

According to the hydrostatic balance equation, the “thickness”, $\Delta z$, of a layer is proportional to its average temperature, $\langle T \rangle$:

$$\Delta z = - \frac{R \langle T \rangle}{g} \Delta \ln p$$

Check this relation for the layer (850-700 hPa) with the heights indicated in the cross section, shown after the satellite image (next slide). Do this for Brest and Paris (Trappes) (data are plotted in the cross-section).

$R = 287 \text{ J kg}^{-1}\text{K}^{-1}$
Thermal wind in a mid-latitude cyclone

NOAA image in channel 4 (IR) made on March 3, 1995, at 0157 UTC.

Crosses: Brest, Paris, Nancy, Munich
Heights [m], wind direction [°] and wind speed [knots] and temperatures [°C] at standard pressure levels

Cross section of a cold front, 3 March 1995 00 UTC at 48.5°N

Brest
07110
48.4°,-4.4°

Paris (Trappes)
07145
48.7°,2°

Nancy
07180
48.7°,6.2°

Stuttgart
10739
48.8°,9.2°

Munchen
10868
48.2°,11.5°

Wien
11035
48.3°,16.4°

Poprad
11952
49.0°,20.3°

Data: http://weather.uwyo.edu/
Exercise 2b

Consistency with thermal wind balance

(i) Note in the cross section (previous slide) that the wind speed decreases with increasing height above 300 hPa at Paris and Nancy. Why?

(ii) The thermal wind obeys the following equation.

\[ v_T = v_g(p_1) - v_g(p_0) = \frac{R}{f} \int_{p_0}^{p_1} \frac{\partial\langle T\rangle}{\partial x} d\ln p \]

Estimate the average thermal wind in the layers \((p_0=850 \text{ to } p_1=700 \text{ hPa})\), \((p_0=700 \text{ to } p_1=500 \text{ hPa})\) and \((p_0=500 \text{ to } p_1=300 \text{ hPa})\) between Brest and Paris and between Paris and Nancy. Heights, temperatures, wind direction and wind speed at pressure levels are indicated in the cross section (previous slide). Compare the result with the observed meridional component of the wind shear.

(iii) Identify the longitude of the minimum in geopotential height at 1000, 850 and 700. What do you see? Compare with the conceptual model of a mid-latitude cyclone.