Dynamical effects of *stratification* and *rotation*:

**Thermal wind balance and atmospheric circulation**

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Content of this lecture

• Zonal mean zonal wind and jet-streams
• Hydrostatic and geostrophic balance
• Pressure coordinate
• Thermal wind balance and jets
• Thermal wind balance in a cyclone
• Exercise: checking the thermal wind balance equation using radiosonde observations
The general circulation of the atmosphere

- Downward motion of potentially cold air masses in baroclinic waves (cold air outbreaks)
- Upward motion of potentially warm air masses in baroclinic waves

- Eddies
- Pole
- Polar tropopause
- TRADE WINDS
- Hadley cell
- ITCZ
- TRADE WINDS
- 0 5 10 15 km
longitudinal mean value of $u$

*Pressure is the vertical coordinate!*

$u = \frac{dx}{dt}$

Zonal mean wind

Annual mean

Pressure (hPa)

Zonal mean wind

Annual mean

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**Subtropical jets**
Hydrostatic balance

Air parcel in hydrostatic balance:

\[ F_p \]  \text{(pressure gradient force)}

\[ F_g \]  \text{(gravitational force)}

Hydrostatic equilibrium:

\[ \rho g = -\frac{\partial p}{\partial z} \]
Geostrophic balance

Air parcel in geostrophic balance:

\[
F_p \quad \text{(pressure gradient force)}
\]

\[
F_{\text{Cor}} \quad \text{(Coriolis force)}
\]

Geostrophic equilibrium:

\[
v_g = \frac{1}{f \rho} \left( \frac{\partial p}{\partial x} \right)_z ; u_g = -\frac{1}{f \rho} \left( \frac{\partial p}{\partial y} \right)_z
\]
Pressure as a vertical coordinate

\[(\delta A)_{1-3} = (\delta A)_{1-2} + (\delta A)_{2-3}\]
\[= (\delta A)_{yz, t} + (\delta A)_{yx, t}\]
\[= \left(\frac{\delta A}{\delta z}\right)_{yz, t} \delta x + \left(\frac{\delta A}{\delta z}\right)_{yx, t} \delta z\]

Let \( A = p \):
\[0 = \left(\frac{\delta p}{\delta x}\right)_{yz, t} + \left(\frac{\delta p}{\delta z}\right)_{yx, t} \left(\frac{\delta z}{\delta x}\right)_{y, p, t}\]

Assuming hydrostatic equilibrium, \( \partial p / \partial z = -\rho g \):

\[\left(\frac{\delta p}{\delta x}\right)_{yz, t} = \rho g \left(\frac{\delta z}{\delta x}\right)_{y, p, t}\]
Pressure gradient term:

\[
\left( \frac{\partial p}{\partial x} \right)_{y,z,t} = \rho g \left( \frac{\partial z}{\partial x} \right)_{y,p,t}
\]

In short:

\[
\left( \frac{\partial p}{\partial x} \right)_{z} = \rho g \left( \frac{\partial z}{\partial x} \right)_{p}
\]
Geostrophic balance in pressure coordinates

\[
\left( \frac{\partial p}{\partial x} \right)_z = \rho g \left( \frac{\partial z}{\partial x} \right)_p \\
\text{and} \\
\left( \frac{\partial p}{\partial y} \right)_z = \rho g \left( \frac{\partial z}{\partial y} \right)_p
\]

Geostrophic wind:

\[
v_g = \frac{1}{f \rho} \left( \frac{\partial p}{\partial x} \right)_z; \quad u_g = -\frac{1}{f \rho} \left( \frac{\partial p}{\partial y} \right)_z
\]

\[
v_g = \frac{g}{f} \left( \frac{\partial z}{\partial x} \right)_p; \quad u_g = -\frac{g}{f} \left( \frac{\partial z}{\partial y} \right)_p
\]
Geostrophic & Hydrostatic Balance in pressure coordinates

Geostrophic wind: \[ v_g = \frac{g}{f} \left( \frac{\partial z}{\partial x} \right)_p \quad u_g = -\frac{g}{f} \left( \frac{\partial z}{\partial y} \right)_p \]

Hydrostatic balance: \[ \frac{\partial p}{\partial z} = -\rho g \quad \rightarrow \quad p = R\rho T \quad \frac{\partial z}{\partial \ln p} = -\frac{RT}{g} \]

Thermal wind equation: \[ \frac{\partial v_g}{\partial \ln p} = -\frac{R}{f} \frac{\partial T}{\partial x} \quad ; \quad \frac{\partial u_g}{\partial \ln p} = \frac{R}{f} \frac{\partial T}{\partial y} \]
Thermal wind balance

\[
\frac{\partial z}{\partial \ln p} = -\frac{RT}{g}
\]

Warm air: \( \frac{\partial z}{\partial \ln p} \) large

Cold air: \( \frac{\partial z}{\partial \ln p} \) small

- \( p_0 \) at NP
- \( p_1 \) at EQ
- \( p_2 \)

\( z \) and \( -y \) axes
Thermal wind balance

\[ \frac{\partial z}{\partial \ln p} = -\frac{RT}{g} \]

Warm air: \[ \frac{\partial z}{\partial \ln p} \text{ large} \]

Cold air: \[ \frac{\partial z}{\partial \ln p} \text{ small} \]

\[ u_g = -\frac{g}{f} \left( \frac{\partial z}{\partial y} \right)_p \]

\( u_g \) increases with height if \( f > 0 \)
Isentropes (thin solid lines, labelled in Kelvin) and isotachs (isopleths of the velocity) (dashed lines, $m \ s^{-1}$) in a vertical section through a cold front. The $\gamma$-coordinate is positive towards the left. Heavy lines mark the tropopause and frontal boundaries. The section extends approximately 1200 km in the horizontal direction (Palmen, E. and C.W. Newton, 1969: *Atmospheric Circulation Systems*. Academic Press, 603 pp).
Zonal mean temperature

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Zonal mean temperature

Where do you expect the highest windspeeds?

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Zonal mean temperature

Subtropical jets