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Impact of trade on viability and exporter selection with heterogeneous fixed cost in the Melitz model

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Abstract

This paper presents a model aim to reconcile the discrepancy between the theoretical and empirical depiction of the productivity distribution. The Melitz (2003) while being able to reflect on the asymmetric selection of heterogeneous firms in trade, the model strictly truncate the least productive firms leaving the productivity distribution with a distinctive cut-off threshold at the lower end. This contradicts the empirical findings (Mayer and Ottaviano, 2007). The model in this paper proposed that firms are not only heterogeneous in terms of productivity but also in terms of fixed cost. In other words, viability selection in our model is based on firms' efficiency (TFP). This model successfully depicts the productivity distributions of active firms in the market that resemble the empirical findings, for which a great range of productivity distribution of exporters and domestic firms overlap. In addition, we show that only when the fixed exporting cost is *no less* than proportionate to the domestic fixed cost will the ultimate free trade scenario ensures that the weighted productivity in trade be greater than the weighted productivity in autarky. Lastly, trade liberation is always welfare improving, mainly due to the increasing product varieties.

Keywords: Firm heterogeneity; Productivity distribution; Exporting;

JEL classification: F14, D20.

1. Introduction

In the past decade, economists started to incorporate firm heterogeneity into the models of international trade and further analyze the impact of trade on these firms. The seminal work of Melitz (2003) is a particularly tractable model and has stimulated a great deal of research into analyzing the implication of firm heterogeneity for a wide range of issues in international trade. However, the productivity distribution depicted in the Melitz model contradicts the empirical findings. This paper reconciles the discrepancy between the empirical distribution found and the implied distribution from the theoretical work by introducing heterogeneous fixed cost into the Melitz model.

A growing body of literature followed the Meliz modeling structure in incorporating firm level heterogeneous marginal cost, i.e. productivity (Bernard et al., 2003; Helpman et al. 2004; Aw and Lee, 2008). These models show trade induces resource reallocation from the least productive firms to most productive ones. It revealed the unequal impact of trade on heterogeneous firms, while leaving the aggregated outcome comparable to a model with representative firms. Despite its ability to capture many stylized facts that differentiate exporters from non-exporters (Bernard and Jensen, 1999; 2004; Bernard et al., 2006; Greenaway and Kneller, 2007), there are consequences in imposing productivity as the sole heterogeneous dimension among these firms. In particular, the model predicts a strong *pecking order*. The ordering suggests only firms with above a certain productivity threshold can carry out economic production, with another productivity threshold drawing between the exporters and non-exporters (also further between exporters and FDI firms, see Helpman et al. 2004). Based on differences in productivity, the distinction between them is clear. However, the productivity distribution depicted based on the Melitz-type model is inconsistent with the empirical findings (Mayer and Ottaviano, 2007). Their empirical evidence suggests that there is no survival cut-off productivity threshold and there exist no productivity threshold to tell exporter apart from domestic firms.

This paper reconciles the discrepancy between the empirical distribution found and the implied distribution from the theoretical work by introducing heterogeneous fixed cost into the Melitz model. The model show how incorporating heterogeneous fixed cost and heterogeneous exporting fixed cost can successfully depicts the productivity distributions of active firms in the market that resembles empirical findings. In the equilibrium, the profitable firms that remain in operation can have various marginal productivity levels. Especially, there are firms with very low marginal productivity still making economic production. This is because now the selection is not solely base on productivity but efficiency. Thus, firms that are less productive make economic production as long as their fixed cost is sufficiently low, while highly productive firms exit if their fixed cost is excessively high. Modeling with heterogeneous exporting fixed cost further made possible to reproduce the overlapping pattern of the productivity distributions for the exporters and non-exporters. The model while retained the idea analytical features of the Melitz model (resource reallocation to more efficient firms) further point out that different level of exporting fixed cost can affect the weighted productivity in the trade equilibrium. For exporting fixed cost relatively lower than the domestic fixed cost, the weighted productivity in trade is lower than the weighted productivity in autarky. Irrespective of the mean to further liberalize trade, we find that trade is welfare improving.

Identifying and modeling with heterogeneous exporting fixed cost is not new in the literature (Robert and Tybout, 1997; Schmitt and Yu, 2001; Jørgensen and Schröder, 2006). However, it has not been the center of focus compare to other heterogeneous aspects, such as productivity. Nonetheless, there are reasons to taken heterogeneous exporting fixed cost into consideration. First, treating market entry cost to be homogeneous is not realistic. Fixed cost related to exporting included cost on market research, establishing foreign contact and distribution networks, training human resource sent to foreign office and adapting to foreign preference and regulations, ect. Business managerial studies reveal the impact of export barriers on management decisions (Leonidou, 1995; 2000). They found that different types of barrier are associated with different cost. Since different firms take different types of barriers more difficult to tackle than another, the fixed cost incurred for market entry varies substantially among the exporting firms. Econometric evidence based on Italian manufacturing panel data find substantial differences among firms' abilities to collect and operationalize information about foreign markets and consumer tastes, which is the main entry barrier (Bugamelli and Infante, 2003). Das et al. (2007) also report that the sunk entry cost vary considerably across Colombian manufacturers.

Second, organizational literatures point out the change in industry structure with the activities in the value chain broken down and produced by different firms (Fine, 1998; Sturgeon, 2000; 2002). More recent case studies by Linden et al. (2009; 2011) broke down the global value chain network of Apple's products and reveal how production of a single product is fragmented. Along the supply chain, each firm provides different inputs, namely from product innovation, component production and assembly to sells and distribution (see also Hess and Coe, 2006; Dedrick et al., 2010). We see that despite being in the same narrowly defined industry, firms in different parts of the value chain requires different fixed cost invest for the relevant activities (Jørgensen and Schröder, 2008). Thus, the fragmentation of the production activities in the same industry provides additional motivation to model with heterogeneous fixed cost.

In the consecutive sections, we follow the Melitz structure in presenting the model and point out along the way the differences from modeling with heterogeneous fixed cost. The second section present the basic set up of the model, and introduce fixed cost heterogeneity. The third section emphasize the additional uncertainty arise from the additional heterogeneous firm dimension we incorporated in the closed economy. The fourth extended the model with the possibility to trade, in which the firms face additional uncertainty, the heterogeneous exporting fixed cost. The fifth section compares the trade equilibrium with autarky equilibrium. We analyze the effect of trade liberation on exporter and viable firm selection, and the effect of trade liberation on the development of the overall weighted productivity and welfare. The last section concludes.

2. Basic model setup

2.1 Consumption

Consumer preferences are given in equation 1 by a constant elasticity of substitution (CES) utility function *U* with a continuum of goods available for consumption indexed by $\omega \in \Omega$, where $q(\omega)$ denotes the quantity consumed of variety ω and Q denotes the aggregate quantities of varieties demanded. The parameter ρ represents consumers' love of variety effect. It ranges between zero and one to ensure that the product varieties are imperfect substitutes and not complements for each other. This in turn determined the lower bound of the elasticity of substitution, which is always greater than one $\varepsilon \equiv 1/(1-\rho) > 1$.Consumers maximize utility subject to the budget constraint given in equation 2, with $p(\omega)$ and $q(\omega)$ as the price charged and quantity consumed for each variety ω and *R* as the total expenditure (equal to aggregate firms' revenue).

(1) $$U = Q = \left[\int_{\omega\in\Omega} q(\omega)^{\rho}\, d\omega \right]^{1/\rho} \;;\quad 0 < \rho < 1$$ (Utility function)

(2) $$R = \int_{\omega\in\Omega} p(\omega) q(\omega)\, d\omega$$ (Budget restriction)

The price index *P* is defined as given in equation 3. *Technical Note 1* derives the individual demand functions as given in equation 4; also see Dixit and Stiglitz (1977). Equation 5 gives the associated individual consumption constraint, which is also the revenue for the firm producing variety ω. Notice that equations 4 and 5 hold for all $\omega \in \Omega$, as is henceforth implicit.

(3) $$P = \left[\int_{\omega\in\Omega} p(\omega)^{1-\varepsilon}\, d\omega \right]^{1/1-\varepsilon} ,\varepsilon \equiv 1/(1-\rho)$$ (Price index)

(4) $$q(\omega) = R\, p(\omega)^{-\varepsilon} P^{\varepsilon-1} = Q\big(p(\omega)/P\big)^{-\varepsilon}$$ (Demand function)

(5) $$r(\omega) \equiv p(\omega) q(\omega) = R\big(p(\omega)/P\big)^{1-\varepsilon}$$ (Firm revenue)

2.2 Production

There are many firms active in the market. Production involves a fixed and variable cost each period. Due to the fixed cost requirement for production, each firm is capable and chooses to produce a single and unique product ω using labor as the only input in the equilibrium. Firms are heterogeneous in terms of marginal productivity. The parameter $\varphi(\omega)$ is the marginal productivity of the firm producing good ω, which is always greater than zero. Production enjoys increasing returns to scale with firm-specific marginal cost $1/\varphi(\omega)$.

To produce $q(\omega)$ goods, firms have to employ $l(\omega)$ workers. With labor as the only input, firms also have to incur a fixed cost $f(\omega)$ as the basis for production activity:

(6) $$q(\omega) = \varphi(\omega)[l(\omega) - f(\omega)]$$ (Production function)

Rearranging the above equation gives us the labor demand function: $l(\omega) = f(\omega) + q(\omega)/\varphi(\omega)$, where $1/\varphi(\omega)$ is the marginal labor input requirement. The fixed labor input $f(\omega)$ ensures that as production expands, less labor is needed to produce a unit of $q(\omega)$, which means that there are internal economies of scale (increasing return to more output) [1]. The average labor requirement decreases as production increases. Input and output follow a linear yet stochastic relationship since the marginal productivity and fixed cost are determined stochastically.

It must be noted that not only variable cost varies with firm productivity $\varphi(\omega)$, which is different between firms, the per period fixed costs $f(\omega)$ differ between firms that produce differentiated goods ω as well, which are both determined stochastically. This stochastic setting of per period fixed cost is where the model presented in this paper deviates from the original Melitz (2003) model.

Each worker is paid the same wage rate *w*, so firm profits are as given in equation (7). Each firm maximizes profits by choosing its optimal pricing level, subject to its demand function from equation (4), taking the economy-wide price (P) and expenditure levels as given. The solution to this problem is given as the optimal price rule in equation (8), where the last equality results from taking the wage rate as the numéraire (derived in detail in *Technical Note 2*). In view of the constant price elasticity of demand, each firm charges a constant mark-up $1/\rho$ of price over marginal costs. It should be noted that although the stochastic determination of per period fixed cost influenced the profit, it does not influence the optimal pricing rule derived by maximizing the firm's profit.

(7) $$\pi(\omega) = r(\omega) - wl(\omega)$$ (Firm profits)

(8) $$p(\omega) = w/\rho\varphi(\omega) = 1/\rho\varphi(\omega)$$ (Optimal price rule)

With the optimal pricing rule, firms' output and revenue are proportional to its productivity parameters ($\varphi(\omega)$, $f(\omega)$ and $\varepsilon = 1/(1-\rho)$), total expenditure *R*, and the price index *P* (see equation 4′, 5′, 7′).

(4′) $$q(\omega) = Q\left(\rho\varphi(\omega)P\right)^{\varepsilon}$$ (Output function)

(5′) $$r(\omega) = R\left(P\rho\varphi(\omega)\right)^{\varepsilon-1}$$ (Firm revenue)

[1] Implicitly, the Dixit-Stiglitz framework assumes no economies of scope. Therefore, there is no reason for firms to produce multiple varieties. Each firm produces a distinct variety and each variety is only produced by one firm. In this case, firms do not lose profit to competition involved in producing the same type of varieties. Hence, the number of firms is also the number of varieties in equilibrium.

For given firm productivity $\varphi(\omega)$, the revenue earned is increasing in the aggregate expenditure (the aggregated revenue, R), increasing in the price index, P, which is also an inverse measure of the competition intensity in the market, and increasing in the inverse of price mark-up, ρ.

Note that two parameters: the firm's marginal productivity $\varphi(\omega)$ and the elasticity of substitution ε alone determine the ratio of the price, output quantity and revenue of any two firms that produce ω and ω' respectively (see equation 9). In other words, the relative revenue of two firms solely depends on their relative productivity.

(9) $$\frac{p(\omega)}{p(\omega')} = \left(\frac{\varphi(\omega)}{\varphi(\omega')}\right)^{-1}; \quad \frac{q(\omega)}{q(\omega')} = \left(\frac{\varphi(\omega)}{\varphi(\omega')}\right)^{\varepsilon}; \quad \frac{r(\omega)}{r(\omega')} = \left(\frac{\varphi(\omega)}{\varphi(\omega')}\right)^{\varepsilon-1}$$

This ratio can also be explained intuitively; a more productive firm charges a lower price, sells more goods, and earns higher revenue. The latter two effects are magnified by the elasticity of substitution. At this stage, the elasticity of substitution ε plays a central role, as it determines the ratio of sales and revenues.

To determine a firm's viability in the market, the absolute value of the operating profits as given in equation 7′ is crucial. Entrepreneurs must earn a positive profit to prove their ability to produce economically, and those who cannot earn a positive profit will immediately exit the market after knowing their productivity and equivalent fixed cost.

(7′) $$\pi(\omega) = (R/\varepsilon)\left(\rho P \varphi(\omega)\right)^{\varepsilon-1} - f(\omega)$$ (Firm profits[2])

Unlike Melitz (2003), the endogenously determined cut-off productivity will not be unique in our setting. But similar to Melitz' model, the model presented here provides a clear divide between profitable entrepreneurs and those who do not have equivalent productivity to cover the stochastic fixed cost. The relationship equation 7′ suggests that firms do not necessarily need to have high productivity as long as the stochastic fixed cost drawn is sufficiently low to enable a firm to earn a positive profit. Thus, it is no longer the case that the firms remaining in the market must be those with the highest productivity. Instead, they are those firms that earn a non-negative profit, which we call viable firms. So, even those firms with sufficiently high productivity but unfortunately draw a fixed cost high enough to make the firm unprofitable, will *not* join the incumbents and start producing. In short, the stochastic determined fixed cost plays a crucial role.

3. Closed economy viability selection

Given a certain productivity draw, there exists a cut-off fixed cost level such that any firm with fixed cost below this level can make a positive profit, allowing economic production. At the exact point where profit equals zero $\pi(\varphi, f) = 0$, we derived the following relation between productivity and fixed cost by rearranging equation 7'.

[2] $\pi(\omega) = r(\omega) - w[f(\omega) + q(\omega)/\varphi(\omega)] = r(\omega) - 1[f(\omega) + \rho p(\omega)q(\omega)] = (1-\rho)r(\omega) - f(\omega) = [r(\omega)/\varepsilon] - f(\omega)$

$$(10) \qquad f^*(\varphi) = (R/\varepsilon)\left(\rho P \varphi\right)^{\varepsilon-1} \quad \Leftrightarrow \pi(\varphi, f^*(\varphi)) = 0$$

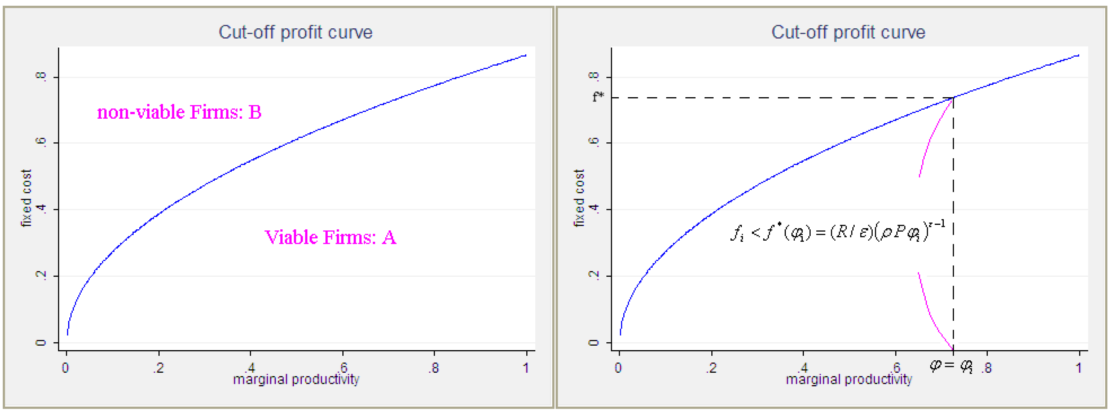
We define this specific productivity and fixed cost combination here as the cut-off profit level. This cut-off profit combination can be expressed in terms of productivity and pinned down a specific fixed cost level. A curve can be drawn on the fixed cost and marginal productivity axes for equation 10. This curve shows combinations of the productivity and fixed cost $(\varphi, f^*(\varphi))$ realizations for which the profit level equals zero $\pi(\varphi, f^*(\varphi)) = 0$, and is later referred to as the *cut-off profit curve*. As shown on the left hand side of figure 1, a clear division between the viable and non-viable firms is distinguished by this curve.

For non-negative profit $\pi(\varphi, f(\varphi)) \geq 0$, productivity and fixed cost have the following relationship:

$$(10') \qquad \pi(\varphi) \geq 0 \quad iff \quad f(\varphi) \leq f^*(\varphi)$$

Simply stated, for a given productivity draw $\varphi(\omega)$, firm ω must have a fixed cost level $f(\omega)$ below the cut-off fixed cost level $f^*(\varphi)$ to become a viable firm and earn a positive profit. These are the firms positioned on the vertical line where $\varphi = \varphi'$ and have fixed cost below the cut-off fixed cost level $f^*(\varphi')$, as shown on the right hand side of figure1. Entrant firms producing ω'' with specific productivity and fixed cost combination (φ'', f'') that generates a profit level smaller than zero: $\pi(\varphi'', f'') < 0$ will be forced to immediately exit the market since economic production is not possible.

Figure 1.The cut-off profit curve



We collect any pairs of productivity and fixed cost combinations that result in positive profit in a set named: **A**. All pairs of productivity and fixed cost combinations in this set allows firms to make non-negative profit: $\forall(\varphi, f) \in \mathrm{A} \Leftrightarrow \pi(\varphi, f) \geq 0$. Along with the cut-off profit condition as given in equations 10 and 10′, set **A** can be formally defined by equation 11 as follows:

$$(11) \qquad \mathrm{A} \equiv \left\{ (\varphi, f) \middle| f \leq f^*(\varphi) \right\}$$

The exiting firms are in set **B**. *Figure 1* shows a division between viable and non-viable firms by the cut-off profit curve, in which the two subsets of productivity and fixed cost combinations are distinguished. The two sets are exhaustive dividing firms by their productivity and fixed cost combination.

(11′) $\mathrm{B} \equiv \left\{ (\varphi, f) \,\middle|\, f > f^{*}(\varphi) \right\}$

Note that the cut-off profit curve is not a fixed curve. With parameters (the elasticity of substitution (ε), aggregate revenue (R) and price (P) in equation 7) at different levels, the cut-off profit curve can have different degrees of curvature, resulting in different realizations of productivity and fixed cost combinations where profit equals zero: $\pi(\varphi, f^{*}(\varphi)) = 0$.

3.1 Uncertainty

Upon paying the entry fixed cost f_{en}, firms draw their fixed cost parameter $f(\omega)$ and their marginal productivity parameter $\varphi(\omega)$ from a common distribution with probability density function: $\kappa(\varphi, f)$. Let Φ be the support of φ and similarly F for f, which by economic sense must be non-negative with support $[0, \infty)$. Then the equivalent marginal probability density functions for productivity and fixed cost are given by: $\kappa_{\varphi}(\varphi) = \int_{F} \kappa(\varphi, f)\, df$ and $\kappa_{f}(f) = \int_{\Phi} \kappa(\varphi, f)\, d\varphi$.

If the productivity parameters and fixed cost are independently distributed, then the joint probability density function would equal the product of the two independent probability density functions: $\kappa(\varphi, f) = \kappa_{\varphi}(\varphi)\kappa_{f}(f)$. However, it would be naïve to assume the productivity and fixed cost to be independent of each other since a higher marginal productivity usually comes at a higher fixed cost. We can also think of higher marginal productivity as a production that produces higher quality. In this sense, it is reasonable that firms which invest a higher per period fixed cost can produce product with higher quality. Therefore, more generally, we expect firms with higher marginal productivity to be associated with higher fixed costs to enable this marginal productivity level. In the extreme case, this would be a one-dimensional one-for-one trade-off, such that knowing $\varphi(\omega)$ implies knowing $f(\omega)$ and *vice versa.*

Nevertheless, without the information regarding the actual distribution of the two, we assume a general distribution form $\kappa(\varphi, f)$ to denote the *ex ante* joint probability density function of φ and *f*, and use $\mu(\varphi, f)$ as the *ex post* joint probability density function to proceed further. Hence, $\mu(\varphi, f)$ is the conditional distribution of $\kappa(\varphi, f)$ on set **A**:

(12) $$\mu(\varphi, f) = \begin{cases} \kappa(\varphi, f)/p_{in}, & (\varphi, f) \in A \\ 0 \qquad\quad , & otherwise \end{cases}, \text{ for } A \equiv \left\{ (\varphi, f) \,\middle|\, f \le f^{*}(\varphi) \right\}$$

$$\text{and } p_{in}(f^{*}) \equiv \iint_{A} \kappa(\varphi, f)\, d\varphi\, df = \int_{0}^{\infty} \int_{0}^{f^{*}(\varphi)} \kappa(\varphi, f) df d\varphi = \int_{0}^{\infty} \int_{\varphi(f^{*})}^{\infty} \kappa(\varphi, f) d\varphi df < 1$$

Since we assume that subsequent firm exit is uncorrelated with the productivity but exogenously given, the exit process will not affect the equilibrium productivity distribution $\mu(\varphi, f)$. Instead, the *ex post* distribution will be determined by the initial joint probability $\kappa(\varphi, f)$, conditioned on successful entry, as in equation 12, where p_{in} is the *ex ante* probability of successful entry for all potential entrants. By definition, p_{in} aggregates the probability over set A. Thus, in the second equality following p_{in}, we first aggregate the probability over the range $[0, f^*(\varphi)]$ for given φ and then aggregate the probability over all possible f, hence $[0,\infty)$, covering all viable firms. In short, p_{in} is a function of f^*.

3.2 Aggregation

We consider only the stationary equilibrium where aggregate variables remain constant over time. An entering firm with marginal productivity φ and fixed cost f will immediately exit if its profit level is negative, while a firm with non-negative profit will enter the market and continue its operation until hit by an adverse shock that forces it to exit with probability δ in each period. In the absence of time discounting, a firm's value function is given as following:

(13) $$v(\varphi, f) = \max\left\{0, \sum_{t=0}^{\infty}(1-\delta)^t \pi(\varphi, f)\right\} = \max\left\{0, \frac{\pi(\varphi, f)}{\delta}\right\}$$ (Value function)

In this setting, the optimal expected profit remain constant unless the firm is hit by a shock. With δM firms simultaneously entering and exiting at the equilibrium in each period, the *ex post* joint probability density function of productivity and fixed cost $\mu(\varphi, f)$ will not be affected. The equilibrium is characterized by a continuous mass of M viable firms (with equivalently M varieties produced).

We calculate here the *ex post* weighted average productivity as the average productivity level for all viable firms. Recall that the cut-off profit level $\pi(\varphi, f^*(\varphi)) = 0$, the weighted average marginal productivity can be therefore written as the expected productivity $\varphi^{\varepsilon-1}$ conditional on a non-negative profit level, and similarly for the weighted average fixed cost (equation 14). Notice that the *ex post* weighted average productivity depends on the specific joint distribution of the productivity and fixed cost, but independent of the number of active firms in the equilibrium market.

(14) $$\tilde{\varphi} \equiv \left[E(\varphi^{\varepsilon-1} \middle| \pi \geq 0)\right]^{1/(\varepsilon-1)} = \tilde{\varphi}(\varphi, f) \quad \tilde{f} \equiv \left[E(f \middle| \pi \geq 0)\right] = \tilde{f}(\varphi, f)$$

Correspondingly, the associated weighted average price, quantity and revenue can be written as simple functions of the weighted productivity $\tilde{\varphi}$ as in equation (15), while the weighted profit is also associated with the weighted fixed cost as in (15′).

(15) $$\tilde{p} \equiv 1/\rho\tilde{\varphi}; \quad \tilde{q} \equiv Q\left(\tilde{p}/P\right)^{-\varepsilon}; \quad \tilde{r} \equiv \tilde{p}\tilde{q};$$

(15′) $\tilde{\pi} = (\tilde{r} / \varepsilon) - \tilde{f} = (R / \varepsilon)\left(\rho P \tilde{\varphi}\right)^{\varepsilon-1} - \tilde{f}$

The weighted $\tilde{\varphi}$ and $\tilde{f}$ also represent the weighted aggregate productivity and fixed cost since they summarize the information in the joint distribution of productivity and fixed cost levels relevant for all aggregate variables. We write the associated aggregate price, quantity, revenue, and profit levels as functions of the weighted marginal productivity $\tilde{\varphi}$ as in equation 16 (see the derivation detail in technical note 4.).

(16) $P = M^{1/(1-\varepsilon)} p(\tilde{\varphi})\,; \quad Q = M^{1/\rho}\, q(\tilde{\varphi})\,; \quad R = PQ = M\, r(\tilde{\varphi})$

At the aggregate level, we see positive externalities from the mass of active firms M as a result of the love-of-variety embedded in the utility function; note the negative power for the price index and the positive power for the quantity index (see Brakman, Garretsen, and van Marrewijk, 2009, chapter 3). The aggregate revenue in turn depends on the number of active firms and the revenue of the firm with the associated weighted average productivity level, which in a sense is the 'representative' firm in this heterogeneous world.

The aggregate profit however is not solely related to the weighted average productivity $\tilde{\varphi}$, but is also related to the fixed cost. Depending on the respective level of fixed cost, even firms with the same marginal productivity can have different profit levels. We estimate the aggregate profit level with the information of the joint distribution of productivity and fixed cost. For any joint distribution of the productivity and fixed cost, we use $\bar{\pi}(f^*)$ to denote the firms' average expected profit level and at the same time use this function to capture the unspecified distribution.

(17)
$$\Pi = \int_0^{\infty} \int_0^{f^*(\varphi)} M\left(\frac{r(\varphi)}{\varepsilon} - f\right) \kappa(\varphi, f)\, df\, d\varphi = M\bar{\pi}(f^*)$$
$$where \quad \bar{\pi}(f^*) \equiv \int_0^{\infty} \int_0^{f^*(\varphi)} \left(\frac{r(\varphi)}{\varepsilon} - f\right) \kappa(\varphi, f)\, df\, d\varphi$$

Substituting the aggregate variables P and R derived in equation 16 into the cut-off profit level stated in equation 10, we rewrite the cut-off fixed cost as a function of the productivity:

(18)
$$f^*(\varphi) = \left(R / \varepsilon\right)\left(\rho P\right)^{\varepsilon-1} \varphi^{\varepsilon-1} = \left(M\, r(\tilde{\varphi}) / \varepsilon\right)\left(\rho M^{1/(1-\varepsilon)} p(\tilde{\varphi})\right)^{\varepsilon-1} \varphi^{\varepsilon-1}$$
$$= \left(M\, r(\tilde{\varphi}) / \varepsilon\right) M^{-1} \left(\rho / \rho\tilde{\varphi}\right)^{\varepsilon-1} \varphi^{\varepsilon-1} = \left(\frac{r(\tilde{\varphi})}{\varepsilon}\right)\left(\frac{\varphi}{\tilde{\varphi}}\right)^{\varepsilon-1}$$

The f^* locus is important for determining the average viable firms' productivity. The location of f^* is independent of the mass of firms M in the stationary equilibrium, but depends on the productivity level and the underlying ex ante joint distribution of productivity and fixed cost. Denoting the part influenced by the distributions as $a = a(\tilde{\varphi})$, we further simplifies f^* as:

(18′) $f^*(\varphi) = a\varphi^{\varepsilon-1}$, where $a = \left(\frac{r(\tilde{\varphi})}{\varepsilon}\right)\left(\frac{1}{\tilde{\varphi}}\right)^{\varepsilon-1} = a(\tilde{\varphi})$ (Cut-off fixed cost)

Notice that from equation 18 and 18′, we implicitly define $a = (R/\varepsilon)(\rho P)^{\varepsilon-1}$. It follows that a is in fact a production parameter since: $q(\varphi) = Q\left(\rho\varphi P\right)^{\varepsilon} = (\varepsilon-1)a\varphi^{\varepsilon}$.

3.3 Entry and Exit

To construct a minimalist dynamic model, we impose an exogenous probability of firm exit (through some shock or bad luck) equal to δ, which is common to all firms. In each period, there are δM number of firms exiting the market. This is counter-balanced by an endogenous determination of firm entry. The active firms in the market are earning zero or positive profits as given in equation 7′. This makes the market attractive for potential entrants to enter.

There are two caveats. First, potential entrants are identical as they do not yet know their productivity level before entry. Second, to enter the market, firm have to incur a one-time fixed entry (labor) cost equal to f_{en}, which is thereafter sunk. The only uncertainty, the level of productivity and fixed cost, is revealed once the entry costs are paid. This productivity and fixed cost level once drawn will do not change. The firms that earn non-negative profits after the uncertainty is resolved will therefore produce and earn the same optimal profit level over time until hit by a shock that forces them to exit. In the equilibrium, there are δM number of firms simultaneously entering and exiting the market.

3.4 Equilibrium in the closed economy

Two conditions are crucial for determining the equilibrium of this economy, (i) the viability condition and (ii) the free entry condition. The two conditions can be represented as surfaces in the (φ, f, π)-space. Since each particular fixed cost suggests different cut-off productivity, the combinations of different productivities and fixed costs provide different profit levels. Therefore, unlike the setting in Melitz model (2003) with a unique fixed cost level and a unique cut-off productivity level, the equilibrium derived in this paper is solved with a continuum level of cut-off fixed cost with respect to different productivity level. Hence, we need a three dimensional space to picture this relationship, since there will be a set of zero cut-off profit curve and free entry curve determine an equilibrium point for each associated levels of fixed cost. The equilibrium condition is underpinned by two equilibrium conditions: the viability condition and the free entry condition.

The viability condition (VC) provides the relationship between the representative firm's profit level $\bar{\pi}$ and the cut-off fixed cost f^*. It states the representative firm's expected profit level, which had already been implicitly defined in equation 17.

(19) $\bar{\pi}(f^*) \equiv \int_0^{\infty} \int_0^{f^*(\varphi)} \left(\frac{r(\varphi)}{\varepsilon} - f\right)\kappa(\varphi, f)\, df\, d\varphi$ (VC)

The free entry condition (FE) provides the relationship between expected profit $\bar{\pi}$ and the attractiveness of entry. Note that a successful entrant will earn an average expected profit $\bar{\pi}$, with probability of termination equal to δ in each period. The associated expected net present value, $\bar{v}$ say, over all time periods t is equal to:

$$\bar{v} = \sum_{t=0}^{\infty} \frac{\tilde{\pi}}{(1-\delta)^t} = \frac{\bar{\pi}}{\delta}.$$

The equilibrium requires the net value of entry equals zero: $\bar{v}\, p_{in} - f_{en} = (\bar{\pi}/\delta) p_{in} - f_{en} = 0$. This is because that no firm will want to enter if the net value of entry is negative and that it cannot be positive either as there is an unlimited pool of potential profit seeking entrants. Thus, the net value of entry must be zero in equilibrium, which again links the average profit level with the cut-off fixed cost. We call this the free entry condition (FE):

$$(19) \qquad \bar{\pi}(f^*) = \frac{\delta f_{en}}{\int\int_A \kappa(\varphi, f) d\varphi df} = \frac{\delta f_{en}}{p_{in}(f^*)} \qquad \text{(FE)}$$

In the equilibrium state, the aggregate variables remain constant over time. To ensure that the same equilibrium quantity is supplied in the market requires a (sufficiently large) mass of potential new entrants M_e with probability of successful entry p_{in} entering in each period to exactly replace the exiting incumbents that are hit by bad luck δM. Since the market is competitive and the incumbents earn non-negative profits, the market remains attractive to enter. With sufficient potential entrants, $p_{in} M_e = \delta M$ always holds.

Labor market requires that the labor supply in the economy is fully matched to the labor demand in the economy. The aggregate labor supply (L) available for production (and demand) reflects the size of the economy and is the major and only factor constraining the size of the entire economic activity. The aggregated labor supply is divided into two parts: $L = L_{in} + L_{en}$, including the labor employed by the incumbent firms (L_{in}) and the labor employed by the potential entrant firms (L_{en}). Labor employed by the incumbent firms are paid with the differences between aggregate revenue and profit: $L_{in} = R - \Pi$. This is also the market clearing condition for these production workers working for the incumbent firms.

The market clearing condition for labor employed by the new entrant firms requires $L_{en} = M_e f_{en}$ to hold; recalling that f_{en} is the one-time fixed (labor) cost a firm had to invest to enter the market before resolving its productivity and fixed cost levels. Using the aggregate stability condition $p_{in} M_e = \delta M$, and the free entry condition, the number of potential entrants L_{en} can be written as: $L_{en} = f_{en} M_e = f_{en} \delta M / p_{in} = M\bar{\pi} = \Pi$. Hence, the aggregate revenue R must equal the total payment to labor L, which is exogenously fixed by the size of the economy: $R = L_{in} + \Pi = L_{in} + L_{en} = L$. The number (mass) of firms in any period can be determined from the average profit level by rearranging equation 15.

(20) $M = R/\bar{r} = L/\varepsilon(\bar{\pi} + \bar{f})$ (Number of firms)[3]

With the number of firms determined, we complete the characterization of the stable equilibrium in the closed economy.

4. Open economy

We now expand the model with trade opportunity and analyze the impact of trade in a world that is composed of two symmetric countries. As in Krugman (1980) and Melitz (2003), if there are no additional costs to trade, then the individual countries can replicate the outcome of the integrated world economy, which is equivalent to an increase in the size of the market such that consumers enjoy welfare gains through an increase in product variety, but there are no firm-level effects.

In line with the empirical evidence (see Roberts and Tybout, 1997; Bugamelli and Infante, 2003) and the heterogeneity literature, we know that expending sale to foreign markets is not without extra costs. Besides per unit cost such as tariff and transportation cost, which we assume to be common for all firms, a huge fixed cost investment is inevitable for firms to become exporters. This include the sunk start-up cost which is needed to learn the bureaucratic procedure, to establish the distribution channel, to adapt the product (taste/packaging…ect) for the foreign market. Moreover, there is the per period fixed cost that needs to be covered, including the cost in maintaining a presence in foreign market and mounting to foreign custom procedure and product standard. The important notion we want to argue here is that per period exporting fixed cost bared by different firms also differs. Besides the differences in ability to cost down in the above mentioned activities, some firms may benefit from having spontaneous connection to foreign buyers in a trade exposition while others have managers experienced with exporting activities. In other words, the advantageous position on firms' exporting fixed cost is unevenly distributed across firms. Under such circumstances, this exporting fixed cost is not necessary related to firm's productivity, but may be positively linked to the domestic fixed cost invested for production.

The standard iceberg transportation cost is applied, whereby a fraction of $\tau > 1$ units of good must be shipped in order for 1 unit to arrive. In addition, there is the fixed export cost per period f_x. Since there is no additional time discounting other than the exit shock with probability δ, this amortized fixed cost per period is equivalent to an up-front entry cost of f_x/δ. We assume that the exporting fixed cost is related to the domestic fixed cost by the following relation $f_x = \eta f$. For $0<\eta<1$ means that the exporting fixed cost firms bear per period to serve the foreign market is less than the fixed cost necessary for the production in the domestic market. With the additional fixed and variable cost associated with export sales, only the most productive firms will find it profitable making additional sales in the foreign market.

We focus on two identical countries (henceforth have the same wage, which is normalized to one). This is because allowing multiple identical countries to trade within the model will not add much unless geographical differences between countries

[3] $M = (P\rho\varphi)^{1-\varepsilon} = R/R(P\rho\varphi)^{\varepsilon-1} = R/\varepsilon(\tilde{\pi} + \tilde{f})$

are being considered. That is, unless we assume the transportation cost between each pairs of countries in trade to be heterogeneous, the value added for introducing additional identical trading countries is limited. While the inclusion of the possibility to trade with multiple countries will only strengthen our assumption in modeling with heterogeneous fixed exporting cost. This is because as firms exporting to more countries, the fixed cost is likely to increase disproportionally as they enter more distant or less familiar markets (due differences in terms of language, culture and business negotiation rules). To focus on the heterogeneous domestic and exporting fixed cost dimension and elaborate the importance of modeling with heterogeneous fixed cost, we model the trade economy with only one other symmetric country.

4.1 Consumption and production

The export decision is made only after uncertainty is resolved (so φ and f are known). For firm with marginal productivity φ, profit maximization implies that equilibrium pricing rule for the domestic market is a constant mark-up over the marginal cost: $p_d(\varphi) = w/\rho\varphi = 1/\rho\varphi$ (for wage normalized to one). The export price with additional variable cost to trade is directly added to the mark-up for the foreign buyer: $p_x(\varphi) = \tau/\rho\varphi = \tau\, p_d(\varphi)$.

Given firms' pricing rule, equilibrium revenue obtained from the domestic sales is $r_d(\varphi) = R(P\rho\varphi)^{\varepsilon-1}$, where *R* and *P* are aggregate revenue and price index in each country. Revenue from the exporting sales is proportional to that in the domestic market due to pricing differences between the two markets $r_x(\varphi) = R(P\rho\varphi/\tau)^{\varepsilon-1} = \tau^{1-\varepsilon} r_d(\varphi)$. In view of the balance of payments condition, *R* is the aggregate revenue of firms, which is equivalent to the aggregate income. The combined revenue for firms with marginal productivity φ is:

(21) $$r_{tr}(\varphi) = \begin{cases} r_d(\varphi) & \text{if does not export} \\ r_d(\varphi) + r_x(\varphi) & \text{if export} \end{cases}$$

Due to the fixed production cost, no firms will export without also producing for the domestic market. Therefore, all viable firms are those that at least produce for the domestic market, while the firms capable of making non-negative profits from exporting will make sales abroad. Accounting for the overhead production cost for the domestic production and export activity, we separate each firm's profit into components earned from the domestic sales π_d and foreign sales π_x; where the share of production fixed cost is deducted from the domestic revenue and the exporting fixed cost is deducted from the foreign revenue. For firm with productivity φ and fixed costs f, the profit from domestic and foreign market is:

(22) $$\pi_d(\varphi, f) = \frac{r_d(\varphi)}{\varepsilon} - f \qquad \text{(Domestic profit)}$$

(23) $$\pi_x(\varphi, f) = \frac{r_x(\varphi)}{\varepsilon} - f_x = \frac{\tau^{1-\varepsilon} r_d(\varphi)}{\varepsilon} - \eta f \qquad \text{(Export profit)}$$

A firm exports to the other market if profit made from export is non-negative ($\pi_x(\varphi, f) \geq 0$). The total firm profit in the open trade economy is therefore:

(24) $$\pi_{tr}(\varphi, f) = \pi_d(\varphi, f) + \max\{0, \pi_x(\varphi, f)\}$$ (Total profit)

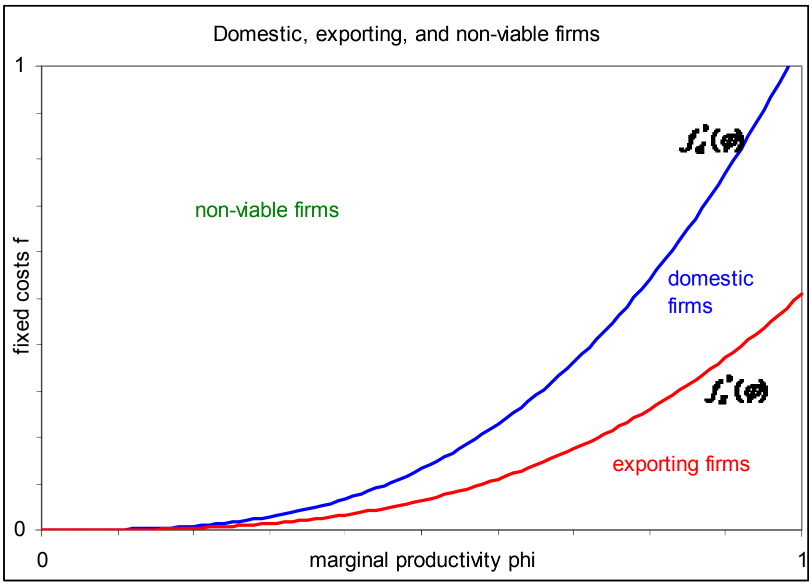
The value function for each firm is: $v(\varphi, f) = \max\{0, \pi_{tr}(\varphi, f)/\delta\}$.

4.2 Entry and exit decision and exporter selection

Similar to the closed economy case, we do not have a unique cut-off productivity level in identifying successful entry firms or exporters. Instead, we have two *zero-profit curves* that pin down a series firms with productivity and fixed cost combinations that earn zero profit in either the domestic market or the exporting market. The first, zero profit *viability* curve identifies those firms that make zero profit from production and are those at the margin of exiting the market. Second, different from the closed economy case, zero profit *exporting* curve further distinguishes the firms that are indifferent between being exporter and non-exporter as they make zero profit from the exporting activity. The zero-profit viability and exporting curves (cut-offs) are both functions of the marginal productivity (equation 25).

(25) $$\begin{array}{lcl} f_d^*(\varphi) = (R/\varepsilon)\left(P\rho\varphi\right)^{\varepsilon-1} & \Leftrightarrow & \pi_d(\varphi, f_d^*(\varphi)) = 0 \\ f_x^*(\varphi) = (\tau^{1-\varepsilon}/\eta) f_d^*(\varphi) < f_d^*(\varphi) & \Leftrightarrow & \pi_x(\varphi, f_x^*(\varphi)) = 0 \end{array}$$

Figure 2.The viability and export selection profit cut-off curve



The relationship identified from equation 25 divides the (φ, f)-plane into three areas (figure 2). Firms with different productivity and fixed cost combination fall in different areas divided by the two zero-profit curves. Those firms with (φ, f) to the left of the viability curve, $f_d^*(\varphi)$, will not be able to make economic profit in the domestic market (non-viable) and therefore exit; while firms with productivity and fixed cost combination to the right of the exporting curve, $f_x^*(\varphi)$, make non-negative profit in both the domestic and foreign market. Firms that lie in the area between the two curves are those with productivity and fixed cost combinations that allow them to make economic production only in the domestic market. The partitioning of firms by exporting status occurs if and only if $f_d^*(\varphi) > f_x^*(\varphi)$. Base on abundant empirical evidence and as in Melitz (2003), only a fraction of viable firms make additional investment to engage in positive export activity, therefore the following parameter restriction: $\eta\tau^{\varepsilon-1} > 1$ must hold[4].

As before, the *ex-ante* probability of successful entry into the market is given by:

(26) $$p_{in} \equiv \int_0^{\infty} \int_0^{f_d^*(\varphi)} \kappa(\varphi, f)\, df\, d\varphi \leq 1$$

Likewise, the *ex-ante* probability of successful entry as an exporting firm is given by equation 27, with the cut-off threshold set to the cut-off fixed exporting cost determine in equation 25.

(27) $$\overline{p}_x \equiv \int_0^{\infty} \int_0^{f_x^*(\varphi)} \kappa(\varphi, f)\, df\, d\varphi < p_{in}$$

The share of exporting firms among the viable firms is thus: $\overline{p}_x / p_{in} \equiv p_x$.

4.3 Uncertainty and aggregation

Provided with the *ex-ante* joint distribution of firm productivity and fixed cost, the *ex-post* distribution of incumbent firms is a subset of firms that survived in the competition.

(28) $$\mu(\varphi, f) = \begin{cases} \kappa(\varphi, f) / p_{in}, & f \leq f_d^*(\varphi) \\ 0 & , \ otherwise \end{cases},$$

Similarly, the *ex-post* probability density function for successful exporters is:

(29) $$\chi(\varphi, f) = \begin{cases} \kappa(\varphi, f) / \overline{p}_x, & f \leq f_x^*(\varphi) \\ 0 & , \ otherwise \end{cases}.$$

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4 $Since\, \tau^{1-\varepsilon} r_d(\varphi) / \varepsilon\eta = f_x^* < f_d^* = r_d(\varphi) / \varepsilon,\, therefore\, \tau^{1-\varepsilon} / \eta < 1$.

In equilibrium, the number of firms in a country is M and the number of exporting firms is $M_x = p_x M$. The total number of active firms selling goods to consumers in either market is thus $M_{tr} = M + M_x = (1 + p_x) M$, which is equivalent to the number of varieties available. Now let $\tilde{\varphi}_d$ denote the weighted average marginal productivity of all viable firms and $\tilde{\varphi}_x$ as the weighted average marginal productivity of exporting firms. As long as the partitioning condition holds, it insures only the most efficient firms make non-negative profit in the exporting market. Therefore, the weighted average marginal productivity of exporters must be greater than the weighted average marginal productivity of all viable firms

When the most efficient firms export their product and enter the foreign market, these exporting goods no longer preserved their most competitive position, for these products being negatively adjusted by the transit cost. Therefore it is important to account for the discounted competitiveness (productivity) of the importing products when they enter the distant market. We define the weighted productivity of all importing goods as the productivity discounted by the marginal exporting cost (equation 30). This term will be used later for ease of comparison. Note that the difference between $\tilde{\varphi}_x$ and $\tilde{\varphi}_i$ can also be understood as the melted/damaged product loss during transportation or as the difference between CIF (cost, insurance and freight) and FOB price (free on board price) in business terminology.

$$\tilde{\varphi}_d = \left[E(\varphi^{\varepsilon-1}) \middle| \pi_d \geq 0 \right]^{1/(\varepsilon-1)} = \left[\int_0^\infty \int_0^{f_d^*} \varphi^{\varepsilon-1} \mu(\varphi, f)\, df\, d\varphi \right]^{1/(\varepsilon-1)} ;$$

$$(30) \qquad \tilde{\varphi}_x = \left[E(\varphi^{\varepsilon-1}) \middle| \pi_x \geq 0 \right]^{1/(\varepsilon-1)} = \left[\int_0^\infty \int_0^{f_x^*} \varphi^{\varepsilon-1} \chi(\varphi, f)\, df\, d\varphi \right]^{1/(\varepsilon-1)} ;$$

$$\tilde{\varphi}_i = \left[E((\varphi/\tau)^{\varepsilon-1}) \middle| \pi_x \geq 0 \right]^{1/(\varepsilon-1)} = \left[\int_0^\infty \int_0^{f_x^*} (\varphi/\tau)^{\varepsilon-1} \chi(\varphi, f)\, df\, d\varphi \right]^{1/(\varepsilon-1)}$$

Taken the effect of allowing foreign varieties in the domestic market, $\tilde{\varphi}_{tr}$ denotes the weighted average marginal productivity of all varieties competing in a market.

$$(31) \qquad \tilde{\varphi}_{tr} = \left\{ \frac{1}{M_{tr}} \left[M \tilde{\varphi}_d^{\,\varepsilon-1} + M_x (\tilde{\varphi}_x / \tau)^{\varepsilon-1} \right] \right\}^{1/(\varepsilon-1)}$$

We rewrite the aggregate price index P and expenditure level R as functions of the weighted productivity $\tilde{\varphi}_{tr}$ and equilibrium numbers of varieties competing in a market:

$$(32) \qquad P_{tr} = M_{tr}^{1/(1-\varepsilon)} p(\tilde{\varphi}_{tr}), \qquad R = M_{tr} r_d (\tilde{\varphi}_{tr})$$

These aggregate expressions of price and revenue are use to rewrite the domestic cut-off fixed cost $f_d^*(\varphi)$ from equation 25.

(33)
$$f_d^*(\varphi)=\left(\frac{R}{\varepsilon}\right)\left(\rho P\right)^{\varepsilon-1}\varphi^{\varepsilon-1}=\left(\frac{M_{tr}\,r_d(\tilde{\varphi}_{tr})}{\varepsilon}\right)\left(\rho M_{tr}^{1/(1-\varepsilon)}p(\tilde{\varphi}_{tr})\right)^{\varepsilon-1}\varphi^{\varepsilon-1}$$
$$=\left(\frac{M_{tr}\,r_d(\tilde{\varphi}_{tr})}{\varepsilon}\right)M_{tr}^{-1}\left(\frac{\rho}{\rho\tilde{\varphi}_{tr}}\right)^{\varepsilon-1}\varphi^{\varepsilon-1}=\left(\frac{r_d(\tilde{\varphi}_{tr})}{\varepsilon}\right)\left(\frac{\varphi}{\tilde{\varphi}_{tr}}\right)^{\varepsilon-1}$$

The relationship implies that the cut-off domestic fixed cost curve is independent of the mass of firms M in a stationary equilibrium, but a function of productivity and the production parameter a_{tr} in the trade equilibrium. Note that the production parameter is constant since the weighted productivity $\tilde{\varphi}_{tr}$ is also a definite value in the equilibrium.

(34)
$$a_{tr}(\tilde{\varphi}_{tr})\equiv\left(\frac{r_d(\tilde{\varphi}_{tr})}{\varepsilon}\right)\left(\frac{1}{\tilde{\varphi}_{tr}}\right)^{\varepsilon-1}$$

As a result, the fixed cost cut-off for viability selection and exporter selection in equation 25 can be further simplified as:

(25')
$$f_d^*(\varphi)=a_{tr}\varphi^{\varepsilon-1}\text{ and }f_x^*(\varphi)=(\tau^{1-\varepsilon}/\eta)f_d^*(\varphi)=(\tau^{1-\varepsilon}/\eta)a_{tr}\varphi^{\varepsilon-1}.$$

Lastly, we make use of the production parameter a_{tr} to express the average profit firms earn from the domestic market $\bar{\pi}_d(a_{tr})$ and from the foreign market $\bar{\pi}_x(a_{tr})$:

(35)
$$\bar{\pi}_d(a_{tr})\equiv\int_0^\infty\int_0^{f_d^*(a_{tr})}\left(\frac{r_d(\varphi)}{\varepsilon}-f\right)\mu(\varphi,f)\,df\,d\varphi$$
$$\bar{\pi}_x(a_{tr})\equiv\int_0^\infty\int_0^{f_x^*(a_{tr})}\left(\frac{r_x(\varphi)}{\varepsilon}-f_x\right)\chi(\varphi,f)\,df\,d\varphi$$

4.4 Equilibrium in the open economy

As in the closed economy case, the viability condition and free entry condition pins down the equilibrium. The viability condition (VC) identifies the relationship between the average profit and the production parameter in trade in the equilibrium:

(36)
$$\bar{\pi}_{tr}(a_{tr})=\bar{\pi}_d+p_x\bar{\pi}_x \qquad \text{(VC)}$$

The average profit for any domestic firm in the open economy is the sum of the average profit from the domestic market and the additional profit from the exporting activity. Since both components in the average trade profit are function of the production parameter a_{tr}, the average profit determined from the viability condition is also a function of a_{tr}.

The free entry condition (FE) is comparable to that of the closed economy but change endogenously in accordance with the probability of entry, where the probability of entry is implicitly defined as function of the production parameter in trade: $p_{in}(f_d^*(a_{tr}))$.

Let $\bar{\upsilon}$ be the present value of average profit flows of viable firms: $\bar{\upsilon} = \bar{\pi}_{tr} / \delta$. With unrestricted entry and an unbounded mass of potential entrants, the equilibrium requires that the cost of entry equals the expected net value of entry, $f_{en} = \bar{\upsilon} p_{in}$, which is the average present value of viable firms multiply by the probability of successful entry. Thus we have the *Free Entry* (FE) condition, in which the average profit is a function of the production parameter a_{tr}, while the exit rate and sunk cost of entry are given exogenously:

$$\bar{\pi}_{tr}(a_{tr}) = \frac{\delta f_{en}}{p_{in}(f_d^*(a_{tr}))}, \qquad (37) \quad \text{(FE)}$$

As in the closed economy case, the numbers of firms entering and the number of firms exiting the market by exogenous shock must equal in the steady-state equilibrium $p_{in}M_e = \delta M$. Likewise, the aggregated fixed entry cost must equal to the aggregated excessive profits incumbents earn $f_{en}M_e = L_{en} = \Pi$, while the aggregated revenue remains exogenously fixed by the size of the economy $R = L$. Therefore, by dividing total revenue by the revenue per firm gives the equilibrium numbers of viable firms:

$$M = \frac{M_{tr}}{1+p_x} = \frac{R}{r_d(\tilde{\varphi}_{tr})}\frac{1}{1+p_x} = \frac{R}{\bar{r}_{tr}} \qquad (38)$$

Note that $r_d(\tilde{\varphi}_{tr})$ denotes the revenue earn per varieties, which is different from the average revenue earn per firm $\bar{r}_{tr}$ (see Technical note 5 for disambiguate).

4.5 Welfare

Welfare defined as the inverse of the aggregated price index $W = P^{-1}$. Since the price index are the weighted averages of the prices charged by individual firms with different productivity (with the weights determined by the *ex post* productivity distribution), the welfare is thus positively related to the weighted productivity in trade and numbers of varieties in the equilibrium.

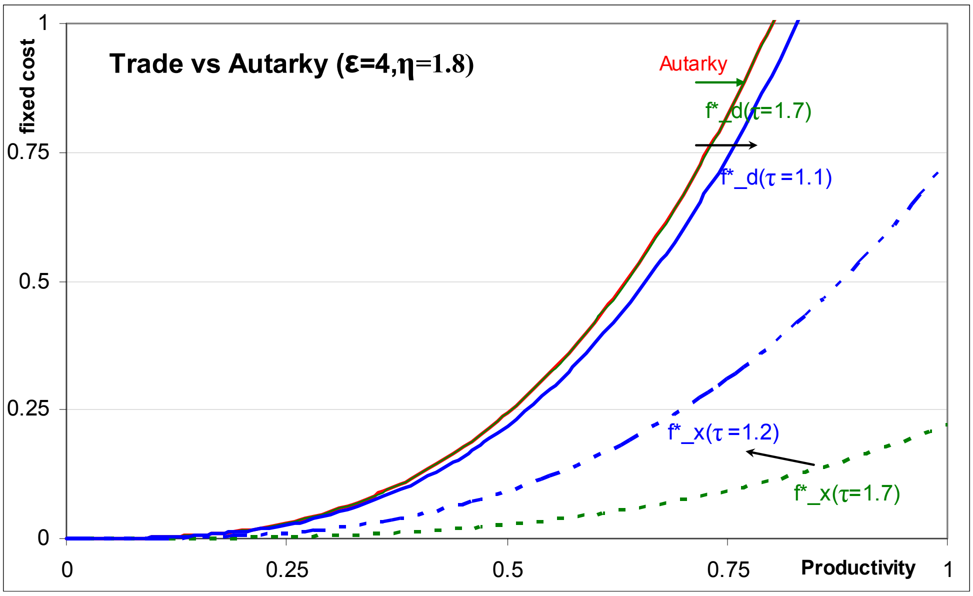
$$W = P^{-1} = M_{tr}^{1/(\varepsilon-1)} p(\tilde{\varphi}_{tr})^{-1} = M_{tr}^{1/(\varepsilon-1)} \rho\tilde{\varphi}_{tr} = (a_{tr}\varepsilon / R)^{1/(1-\varepsilon)} \rho \qquad (39)$$

5. Impact of trade

The current section analyzes the effect of trade by comparing the autarky equilibrium with trade equilibriums. Due to the complexity of the model, simulated numerical solution is provided to analyze the result. We report the changes in the equilibrium production parameter a_{tr} with decreasing trade barriers and compare the result with the equilibrium a in autarky.

Recall that the production parameter is positively associated with the zero-profit viability and exporting curve (equation 25'). For $a_{tr} < a$ imply a stronger viability selection in trade equilibrium since at any level of marginal productivity, the maximum fixed cost viable firms can have in trade is lower than in autarky (figure 3). The consecutive sections show how gradual trade liberalization from decreasing transportation cost at various exporting fixed cost level affect the selection of viable firms and exporting firms. Note that the result presented shows the long-run steady state equilibrium after all mechanism described above is at work. Furthermore, the figures presented in this section are based on simulation results with exogenous parameters $\varepsilon = 4, \delta = 0.1$ and $f_{en} = 2$. We construct the underlying distribution by taking the product of two independent beta distributions for fixed cost and productivity respectively: $\kappa(\varphi, f) = \kappa(f; \alpha_f, \beta_f)\kappa(\varphi; \alpha_\varphi, \beta_\varphi)$ with shape parameters: $\alpha_f = 3, \beta_f = 5$, $\alpha_\varphi = 4$ and $\beta_\varphi = 3$.

Figure 3 the lower the transportation cost, the more stringent the viability selection.



5.1 Trade liberation, decreasing transportation cost

As soon as a country is open to trade, the most productive firms start to export. With the zero profit exporter selection being very stringent at the beginning of opening to trade, only a small percentage of viable firms are able to make non-negative profits from exporting (figure 5). Successful exporters are either those with high productivity or those with low fixed cost investment or the firms with both qualities. At high level of transportation cost, foreign products' competitiveness is heavily discounted when entering the other market. As a result, the weighted productivity of these imported goods is much smaller than the weighted productivity of the domestic goods at high level of transportation cost (fig.6). Despite the weighted productivity of imports being significantly lower than the weighted productivity of the domestic varieties, the negative influence on the overall productivity is small when the transportation cost is high (fig.6; equation 31). In addition, despite that weighted productivity of imports start with a disadvantage, the least productive product among these imports is still more productive than the average productivity of domestic products before exported.

As trade further liberalizes from decreasing transportation cost, the viability selection become more stringent as the competition intensity increased. With more completive products entering and sharing the market, the zero profit viability selection becomes stronger, resulting in a downward pressure in the equilibrium production parameter a_{tr}. As a result, the probability of entry for the domestic firms decreases, while the probability of becoming an exporter increases (movement along different p_{in} and $\overline{p}_x$, figure 5). Thus, the weighted productivity of the domestic products increases (figure 6). The weighted productivity of the exporting products in contrast decreases since the zero profit exporting selection is weakened as trade liberalize. Regardless of the decreasing trend, the weighted productivity of the same goods that become imports in the foreign country is increasing as the transportation cost decreases. Hence, the overall effect on the weighted productivity in trade $\tilde{\varphi}_{tr}$ will first decreases before increasing again. This holds for all levels of exporting fixed cost (figure 6). Despite the non-monotonic development in the weighted productivity, the equilibrium production parameter a_{tr} decreases monotonically as the transportation cost decreases (figure 7).

5.2 Trade liberation, affect of different exporting fixed cost

Having an overview of the general development of the weighted productivities as the transportation cost decreases, we move on to compare the affect of exporting fixed cost on the development of the weighted productivity. As the transportation cost decreases, the decreasing intensity of the exporter selection is relaxed more quickly when the exporting fixed cost is less than proportion to the domestic fixed cost ($\eta < 1$). In other words, at any given level of transportation cost $\overline{p}_{x(\eta<1)} > \overline{p}_{x(\eta=1)} > \overline{p}_{x(\eta>1)}$. On the flip side, a certain level of viability selection intensity (p_{in}) is realized only at a lower level of transportation cost for high level of exporting fixed cost (figure 5). In short, all else equal, the equilibrium a_{tr} is lower for lower level of exporting fixed cost. Thus, as figure 4 shows: $f^*_{autarky} > f^*_{d(\eta>1)} > f^*_{d(\eta=1)} > f^*_{d(\eta<1)}$ and $f^*_{x(\eta<1)} > f^*_{x(\eta=1)} > f^*_{x(\eta>1)}$.

Since the percentage of exporters is the weight used in determining the weighted productivity in trade, the negative pressure of weighted productivity of imports on the overall weighted productivity in trade is stronger when the exporting fixed cost is low. In addition, since the weighted productivity of imports is always lower than the weighted productivity of domestic varieties when the exporting fixed cost is low ($\eta < 1$), the reverse increasing development of the weighted productivity in trade is limited as the transportation decreases (figure 6a). In contrast, the weighted productivity of the imports will at some point become greater than the weighted productivity of the domestic varieties, resulting in an overshoot in the reverse development of the weighted productivity above the weighted productivity in autarky (figure 6c).

To summarize, the overall weighted productivity in trade is greatest when the fixed exporting fixed cost is more than proportion to the domestic fixed cost and when the transportation cost is zero ($\tau = 1$). When the transportation cost is at its lowest level, while insured partitioning of exporters and domestic firms, the weighted productivity in trade for different exporting fixed cost levels have the following relationship: $\tilde{\varphi}_{\eta>1} > \tilde{\varphi}_{\eta=1} > \tilde{\varphi}_{autarky} > \tilde{\varphi}_{\eta<1}$ (figure 6). The equilibrium development of the production parameter a_{tr} decreases monotonically as the transportation cost decreases irrespective of the exporting fixed cost. The smallest a_{tr} is achieved when there is no transportation cost and when the exporting fixed cost equal to the domestic fixed cost (figure 7).

Figure 4 the lower the exporting fixed cost, the more stringent the viability selection

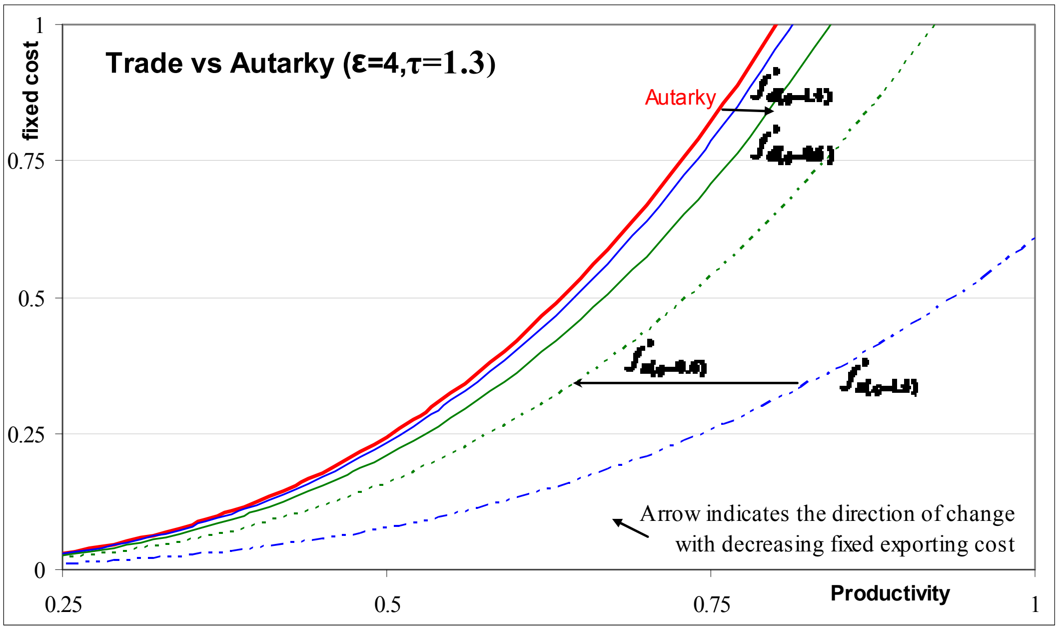
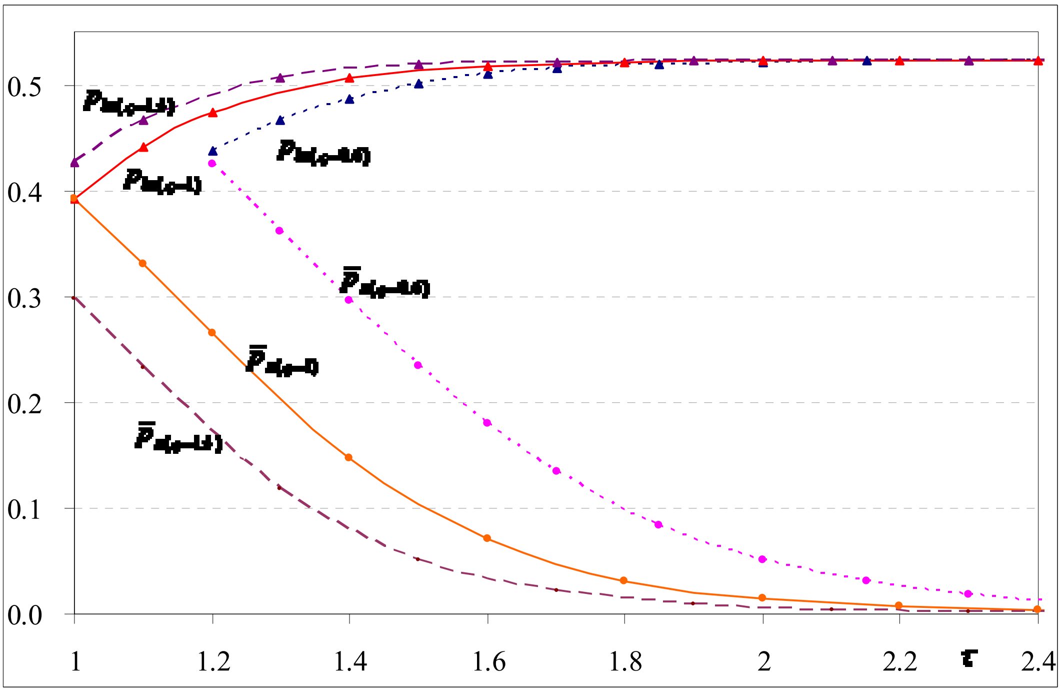


Figure 5 Development of probability of entry and the ex ante probability of successful entry of exporting firms for different exporting fixed cost



Given $\varepsilon = 4, \delta = 0.1, f_{en} = 2$ and the joint distribution as $\kappa(\varphi, f) = \kappa(f; \alpha_f, \beta_f)\kappa(\varphi; \alpha_\varphi, \beta_\varphi)$, which is the product of two independent beta distributions for the fixed cost and productivity, with shape parameters: $\alpha_f = 3, \beta_f = 5, \alpha_\varphi = 4, \beta_\varphi = 3$.

Figure 6 Development of weighted productivity and probability of entry

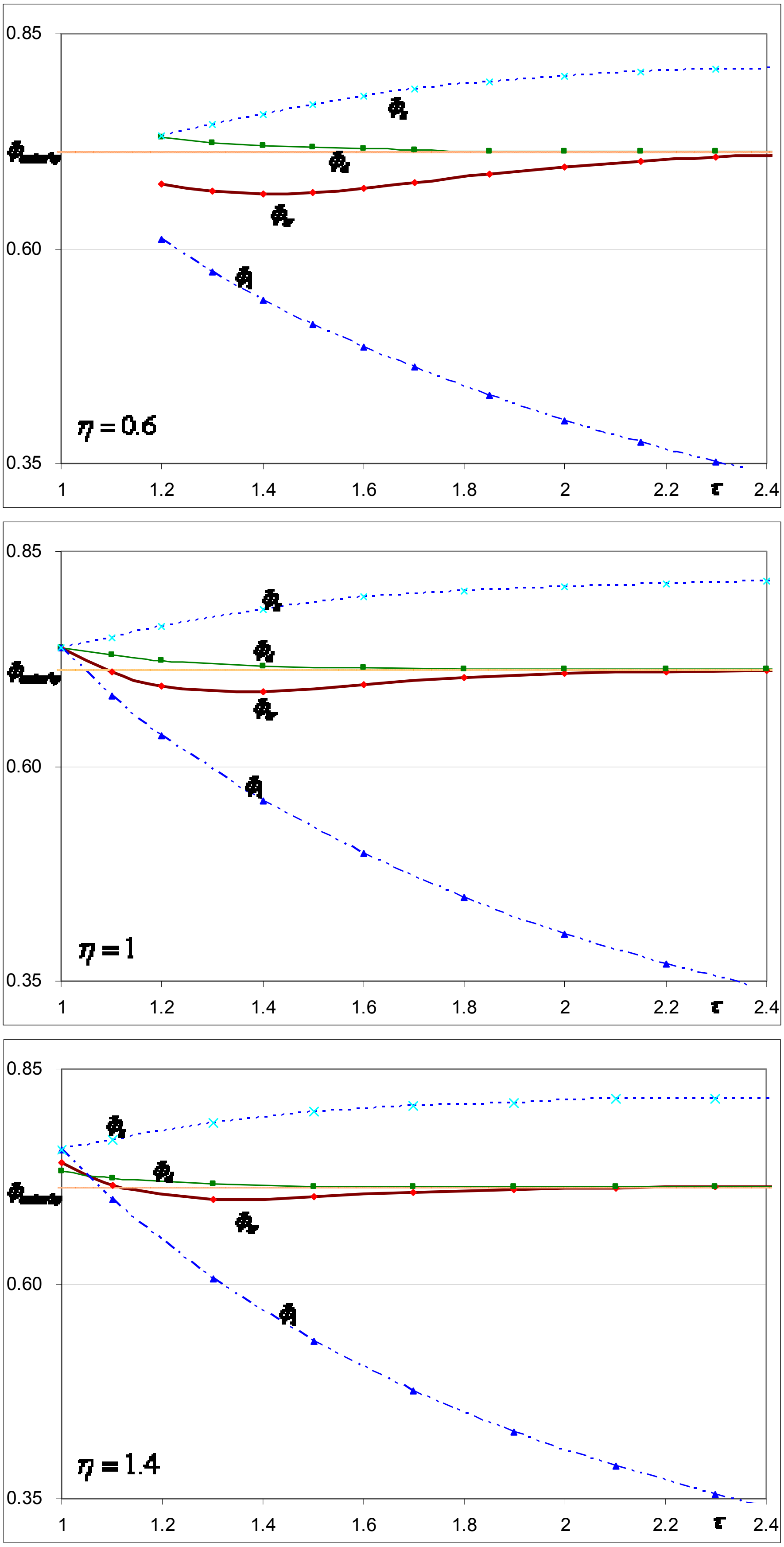
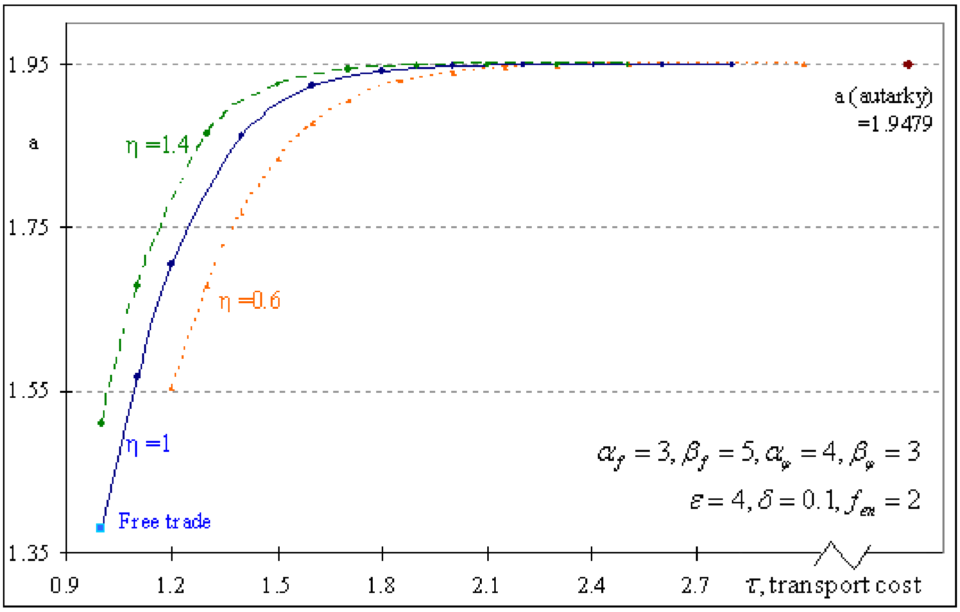


Figure 7 Equilibrium trajectory of a_{tr} at various level of exporting fixed cost.



5.3 Welfare implication

Results above suggest that free trade environment as compared to autarky always imposes a more stringent viability selection, while allowing more firms to become exporters. During the transition of trade liberation, we observe an optimizing intra-sector resource reallocation as market share is reallocated from the less competitive firms to the more competitive ones. The numbers of domestic firms in the trade equilibrium is less than in autarky. The decreasing number of less cefficient product varieties produced by domestic firms is replaced by an increasing number of foreign varieties.

The overall welfare is evaluated based on a combined measure of the price and varieties consumers enjoy. With the price as the inverse of the weighted productivity, consumers experience an increase in overall price level before it decreases again as trade liberalize. At the same time, consumers observe increasing product varieties as the country move from autarky to an open economy. Despite the less favorable price change, the gain from the increase in product variety out-weights the loss during the transition. As a result, the welfare in trade is always greater than in autarky and trade liberalization is always welfare improving.

6. Conclusion

The model in this paper reconciles the discrepancy in productivity distribution depicted between the theoretical Melitz type model and the empirical findings. By introducing heterogeneous fixed cost into the Melitz (2003) model, the cut-off productivity is no longer unique but depends on the level of fixed cost. The resulting equilibrium from this adapted model retains the idea quality of the original model. It shows that trade liberalization will always induce more stringent selection and provide welfare improvement. In addition, the model more accurately reflects the empirical productivity distribution of viable firms and the overlapping productivity distribution of the domestic and exporting firms. The resource (market share) is not reallocated from low productive firms to highly productive firms but from less to more efficient ones. Here, the most efficient firms are those who make cheap product (by their high marginal productivity) with a fixed cost investment lower than their peers.

Introducing heterogeneous fixed cost not only made the model more closely fitting to the empirical distribution, but it also unravel how the level of exporting fixed cost relative to the domestic fixed cost may affect the equilibrium weighted productivity. We see that only when the fixed exporting cost is *no less* than proportionate to the domestic fixed cost will the ultimate free trade scenario ensures that the weighted productivity in trade be greater than the weighted productivity in autarky. Despite resource reallocating towards imports discounted the benefit that could otherwise be given to the consumers (for the utility is melted during the transportation), we observe welfare gain.

7. References

Bernard, A.B. and B.J. Jensen (1999), "Exceptional exporter performance: cause, effect, or both?", *Journal of International Economics*, **47**(1): 1–25.

Bernard, A., J. Eaton, J.B. Jensen and S. Kortum (2003), "Plants and productivity in international trade", *American Economic Review*, **93**: 1268-1290.

Bernard, A. and J.B. Jensen (2004), "Why some firms export", *Review of Economics and Statistics*, **86**: 561–9.

Bernard, A. and J. Jensen and P. Schott (2006), "Trade costs, firms and productivity**",** *Journal of Monetary Economics*, 53(5): 917–937.

Bugamelli, M. and L. Infante (2003), "Sunk costs to exports", Bank of Italy Discussion papers, No. 469.

Das, S., M.J. Roberts and J.R. Tybout (2007), "Market entry costs, producer heterogeneity and export dynamics" *Econometrica*, **75**(3): 837-873.

Dedrick, J., K.L. Kraemer and G. Linden (2010), "Who profits from innovation in global value chains? A study of the iPod and notebook PCs", *Industrial and Corporate Change* 19(1), 81-116.

Greenaway, D. and R. Kneller (2007), "Firm heterogeneity, exporting and foreign direct investment", *The Economic Journal*, **117** :134–161

Helpman, E., M.J. Melitz and S.R. Yeaple (2004) "Export versus FDI with Heterogeneous Firms", *The American Economic Review*, **94**(1): 300-316.

Hess, M. and N.M. Coe (2006), "Making connections: global production networks, standards, and embeddedness in the mobile telecommunications industries", *Environment and Planning*, 38(7): 1205-1227.

Krugman, Paul R. (1980), "Scale Economies, Product Differentiation, and the Pattern of Trade", *American Economic Review*, **70**, 950-959.

Jørgensen, J.G. and P.J.H Schröder (2006), "Tariffs and firm-level heterogeneous fixed export costs", *Contributions to Economic Analysis and Policy*, 5(1): 1–15.

Jørgensen, J.G. and P.J.H. Schröder (2008), "Fixed export cost heterogeneity, trade and welfare", *European Economic Review*, **52**: 1256-1274.

Leonidou, L.C. (1995), "Empirical research on export barriers: Review, assessment, and synthesis", *Journal of International Marketing*, **3**(1): 29-43.

Leonidou, L.C. (2000), "Barriers to export management: An organizational and internationalization analysis", *Journal of International Management*, **6**(2): 1-28.

Leonidou, L.C. (2004), "An analysis of the barriers hindering small business export development", *Journal of Small Business Management*, **24**(3): 279-302.

Linden, G., K.L. Kraemer and J. Dedrick (2009), "Who captures value in a global innovation network? The case of Apple's iPod", *Communications of the ACM*, **52**(3): 140-144.

Linden, G., J. Dedrick and K. L. Kraemer (2011) "Innovation and job creation in a global economy: The case of Apple's iPod", *Journal of International Commerce and Economics*, 3(1): 223-239.

Melitz, M. J. (2003), "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity", *Econometrica,* Econometric Society, **71**(6): 1695-1725.

Mayer, T and G. Ottaviano (2007), "The happy few: new facts on the internationalisation of European firms," Bruegel Blueprint 3.

Roberts, M.J. and J.R. Tybout (1997), "The decision to export in Colombia: an empirical model of entry with sunk costs", *American Economic Review*, **87**: 545-564.

Schmitt, N. and Z. Yu (2001), "Economics of scale and the volume of intra-industry trade", *Economics Letters*, **74**: 127–132.

Yeaple, S.R. (2009), "Firm heterogeneity and the structure of U.S. multinational activity", *Journal of International Economics,* **78**(2): 206-215.

8. Appendix: technical notes

***Technical Note* 1.Derivation of demand function**

To maximize equation 1 subject to the budget constraint given in equation 2, we define the Lagrangian Γ, using the multiplier λ:

$$\Gamma = \left[\int_{\omega\in\Omega} q(\omega)^{\rho}\, d\omega \right]^{(1/\rho)} + \lambda \left[R - \int_{\omega\in\Omega} p(\omega) q(\omega)\, d\omega \right]$$

Differentiating Γ with respect to $q(\omega)$ and equating to 0 gives the first order conditions. Taking the ratio of the first order conditions for two varieties ω and ω', cancels the multiplier λ. Define $\varepsilon \equiv 1/(1-\rho)$ as discussed in the main text. Then:

$$\frac{q(\omega)^{\rho-1}}{q(\omega')^{\rho-1}} = \frac{p(\omega)}{p(\omega')}, \text{ or } q(\omega) = p(\omega)^{-\varepsilon} p(\omega')^{\varepsilon} q(\omega')$$

It is evident that the elasticity of substitution is constant $\varepsilon = \frac{-d\ln q(\omega)/q(\omega')}{d\ln p(\omega)/p(\omega')}$, hence a CES demand function. Substituting the utility maximizing quantity back to the budget constrain (R) and aggregated quantity (Q) function, while noting that $-\varepsilon\rho = 1-\varepsilon$ and $1/\rho = -\varepsilon/(1-\varepsilon)$, the definition of the price index *P* is then derived by dividing the aggregated expenditure by the aggregated quantity consumed:

$$R = \int_{\omega\in\Omega} p(\omega)q(\omega)d\omega = \int_{\omega\in\Omega} p(\omega)^{1-\varepsilon} p(\omega')^{\varepsilon} q(\omega')d\omega = p(\omega')^{\varepsilon} q(\omega') \int_{\omega\in\Omega} p(\omega)^{1-\varepsilon} d\omega$$

$$Q = \left[\int_{\omega\in\Omega} q(\omega)^{\rho}\, d\omega\right]^{1/\rho} = p(\omega')^{\varepsilon} q(\omega') \left[\int_{\omega\in\Omega} p(\omega)^{-\varepsilon\rho} d\omega\right]^{1/\rho}$$

$$P = \frac{R}{Q} = \left[\int_{\Omega} p(\omega)^{1-\varepsilon}\, d\omega\right]^{1/(1-\varepsilon)}$$

The optimal consumption and expenditure decision for individual varieties can thus be expressed by the aggregated term as in equation 4 and 5.

The aggregate price index *P* is a perfect reflection of consumer welfare: if income *R* increases by more than the price index *P*, welfare rises, and *vice versa* if this is not the case (see Diewert (1976) on the exact indices).

Technical Note 2.**Deriving the optimal pricing rule**

Substituting the demand $q(\omega)$ for good ω from equation 4 and the production function 6 in the profit function (7) gives:

$$\pi(\omega) = p(\omega)Q\left(p(\omega)/P\right)^{-\varepsilon} - w\left[f(\omega) + Q\left(p(\omega)/P\right)^{-\varepsilon} / \varphi(\omega)\right]$$
$$= p(\omega)^{1-\varepsilon} QP^{\varepsilon} - wf(\omega) - wp(\omega)^{-\varepsilon} QP^{\varepsilon} / \varphi(\omega)$$

Differentiating with respect to price $p(\omega)$ and equating to zero gives:

$$\partial\pi(\omega)/\partial p(\omega) = (1-\varepsilon)p(\omega)^{-\varepsilon} QP^{\varepsilon} + \varepsilon wp(\omega)^{-\varepsilon-1} QP^{\varepsilon} / \varphi(\omega) = 0$$

Cancelling the term $p^{-\varepsilon} QP^{\varepsilon}$ and solving the above equation gives the optimal pricing rule: $p(\omega) = [\varepsilon/(\varepsilon-1)]w/\varphi(\omega) = w/\rho\varphi(\omega)$.

Technical Note 3.Derivation of the ex post weighted average productivity

The joint probability density function of the productivity and fixed cost is $\kappa(\varphi, f)$. To calculate the weighted average over the domain in which profit is non-negative, we multiply $\varphi^{\varepsilon-1}$ by its *ex ante* distribution and integrate over all viable firms in set **A**.

$$\tilde{\varphi}(\varphi, f) \equiv \left[E(\varphi^{\varepsilon-1} \middle| \pi \geq 0)\right]^{1/(\varepsilon-1)} = \left[\frac{1}{\int\int_A \kappa(\varphi, f) d\varphi df} \int\int_A \varphi^{\varepsilon-1}\kappa(\varphi, f) d\varphi df\right]^{1/\varepsilon-1}$$

$$= \left[\frac{1}{p_{in}}\int\int_A \varphi^{\varepsilon-1}\kappa(\varphi, f) d\varphi df\right]^{1/\varepsilon-1} = \left[\int_0^\infty\int_0^\infty \varphi^{\varepsilon-1}\mu(\varphi, f) d\varphi df\right]^{1/(\varepsilon-1)} = \left[\int_0^\infty \varphi^{\varepsilon-1}\mu_\varphi(\varphi) d\varphi\right]^{1/(\varepsilon-1)}$$

Technical Note 4.Derivation of aggregate price

With a continuum of active firms M in the market and a marginal distribution $\mu_\varphi(\varphi)$ of marginal productivity level, the price index *P* is then given by:

$$P = \left[\int_\Omega p(\omega)^{1-\varepsilon} d\omega\right]^{\frac{1}{(1-\varepsilon)}} = \left[\int_0^\infty p(\varphi)^{1-\varepsilon} M\mu_\varphi(\varphi) d\varphi\right]^{\frac{1}{(1-\varepsilon)}} = M^{\frac{1}{(1-\varepsilon)}}\left[\int_0^\infty (\rho\varphi)^{\varepsilon-1}\mu_\varphi(\varphi) d\varphi\right]^{\frac{1}{(1-\varepsilon)}}$$

$$= \frac{M^{1/(1-\varepsilon)}}{\rho\tilde{\varphi}} = M^{1/(1-\varepsilon)} p(\tilde{\varphi}); \quad where \quad \tilde{\varphi} \equiv \left[\int_0^\infty \varphi^{\varepsilon-1}\mu_\varphi(\varphi) d\varphi\right]^{\frac{1}{(\varepsilon-1)}}$$

Define $\tilde{\varphi}$ as the weighted average of the marginal productivity levels. All firms with the same marginal productivity sell the same quantity, such that:

$$Q = \left[\int_\Omega q(\omega)^\rho d\omega\right]^{1/\rho} = \left[\int_0^\infty q(\varphi)^\rho M\mu_\varphi(\varphi) d\varphi\right]^{1/\rho} = M^{\frac{1}{\rho}}\left[\int_0^\infty q(\tilde{\varphi})^\rho \left(\frac{\varphi}{\tilde{\varphi}}\right)^{\rho\varepsilon} \mu_\varphi(\varphi) d\varphi\right]^{1/\rho}$$

$$= M^{\frac{1}{\rho}} q(\tilde{\varphi})\left[\int_0^\infty \frac{\varphi}{\tilde{\varphi}}^{\varepsilon-1} \mu_\varphi(\varphi) d\varphi\right]^{\frac{\varepsilon}{\varepsilon-1}} = M^{\frac{1}{\rho}} \frac{q(\tilde{\varphi})}{\tilde{\varphi}^\varepsilon}\left[\int_0^\infty \varphi^{\varepsilon-1}\mu_\varphi(\varphi) d\varphi\right]^{\frac{\varepsilon}{\varepsilon-1}} = M^{\frac{1}{\rho}} \frac{q(\tilde{\varphi})}{\tilde{\varphi}^\varepsilon}\tilde{\varphi}^\varepsilon = M^{\frac{1}{\rho}} q(\tilde{\varphi})$$

Similarly, firms with the same marginal productivity have the same revenue:

$$R = \int_\Omega p(\omega)q(\omega) d\omega = \int_\Omega r(\omega) d\omega = \int_0^\infty r(\varphi) M\mu_\varphi(\varphi) d\varphi = M\int_0^\infty r(\tilde{\varphi})\left(\frac{\varphi}{\tilde{\varphi}}\right)^{\varepsilon-1} \mu_\varphi(\varphi) d\varphi$$

$$= M\frac{r(\tilde{\varphi})}{\tilde{\varphi}^{\varepsilon-1}}\int_0^\infty \varphi^{\varepsilon-1}\mu_\varphi(\varphi) d\varphi = M\, r(\tilde{\varphi})$$

Technical Note 5.Disambiguite between the average revenue per firm and average revenue per product

Average revenue earn per product variety is denoted as: $r_d(\tilde{\varphi}_{tr})$.

$$\begin{aligned} r_d(\tilde{\varphi}_{tr}) &= R(P\rho\tilde{\varphi}_{tr})^{\varepsilon-1} \\ &= R(P\rho)^{\varepsilon-1}\left\{\left(\frac{1}{1+p_x}\tilde{\varphi}_d{}^{\varepsilon-1}+\frac{p_x}{1+p_x}(\tilde{\varphi}_x/\tau)^{\varepsilon-1}\right)^{1/(\varepsilon-1)}\right\}^{\varepsilon-1} \\ &= R(P\rho)^{\varepsilon-1}\left(\frac{1}{1+p_x}\tilde{\varphi}_d{}^{\varepsilon-1}+\frac{p_x}{1+p_x}(\tilde{\varphi}_x/\tau)^{\varepsilon-1}\right) \end{aligned}$$

Average revenue earn per firm is denoted as: $\overline{r_{tr}}$.

$$\begin{aligned} \overline{r_{tr}} &= r_d(\tilde{\varphi}_d)+p_x r_x(\tilde{\varphi}_x) \\ &= R(P\rho\tilde{\varphi}_d)^{\varepsilon-1}+p_x R(P\rho\tilde{\varphi}_x/\tau)^{\varepsilon-1} \\ &= R(P\rho)^{\varepsilon-1}\left[\tilde{\varphi}_d{}^{\varepsilon-1}+p_x(\tilde{\varphi}_x/\tau)^{\varepsilon-1}\right] \end{aligned}$$

Therefore, to account for the numbers of firms

$$M = M_{tr}\frac{1}{1+p_x} = \frac{R}{r_d(\tilde{\varphi}_{tr})}\frac{1}{1+p_x}$$