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Employment Protection, Product Market Regulation and Firm Selection

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Abstract

Why are firm and job turnover rates so similar across OECD countries? We argue that this may be due to the joint regulation of product and labor markets. For our analysis, we build a stochastic equilibrium model with search frictions and heterogeneous multiple-worker firms. This allows us to distinguish firm entry and exit from hiring and firing in a model with equilibrium unemployment. We show that firing costs, sunk entry costs and bureaucratic flow costs have countervailing effects on firm and job turnover as different types of firms select to operate in the market.

Keywords: Firing Cost, Product Market Regulation, Firm Selection, Firm Turnover, Job Turnover, Unemployment

JEL classification: E24, J63, J64, J65

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1 Introduction

Anglo-Saxon countries have more flexible labor markets *and* less regulated product markets compared with continental European countries.¹ One would expect that the large differences in regulation across countries change the decisions of firms at both the hiring-and-firing and entry-and-exit margin. Empirically, however, firm and job turnover rates are rather similar across OECD countries.² We argue that the joint regulation of the labor and product market helps to explain this apparent puzzle. We show that firing costs, sunk entry costs and bureaucratic flow costs have countervailing effects on firm turnover and job turnover as different types of firms select to operate in the market.

For our analysis, we propose a model with search frictions and heterogeneous multiple-worker firms wherein stochastic shocks generate endogenous firm and job turnover. We model employment protection legislation (EPL) as wasteful firing costs³ and allow for two components of product market regulation (PMR): a fixed flow cost that captures the bureaucratic burden which firms have to bear every period and a sunk entry cost that captures administrative procedures and costs for licenses which firms need to acquire before they can start to produce.

We find that EPL and PMR have potentially countervailing effects on *firm* turnover. Firing costs reduce firm entry and increase firm exit. These opposite effects imply that the overall impact on firm turnover is small unless firms can default on firing costs upon exit, in which case firing costs tend to increase firm turnover. Concerning PMR, we find that bureaucratic flow costs increase firm turnover as exit becomes more attractive in bad times. Sunk entry costs, instead, decrease firm turnover since the firms that would find it attractive to exit in bad times no longer enter in good times.

The model also predicts that EPL and PMR influence *job* turnover in

¹The joint regulation of product and labor markets is borne out in the correlation between OECD indicators for the stringency of employment protection legislation (EPL) and product market regulation (PMR) which equals 0.72 and is highly significant (Nicoletti et al., 1999). The indicator of PMR includes bureaucratic flow costs to operate a firm in each period as well as sunk entry costs to set up a firm.

²Similar firm turnover rates across OECD countries are documented by OECD (2003), ch. 4, and Bartelsman et al. (2004). The similarity of job turnover rates of operating firms across OECD countries is discussed in Bertola and Rogerson (1997) and Pries and Rogerson (2005).

³In reality, transfers between firms and workers are also an important component of employment protection legislation. For a recent discussion on the effects of severance payments see Garibaldi and Violante (2005) and their references. All that matters for our purposes here is that some component of firing costs is “non-Coasean”.

opposite ways. Bureaucratic flow costs and sunk entry costs increase job turnover per firm through an equilibrium effect. Both costs lower the number of operating firms and so alleviate the congestion in the labor market. As search frictions decrease, firms post more vacancies and these vacancies are filled more quickly. Hence, the average firm size increases in good times although smaller search frictions also imply less labor hoarding in bad times. This positive effect of PMR on the job turnover per firm is opposite to the standard negative effect of firing costs. It illustrates how regulations in labor and product markets interact: more stringent PMR implies that operating firms are larger and thus modifies the effect of EPL on labor-hoarding. Because of this selection effect, it is not innocuous to neglect differences in PMR across countries when analyzing the impact of EPL in empirical research. In the light of our findings, empirical analyses based on firm-level panel data, as Autor et al. (2007) in this feature, seem particularly promising because such data allows one to control for firm selection by inserting plant or firm fixed effects in the regressions.

Related literature. We are not aware of an equilibrium model of unemployment in which barriers to entry and firing restrictions are meaningfully different. This is because standard search-and-matching models are based on the premise that “firms” employ only one worker. Since a firm is equivalent to a job in this case, firm entry and exit cannot be distinguished from job creation and destruction.⁴ In order to make that distinction, we build on the model with multiple-worker firms of Bertola and Caballero (1994). By introducing firm heterogeneity into their framework, we are able to characterize an economy where idiosyncratic shocks generate both firm and job turnover.

In a seminal paper, Hopenhayn and Rogerson (1993) analyze the effect of firing costs in a model with an intensive hiring-and-firing and extensive exit-and-entry margin. The main difference to our paper is that they provide a neoclassical model of *employment* whereas we embed our analysis in the Diamond-Mortensen-Pissarides (DMP) framework with search frictions and equilibrium *unemployment*. Our paper therefore fills the gap in the literature between the neoclassical models with job and firm turnover but without unemployment and the search-and-matching models with unemployment but without firm turnover.⁵

⁴For an analysis of hiring subsidies and firing costs using models with one-worker firms see Mortensen and Pissarides (2003) and Pissarides (2000), chapter 9. In these models, both policies have a similar effect on the match surplus and so hiring subsidies can be designed to offset the effects of firing costs. See also Fonseca et al. (2001) for an analysis of entry costs in such models.

⁵Smith (1999) allows for both margins in a search-and-matching model. He does not

Adding an exit-and-entry margin to the DMP model affords a number of new insights. Firstly, as discussed above, it allows us to derive predictions about the relationship between regulations and firm turnover. This differentiates our explanation from previous research about the similarity of job turnover rates across countries⁶ since our hypothesis, that this is due to the joint regulation of product and labor markets, helps to explain the similarity of both firm and job turnover rates. Secondly, our model naturally relates to recent empirical evidence in Haltiwanger et al. (2006) according to which 30 to 40% of job flows in OECD countries are due to firm entry and exit. We show that adjustments at the extensive margin induced by firing costs are unambiguously detrimental to employment since they reduce firm entry and increase firm exit. Moreover, for plausible parameter values, this negative effect dominates the well-known ambiguous impact of EPL on employment at the hiring-and-firing margin.

Finally, our paper contributes to the growing literature on the relationship between PMR and labor market outcomes. For example, Blanchard and Giavazzi (2003) argue that higher rents in regulated product markets make it more attractive for workers to increase their bargaining power in order to appropriate some of the rents. Ebell and Haefke (2006) extend their model to a dynamic context and determine the type of bargain (individual or collective) as a function of PMR. In this paper, we take the type of bargain as exogenous and focus instead on the interactions between EPL and PMR.

The rest of the paper is structured as follows. In Section 2, we present the basic model. Section 3 characterizes analytically how the firms' optimal behavior at both the extensive and intensive margin depends on EPL and PMR. We then solve numerically for the equilibrium in Section 4 and analyze the quantitative effects of regulations in Section 5. We conclude in Section 6.

2 Model

The model builds on Bertola and Caballero (1994), henceforth BC. The non-trivial innovation is that we add permanent productivity differences between

consider stochastic shocks, however, so that no firm turnover occurs in the steady state.

⁶Alternative explanations for the similarity of job turnover across countries are that (i) countries with strict EPL also have higher wage floors (Bertola and Rogerson, 1997) or (ii) strict EPL for permanent contracts is complemented with a segment of the labor market with temporary contracts and high turnover (Blanchard and Landier, 2002, Abowd et al., 1999, and Kahn, 2007) or (iii) the difficult measurement of job and especially firm turnover across countries.

firms. As shown below, this extension allows us to generate endogenous firm turnover. In the following, we first discuss the set-up of the model and show how wages and labor demand are determined. Then we characterize the optimal exit and entry decision of firms.

2.1 Set-up

The economy is populated by a continuum of workers which are homogeneous and infinitely-lived. They are employed by a continuum of firms whose mass is endogenously determined in equilibrium by the optimal entry and exit decisions characterized below. Firms are heterogeneous (indexed by the subscript i) as they differ with respect to their permanent productivity a_i . Each firm i also receives idiosyncratic transitory shocks to its business conditions. Both firms and workers are risk neutral and discount the future at rate r .

Technology. Firms use labor as their only input. The production technology has a fixed overhead component f and a variable component. The marginal-revenue product of labor at firm i is given by the function $\rho(a_i, l, \eta)$ which depends on the firm's permanent productivity a_i , the current employment level l and the idiosyncratic business condition of the firm η . In the remainder of the text, we simplify notation using $\rho_i(l, \eta)$ to denote firm i 's marginal-revenue product. We focus on idiosyncratic shocks η at the firm level since aggregate net changes in the number of firms or employment account for only a very small fraction of the observed gross turnover in OECD countries (see Bartelsman et al., 2004, and Haltiwanger et al., 2006).

We will consider two different specifications of $\rho(\cdot)$. If labor has constant returns to scale, so that $\rho(\cdot)$ is independent of l , we can characterize the solution of the model analytically. Constant returns, however, imply unrealistic hiring-and-firing behavior. Thus we also solve numerically the decreasing returns to scale model where $\partial\rho(\cdot)/\partial l < 0$.

Without loss of generality, we consider that $\rho_i(l, \eta)$ is increasing in a_i and η . We assume that η jumps discretely and infrequently between a good state and a bad state, which we denote η^1 and η^0 , respectively. Restricting our attention to only two temporary states allows us to characterize analytically the firms' optimality conditions. The permanent productivity shifter a_i then determines whether or not firms remain in the market when their business conditions are bad. In the steady state, only firms with a permanent productivity below a critical level exit in the bad state and enter in the good state. This implication of the model captures in a stylized way the empirical

fact that small and young firms have a higher hazard rate of exit than large firms.

Institutions. Firms are constrained by institutions in the product and labor market. We model EPL in the labor market as a wasteful firing cost F which impedes firms' layoff decisions. In the product market we allow for two components of wasteful PMR: (i) a higher fixed flow cost f that captures the bureaucratic burden which firms have to bear every period, (ii) a sunk entry cost C that captures administrative procedures and costs for licenses which firms need to acquire before they can start to produce. We distinguish between these two components of PMR for two reasons. Firstly, both components are relevant empirically so that indices of PMR contain both fixed-cost and sunk-cost components (see Nicoletti et al., 1999). Secondly, we will see below that the effect of PMR on firm turnover crucially depends on whether costs are fixed or sunk.

The labor market. Search frictions impede trade in the labor market. We assume a Cobb-Douglas matching technology with constant returns, so that every vacancy is matched to an unemployed worker at Poisson rate

$$q(\theta) = \xi\theta^\gamma, \quad -1 < \gamma < 0,$$

where θ is the steady state vacancy-unemployment ratio and ξ is the scaling factor of the matching function.⁷ The hiring process consumes time and resources. We assume that opening v vacancies entails a flow cost $cv^2/2$ so that the marginal vacancy cost is cv . Convex costs imply that firms do not find it optimal to post an infinite number of vacancies but instead gradually converge to their optimal level of employment.

A firm-worker match can be destroyed for two reasons: endogenous firing of the worker by the firm or exogenous quitting of the worker at the Poisson rate χ . As shown below, exogenous quits together with gradual hiring imply a finite firm size even when returns to labor are constant.

2.2 Hiring and firing

The assumptions above imply that the adjustment costs are a convex function of positive employment changes and a linear function of negative changes.

⁷The scaling factor helps us to match a plausible θ in the numerical part below. Note that in our model with multiple-worker firms, θ is not “intrinsically meaningless” as in the one-worker firm model of Shimer (2005). In our model, changes in the scaling factor ξ cannot be perfectly offset by changes in the vacancy-posting cost c .

This asymmetry is reflected in the firms' optimal labor demand: whereas upward employment adjustments are gradual, downward adjustments are lumpy and instantaneous.

In order to derive analytic solutions, we restrict our attention to the case where idiosyncratic shocks are such that firms hoard labor and/or fire workers if business conditions are bad, and hire workers if conditions are good. The labor demand schedule in good times maximizes the firms' asset value $A_i(l, \eta^1)$. In order to define that value, we introduce the firm i 's profit flow $\pi_i(l, v, \eta)$ as a function of employment l , posted vacancies v and business conditions η

$$\pi_i(l, v, \eta) \equiv \underbrace{\int_0^l \rho_i(x, \eta) dx}_{\text{revenues}} - \underbrace{w_i(l, \eta)l}_{\text{vacancy posting cost}} - \underbrace{\frac{cv^2}{2}}_{\text{fixed cost}} - f,$$

where the expression of the wage $w_i(l, \eta)$ is derived in the next subsection. Given that vacancies are filled at rate $q(\theta)$ and workers quit at rate χ , the law of motion of employment is $\dot{l} = q(\theta)v - \chi l$, where a dot denotes a time derivative. Hence, the Bellman equation of firm i in good times η^1 reads

$$\begin{aligned} rA_i(l, \eta^1) &= \max_v [\pi_i(l, v, \eta^1) + \frac{\partial A_i(l, \eta^1)}{\partial l} \overbrace{(q(\theta)v - \chi l)}^i] \\ &\quad + \delta^1 [A_i(l_i^0, \eta^0) - A_i(l, \eta^1) - (l - l_i^0)F] \\ \text{s.t. (i) } v &\geq 0; \text{ (ii) } l \geq l_i^0 \geq 0, \end{aligned} \quad (1)$$

where δ^j is the rate at which the firm's business condition η changes from state j to state $|j - 1|$ and l_i^0 is the optimal amount of hoarded labor in the bad state which will be determined below. The asset value has a straightforward interpretation: it is the sum of profit flows and expected capital gains which result from changes of the exogenous state variable η and of the endogenous state variable l . Upon transition from the good to the bad state, firms also pay the firing costs F multiplied by the number, $l - l_i^0$, of fired workers.

Differentiating equation (1) with respect to v gives the optimality condition for vacancy posting

$$\frac{cv}{q(\theta)} = \frac{\partial A_i(l, \eta^1)}{\partial l} \equiv S_i(l, \eta^1) \text{ for } S_i(l, \eta^1) \geq 0, \quad (2)$$

where $S_i(l, \eta)$ denotes the shadow value of labor. As in standard dynamic models, the optimal labor demand schedules can be characterized considering this shadow value. We assume that idiosyncratic shocks are such that

$S_i(l, \eta^0) < 0$ and $S_i(l, \eta^1) > 0$ for all firms i , so as to ensure that they do not post vacancies when business conditions are bad and that they hire workers when business conditions are good.⁸

Condition (2) shows that, in good times, the firm posts vacancies until the expected marginal recruitment costs $cv/q(\theta)$ equal the shadow value $S_i(l, \eta^1)$ of an additional worker. Since the marginal cost of posting vacancies is increasing, there always exists an interior optimum. Optimal recruitment efforts are smoothed over time and the firm converges gradually to its employment target. Note that a firm never posts vacancies and fires workers at the same time because workers are homogenous and thus the shadow value in a given firm has the same sign for all workers.

Differentiating equation (1) with respect to labor l shows that the shadow value of labor satisfies the following asset equation

$$\begin{aligned}
 rS_i(l, \eta^j) = & \underbrace{\rho_i(l, \eta^j) - w_i(l, \eta^j) - \frac{\partial w_i(l, \eta^j)}{\partial l} l}_{\text{Flow value of the marginal worker}} \\
 & \underbrace{-\chi S_i(l, \eta^j) + \frac{\partial S_i(l, \eta^j)}{\partial l} \dot{l} + \delta^j [S_i(l, \eta^{j-1}) - S_i(l, \eta^j)]}_{\text{Expected change of the shadow value of labor}}.
 \end{aligned} \tag{3}$$

The shadow value consists of two components: the flow value of the marginal worker and the expected change in the shadow value of labor. The flow value of hiring an additional worker in the first line of equation (3) is standard but for the term $-\partial w_i(\cdot)/\partial l l$. In our model with multiple-worker firms, the marginal cost of employment is not equal to the wage. Multiple-worker firms with monopsony power take into account the effect of their hiring decision on the wages of all previously employed (infra-marginal) workers. If labor has constant returns this effect vanishes. With decreasing returns, however, the term $-\partial w_i(\cdot)/\partial l l$ is positive and increases the shadow value of labor. Consequently firms hire more workers than without this hiring externality.⁹

The second line of equation (3) collects the expected changes in the shadow value of labor. Firstly, the worker quits with Poisson hazard χ in which case the shadow value falls to zero. Secondly, the shadow value changes as more workers are hired. Finally, the current business condition η^j changes state at Poisson rate δ^j .

⁸In the numerical part these two inequalities are satisfied for the choice of parameter values.

⁹This outcome of intra-firm bargaining has been derived in deterministic models such as the partial equilibrium analyses of Stole and Zwiebel (1996), the equilibrium analysis of Smith (1999), and Cahuc et al. (2004) with multiple types of workers and capital.

If firms are at the firing margin in bad times, as assumed in equation (1),

$$S_i(l, \eta^0) = -F. \quad (4)$$

When the shadow value is a decreasing function of employment, the optimal amount of labor hoarded in bad times l_i^0 is such that equation (4) holds.¹⁰

2.3 Wage determination

Wages are determined by Nash-bargaining between the firm and each employee. In good times the marginal worker generates a positive surplus. Then the Nash-bargaining problem has an interior optimum. Conversely, the shadow value of labor in bad times is negative so that the firm can credibly threaten to layoff its marginal worker. Then wages are set to make employees indifferent between staying in the firm and being unemployed.

We assume that all contracts can be renegotiated immediately so that all employees *in a given firm* earn the same wage. On the other hand, wages of workers *across firms* differ since firms' marginal surpluses are not the same and workers appropriate some of those surpluses when they have positive bargaining power. This is another major difference to the neoclassical model of Hopenhayn and Rogerson (1993) where labor markets are competitive so that homogenous workers earn identical wages.

To determine wages we need to define the workers' asset values. The value of being employed in firm i , $W_i(l, \eta^j)$, is given by

$$\begin{aligned} rW_i(l, \eta^j) = & w_i(l, \eta^j) + \chi [W^u - W_i(l, \eta^j)] \\ & + \frac{\partial W_i(l, \eta^j)}{\partial l} l + \delta^j [W_i(l, \eta^{j-1}) - W_i(l, \eta^j)] , \end{aligned} \quad (5)$$

where W^u denotes the asset value of an unemployed worker. The right-hand side of the equation has a straightforward interpretation analogous to the shadow value of labor above. Similarly, the asset value of an unemployed worker W^u can be decomposed as follows

$$rW^u = b + \theta q(\theta) (E[W_i(l, \eta^1)] - W^u) , \quad (6)$$

where b denotes the utility flow during unemployment, $\theta q(\theta)$ is the Poisson rate at which unemployed workers find a job and $E[W_i(l, \eta^1)]$ is the expected value of being matched to one of the posted vacancies. The expectation operator is required to compute the average capital gain of an unemployed

¹⁰Note that firing costs can generate an inaction range if $S_i(l, \eta^0) \in (-F, 0]$. In that range, firm i lets its labor force deplete at the exogenous rate χ of worker attrition.

worker because potential employers differ in terms of their permanent productivity a_i and current level of employment l . Wages are then set to solve the Nash-bargaining problem

$$\max_{w_i} (W_i(l, \eta) - W^u)^\beta (S_i(l, \eta))^{1-\beta}, \quad (7)$$

where β is the bargaining power of the worker.¹¹ Notice that individual bargaining implies that the worker bargains on the marginal revenue and not on the average revenue per worker. Since the shadow value of labor $S_i(l, \eta^0)$ is negative in bad times, the Nash-bargaining problem does not have an interior optimum. In this case, the firm only pays a wage such that workers are indifferent between being employed and being unemployed, i.e. $W_i(l, \eta^0) = W^u$. In good times instead, the shadow value of labor is positive and it follows from (7) that

$$\beta S_i(l, \eta^1) = (1 - \beta)(W_i(l, \eta^1) - W^u). \quad (8)$$

Using equations (5), (8), $W^u = W_i(l, \eta^0)$, $(1 - \beta)\dot{W}_i(l, \eta^1) = \beta\dot{S}_i(l, \eta^1)$ where dots denote time derivatives and $S_i(l, \eta^0) = -F$, we find that the bargained wage is given by

$$w_i(l, \eta^1) = \beta \left(\rho_i(l, \eta^1) - \frac{\partial w(l, \eta^1)}{\partial l} l - \delta^1 F \right) + (1 - \beta)rW^u.$$

Intuitively, a fraction β of the expected firing costs is passed on to the worker. The expression of the wage is rather standard but for the term $-\partial w_i(\cdot)/\partial l l$ due to the hiring externality. Because of this term, wages of workers in firms with good business conditions are determined by a non-homogenous first-order differential equation with the following solution¹²

$$w_i(l, \eta^1) = l^{-1/\beta} \int_0^l x^{\frac{1-\beta}{\beta}} \rho_i(x, \eta^1) dx - \beta \delta^1 F + (1 - \beta)rW^u. \quad (9)$$

¹¹In the literature the firing cost is sometimes subtracted from the threat point of the firm. This assumes that firms have to pay the firing cost if they cannot agree in the bargain with the marginal worker. The assumption matters for the effect of firing cost on wages which is negative under our assumption but positive otherwise. See Ljungqvist (2002) for a discussion of these different assumptions in the context of one-worker firm matching models.

¹²See for example Cahuc and Wasmer (2001) or Bertola and Garibaldi (2001). In canonical form the equation can be written as $x'(l) + x(l)p(l) + q(l) = 0$, where $x(l) = w(l, \eta)$, $p(l) = 1/(\beta l)$ and $q(l) = -(\rho(l, \eta) - \delta^1 F + (1 - \beta)rW^u/\beta)/l$. All solutions are then in the form $x(l) = \left(c - \int_0^l \frac{q(u)}{z(u)} du \right) z(l)$, where c is a constant of integration and $z(\cdot)$ is the solution of the homogenous differential equation $\frac{dz(l)}{dl} + p(l)z(l) = 0$. Homogenous solutions are proportional to $z(l) = l^{-1/\beta}$ and $c = 0$ to ensure that wages are bounded as $l \rightarrow 0$.

Wages in firms with good business conditions are a weighted sum of the worker's outside option, expected firing costs and an harmonic average of the employees' infra-marginal productivities.

2.4 Firm entry and exit

We now characterize the optimal behavior of firms at the exit-and-entry margin. More precisely, we determine the permanent productivity threshold a^* below which firms exit the market and the threshold a^{**} above which firms enter the market.

Exit threshold. To simplify matters, we assume that the “ownership” of the production opportunity a_i is lost after exit. In other words, a firm which leaves the market in the bad state does not retain the option to exploit the production opportunity when its business conditions switch back to the good state. Although exit of a given “owner” is definite, the production opportunity is still available and will be exploited again in good times if its asset value is high enough to make entry profitable.¹³

Accordingly, firm i is indifferent whether to exit the market or not when $A_i(l_i^0, \eta^0) = 0$. As the asset value is increasing in a_i , there exists a unique threshold productivity a^* such that $A_i(l_i^0, \eta^0) \geq 0$ if $a_i \geq a^*$. In other words, the firms with a permanent productivity below a^* are always better off by exiting in the bad state. The value of a^* can be formally characterized considering the asset values in the bad state

$$(r + \delta^0)A_i(l_i^0, \eta^0) = \pi_i(l_i^0, 0, \eta^0) + \delta^0 A_i(l_i^0, \eta^1) , \quad (10)$$

where employment is evaluated at its optimal value l_i^0 in the bad state.¹⁴ The exit condition is therefore satisfied when

$$\pi_i(l_i^0, 0, \eta^0 | a_i = a^*) + \delta^0 A_i(l_i^0, \eta^1 | a_i = a^*) = 0 . \quad (11)$$

¹³Alternatively, one could interpret a_i as managers' abilities. In this case the production opportunities are retained and entry costs deter firms from exiting the market. If entry costs are independent of a_i and firms can default on firing costs upon exit (an extension discussed in Section 5), it also follows that larger firms are more likely to exit in bad business conditions than small firms. This contrasts with empirical evidence, so that we stick to the more parsimonious modeling proposed in the text.

¹⁴Employment in bad times is constant at l_i^0 if firms do not hoard labor in bad times, $l_i^0 = 0$ (which is the case for firms at the exit-and-entry margin in our analysis with constant returns to labor), or if workers do not quit for exogenous reasons, $\chi = 0$ (which we assume for our analysis with decreasing returns to labor). Allowing for varying employment levels in bad times would substantially complicate the analysis as we discuss further below.

Note that optimal vacancy posting implies that $\partial A_i(l, \eta^1) / \partial l = cv_i(l, \eta^1) / q(\theta)$, so that equation (1) evaluated at a^* simplifies to

$$(\delta^1 + r)A_i(l_i^0, \eta^1 | a_i = a^*) = \int_0^{l_i^0} \rho(x, \eta^1, a^*) dx - w_i(l_i^0, \eta^1)l_i^0 - f + \frac{cv_i(l_i^0, \eta^1)^2}{2} - \frac{cv_i(l_i^0, \eta^1)\chi l_i^0}{q(\theta)}. \quad (12)$$

This equation enables us to solve for a^* using the exit condition (11) and the optimal labor demand schedules derived above. Equation (12) shows explicitly that bureaucratic flow costs which increase the fixed costs f directly lower the asset value of the marginal firm and thus increase firm exit. Firing costs instead matter through their effect on the vacancy-posting policy $v_i(\cdot)$ and the amount of labor hoarded l_i^0 . We will derive these effects in closed form when we assume constant returns to labor in the next section.

Entry threshold. A firm is indifferent whether to enter the market in good times if its value just covers the entry cost. The condition $A_i(0, \eta^1 | a_i = a^{**}) = C$ implicitly determines the critical value a^{**} of the indifferent firm. For brevity, we focus on the interesting case where $a^{**} < a^*$, so that the economy exhibits positive firm turnover. Then, replacing $l = 0$, $l_i^0 = 0$, $A_i(l_i^0, \eta^0) = 0$ in equation (1) and $\partial A_i(l, \eta^1) / \partial l = cv_i(l, \eta^1) / q(\theta)$ as above, we can simplify the entry condition to obtain

$$v_i(0, \eta^1 | a_i = a^{**}) = \sqrt{\frac{2((r + \delta^1)C + f)}{c}}. \quad (13)$$

Given that more productive firms post more vacancies ($v_i(\cdot)$ is increasing in a_i), the entry threshold a^{**} is unique. Equation (13) shows that entry costs C and bureaucratic flow costs which increase f directly change a^{**} , whereas firing costs F matter through their indirect influence on the firms' vacancy schedules $v_i(\cdot)$. Depending on the model's parameters (especially C), a^{**} might be larger than a^* . Then, there is no firm turnover. The interesting case is when the equilibrium is characterized by the following cross-sectional distribution: the production opportunities below a^{**} are vacant, those between a^{**} and a^* are exploited in the good state and left unused in the bad state, while production opportunities above a^* always remain in operation.

We now characterize the optimal behavior of firms in more detail by specifying $\rho_i(l, \eta)$. We first start with the simple case in which returns to labor

are constant so that the hiring externality vanishes and the wage determination is simplified. This allows us to derive analytic insights on optimal firm behavior at the hiring-and-firing and exit-and-entry margin. We then extend our model to decreasing returns to labor and allow for equilibrium effects by solving the model numerically.

3 Partial equilibrium with constant returns

When returns to labor are constant, the marginal revenues $\rho_i(\cdot)$ and consequently the shadow value of labor $S_i(\cdot)$ are independent of the employment level. Nevertheless, firm size is well defined because of the convex vacancy-posting costs and worker quits. For concreteness we assume that¹⁵

$$\rho_i^1 = a_i \eta^1 \text{ and } \rho_i^0 = a_i \eta^0 .$$

The permanent components a_i are distributed on the interval $[\underline{a}, \bar{a}]$, where $\underline{a} \geq 0$. The density of the available “production opportunities” a_i is exogenous. Which of these production opportunities are exploited in equilibrium is determined by the entry and exit conditions.

Since the shadow value of labor is increasing in a_i and independent of l , there exists a threshold $\tilde{a}$ at which $S_i(l, \eta^0 | a_i = \tilde{a}) = -F$ for all l . Hence, firms with $a_i > \tilde{a}$ never fire workers and firms with $a_i < \tilde{a}$ fire all their employees when business conditions turn bad. Constant returns to labor therefore imply that firms either fire all their employees or none at all. As shown in Remark 1, taking the outside option of workers W^u as given allows us to derive analytic results for the labor-hoarding threshold as well as the wage schedule and stationary level of employment.

Remark 1: *If returns to labor are constant,
(i) firms ($a_i \leq \tilde{a}$) that do not hoard labor pay the wage*

$$w_i(\eta^1) = \beta (a_i \eta^1 - \delta^1 F) + (1 - \beta) r W^u$$

when business conditions are good. Their shadow value of labor is

$$S_i(\eta^1) = \frac{1 - \beta}{r + \chi + \delta^1} (a_i \eta^1 - \delta^1 F - r W^u)$$

¹⁵In this specification marginal revenues in bad business conditions decrease by the common fraction η^0/η^1 independently of firm type a_i . This implies that the absolute difference between marginal revenues in the good and bad state is larger for firms with high a_i . Alternatively, one could consider an additive specification $a_i + \eta$ without changing the main insights.

and their labor demand converges to

$$\bar{l}_i(\eta^1) = q(\theta)^2 S_i(\eta^1) / (c\chi) .$$

(ii) the labor-hoarding threshold $\tilde{a}$ depends negatively on F and positively on the workers' outside option W^u .

Proof: See Appendix A.

The results contained in Remark 1 are intuitive. Firms in the good state pay a wage that is a weighted average of the surplus and of the worker's outside option. As shown formally in the proof, wages of labor-hoarding firms in the bad state are less than the annuity value of being unemployed because workers are willing to take wage cuts in bad times if they are compensated in good times. That option value of waiting is larger for firms with a higher permanent productivity a_i so that wages in such firms are relatively lower when business conditions are bad. Note that firing costs lower the shadow value of labor in good times $S_i(\eta^1)$. As we will see below, this is the channel through which firing costs affect the entry and exit thresholds.

Employment converges to a stationary level in good times due to our assumptions of convex vacancy-posting costs and positive worker quit rate ($\chi > 0$). Intuitively, firms attain larger employment levels if the vacancy-posting cost c is smaller and the surplus of the match $S_i(\eta^1)$ is larger. Since the surplus increases with a_i , firms with permanently higher marginal revenues are also larger.

Concerning the labor-hoarding threshold $\tilde{a}$, it is very intuitive that more firms decide to hoard labor as firing costs increase. Conversely, if the workers' outside option W^u increases, wages are higher, labor hoarding is more expensive and so $\tilde{a}$ increases. Interestingly, entry costs C and fixed costs f have no direct effect on the labor hoarding threshold $\tilde{a}$.

Turning our attention to the extensive margin, we notice that endogenous firm and job turnover arise when $a^{**} < a^* < \tilde{a}$.¹⁶ In this case, firms just above the exit margin a^* do not hoard any labor. These firms entail losses during bad times because they employ no worker, and thus generate no revenues, but pay the flow cost f . Yet, these firms stay in the market due to the positive option value of waiting for good times. Remark 2 shows that the threshold a^* below which firms exit the market and the threshold a^{**} above which firms decide to enter the market can be derived in closed form.

¹⁶For endogenous job turnover of continuing firms we need $a^* < \tilde{a}$ and for endogenous firm turnover $a^{**} < a^*$.

Remark 2:

(i) The exit threshold a^* is given by

$$a^* = \left(\frac{1}{\eta^1} \right) \left[rW^u + \delta^1 F + \left(\frac{r + \chi + \delta^1}{q(\theta)(1 - \beta)} \right) \sqrt{2c \left(\frac{r + \delta^0 + \delta^1}{\delta^0} \right) f} \right].$$

(ii) The entry threshold a^{**} is given by

$$a^{**} = \left(\frac{1}{\eta^1} \right) \left[rW^u + \delta^1 F + \left(\frac{r + \chi + \delta^1}{q(\theta)(1 - \beta)} \right) \sqrt{2c \left((r + \delta^1) C + f \right)} \right].$$

Hence, there is positive firm turnover ($a^{**} < a^*$) when entry costs are smaller than the expected costs of waiting for a good shock ($C < f/\delta^0$).

Proof: see Appendix A.

Remark 2 shows that the exit threshold a^* is increasing in fixed costs f : less firms find it optimal to remain in the market in bad times as the cost of doing so increases. Firing costs F also increase a^* because they reduce the option value of waiting by lowering the shadow value of labor. The effect of firing costs and fixed costs on the entry threshold a^{**} is qualitatively the same. Entry costs C instead affect the exit and entry thresholds differently. On the one hand, entry costs raise the entry threshold a^{**} as expected. On the other hand, they leave the exit threshold a^* unchanged since set-up costs are sunk.

We now summarize the effect of EPL and PMR on job and firm turnover in Table 1 where the results on firm turnover implicitly assume a uniform distribution of a_i .¹⁷ The comparative statics results immediately follow from Remarks 1 and 2.¹⁸ The main message from Table 1 is that firing costs change the behavior of firms at both extensive and intensive margins, whereas PMR only matters for the entry-and-exit margin if we abstract from equilibrium effects.

¹⁷With a non-constant density, additional conditions are necessary to sign the effects. For example, an upward shift of the thresholds a^{**} and a^* which increases the interval $[a^{**}, a^*]$ then does not imply necessarily a larger probability mass of firms in that interval and thus higher firm turnover.

¹⁸Firing costs F are neutral because $\partial a^{**}/\partial F = \partial a^*/\partial F$. To establish the positive effect of fixed costs f , we notice that $\partial a^*/\partial f > \partial a^{**}/\partial f$ because

$$\frac{r + \delta^0 + \delta^1}{\delta^0} \sqrt{2c \left((r + \delta^1) C + f \right)} > \sqrt{2c \left(\frac{r + \delta^0 + \delta^1}{\delta^0} \right) f}.$$

	Labor-hoarding threshold $\tilde{a}$	Exit threshold a^*	Entry threshold a^{**}	Firm Turnover $[a^{**}, a^*]$
<i>Compar. Static</i>	(1)	(2)	(3)	(4)
Increase in F	↓	↑	↑	No effect
Increase in f	No effect	↑↑	↑	↑
Increase in C	No effect	No effect	↑	↓

Table 1: Partial equilibrium effects when returns to labor are constant.

Concerning firm turnover, columns (2)-(4) in Table 1 show that firing costs, sunk entry costs and bureaucratic flow costs have quite different and countervailing effects. Firing costs leave firm turnover unchanged unless firms that exit the market default on firing costs. This seems to be a plausible assumption for some countries with strict EPL, like Portugal or Spain, where firing costs may not be paid by the firm in case of insolvency because either firms are granted exemptions in the event of plant closures or firms are simply not able to cover payments to workers in case of bankruptcy (see Samaniego, 2006, Section 2). The option to default implies that firms exit when their asset values in the bad state are below the total firing costs which they would need to pay to remain in business. Clearly, this increases the incentive to exit and thus firm turnover (see Appendix C for formal derivations). Whereas the effect of firing costs on firm turnover is either positive or zero, sunk entry costs decrease it. These results suggest that it is not so surprising that Portugal or France, where both labor and product markets are more heavily regulated, have similar firm turnover rates as Canada or the US.

Concerning job turnover, column (1) in Table 1 shows the standard positive effect of firing costs on labor hoarding. A fall of the threshold $\tilde{a}$ implies that more firms hoard labor in bad times so that job turnover falls. Product market regulation instead does not matter for job turnover of operating firms. This is no longer true, however, in the equilibrium analysis below. Then the selection effect of PMR feeds back into the hiring and firing decisions because changes in the distribution of active firms affect the outside option of workers W^u and labor market tightness θ .

Unfortunately, these interactions cannot be characterized analytically. Since we have to resort to numerical methods, we first relax the constant returns assumption. Although it has allowed us to derive analytical results, its rather unrealistic implication that firms either fire all workers, or none at all, motivates us to consider decreasing returns to labor.

4 Equilibrium analysis with decreasing returns

With decreasing returns to labor, the marginal revenues $\rho_i(\cdot)$ and the shadow value of labor $S_i(\cdot)$ are decreasing functions of the level of employment. Firm size is therefore well defined so that we can simplify matters by ruling out worker quits, $\chi = 0$.¹⁹ We characterize the firm's labor-demand schedule using the linearization²⁰

$$\rho_i^1(l) = a_i \eta^1 - \sigma l \quad \text{and} \quad \rho_i^0(l) = a_i \eta^0 - \sigma l .$$

The main difference compared with the constant-returns model is that it can be optimal for firms to fire a share, but not all, of their labor force. As firms reduce their employment level, marginal revenues increase until the shadow value of labor $S_i(l_i^0, \eta^0) = -F$, where l_i^0 is the optimal amount of hoarded labor. In good times, the firm gradually increases the size of its labor force. The shadow value of labor and consequently the number of posted vacancies fall with the time τ spent in the good state. Accordingly, introducing τ as a state variable enables us to keep track of the number of hired workers and posted vacancies. For brevity, we use hereafter the notation $l_i(\tau)$ and $v_i(\tau)$ for employment and posted vacancies in the good state. The explicit solution of the optimal vacancy schedule $v_i(\tau)$ is similar to the one in BC and can be found in Appendix B.

The optimal entry and exit decisions are computed using the general expressions in Section 2.4. Compared with the constant-returns case, the more complicated wage setting prevents us from obtaining an analytical expression for hoarded labor, $l_i(0)$, and vacancies posted, $v_i(0)$, when a good shock arrives. This is why we have to solve the model numerically.

Equilibrium definition. We define an equilibrium for the economy as a set of aggregate quantities for employment and vacancies $\{L, V\}$, matching rate $q(\theta)$, permanent productivity thresholds $\{a^{**}, a^*, \tilde{a}\}$, employment distribution $\mu(l|a_i)$ and infinite sequences for quantities $\{l_i(\tau), v_i(\tau)\}_{\tau=0}^{\infty}$ and prices $\{w_i^0, w_i^1(\tau)\}_{\tau=0}^{\infty}$, where τ denotes the time spent in the good state, so that:

¹⁹Positive quit rates substantially complicate the analysis with decreasing returns since the labor hoarded in the bad state is no longer constant: firms use worker quits to save on firing costs. This makes the employment level in the bad state a function of the time spent in that state, which in turn implies that employment in the good state is a function of how much time the firm has previously spent in the bad state. These complications are ruled out by setting $\chi = 0$.

²⁰Note that a_i only shifts the intercept but σ , the parameter which governs how much marginal revenues decrease in employment, does not depend on a_i . This assumption could be relaxed but would complicate the interpretation of the results, as the convergence speed of firms towards their target employment level would differ.

- Given the matching rate and prices, $\{l_i(\tau), v_i(\tau)\}_{\tau=0}^{\infty}$ solve firm i 's optimization problem.
- Wages in the bad state w_i^0 make employees indifferent between staying in the firm and being unemployed. Wages in the good state $\{w_i^1(\tau)\}_{\tau=0}^{\infty}$ solve the Nash-bargaining problem.
- Permanent productivity thresholds $\{a^{**}, a^*, \tilde{a}\}$ are determined by the optimal entry, exit and labor-hoarding decisions of firms.
- Aggregate quantities $\{L, V\}$ result from the aggregation of firms' optimal labor demand schedules.
- The matching rate $q(\theta)$ is given by the aggregate matching function.
- The flows into and out of the employment distribution $\mu(l|a_i)$ balance out.

The equilibrium is solved for quite similarly to BC so that we collect all the derivations in Appendix B. The model with decreasing returns to scale can be solved largely analytically but for the two conditions that determine $l_i(0)$ and $v_i(0)$. The decentralized equilibrium is not efficient because of the congestion externality, as the solution of the social planner's problem makes clear (results are available on request). As in BC, the standard Hosios condition is not enough to restore efficiency because of intra-firm bargaining distortions and firm heterogeneity. For the purposes of this paper, however, it is important to note that the congestion externalities are rather unimportant for the parameter values which we use below.

Numerical algorithm. The algorithm proceeds in three steps. In Step 1, we set starting values for labor market tightness θ and the exit threshold a^* . In Step 2, we solve for $v_i(\tau)$, $l_i(\tau)$, determine the labor-hoarding threshold $\tilde{a}$ and use the optimal value of $v_i(0)$ to solve for a^{**} . We then update θ and repeat Step 2 until convergence at precision of 10^{-6} . Otherwise we continue with Step 3 and update a^* using the condition $A_i(l, \eta^0 | a_i = a^*) = 0$. Unless a^* has converged at a numerical precision of 10^{-6} , we update θ and restart the algorithm at Step 2. Our numerical results indicate that the equilibrium labor market tightness θ is locally unique.²¹

²¹We calculate the slope of the feedback locus $\theta'(\theta)$ when we compute θ (in each iteration for each given value of a^*). In the program we check that the locus $\theta'(\theta)$ intersects only once with the 45-degree line. We have always found a unique equilibrium for positive θ given the parameter values we considered.

Parameterization. We keep the stylized shock structure of the model to ensure comparability of the numerical results with the analytical results in Section 3. We leave it to future research to calibrate a more ambitious model with more than two stochastic states. In the following section, we rather provide quantitative results that illustrate: (i) how the joint regulation of labor and product markets implies similar firm turnover and job turnover per firm in unregulated and regulated economies; (ii) how firing costs affect unemployment and productivity by changing the firms' decisions at the exit-and-entry and hiring-and-firing margin. These numerical results complement the analytical predictions for the constant-returns case presented in the previous section.

As a benchmark, we choose a flexible economy without firing and entry costs ($F = C = 0$). This is a reasonable approximation for the US where entry barriers and firing costs are quite small compared with continental European countries. Instead, the fixed costs f in the benchmark are positive to generate firm turnover. We set f to match a firm exit rate of 10%, as observed in the US (see OECD, 2003, and Bartelsman et al., 2004). The other parameter values for the benchmark economy in Table 2 are set to match US labor market statistics.

We assume that production opportunities are uniformly distributed, $a_i \sim U(0, \bar{a})$, since the constant density facilitates the interpretation of the numerical results. The upper bound of the uniform distribution is determined normalizing total employment in the frictionless economy to 1.²² We set the annual interest rate r to 0.05. We choose a matching efficiency $\gamma = -0.6$ which is in the middle of the range of estimates reported by Petrongolo and Pissarides (2001). Furthermore, in the absence of well established values, we follow the common practice of setting $\beta = -\gamma$, although the Hosios condition does not ensure efficiency in our model.

²²We restrict our attention to the case where firms in the frictionless economy operate solely in the good state so that all labor is shed if a bad shock occurs. Since the workforce in the good state is equal to $l_i = \frac{a_i \eta^1 - b}{\sigma}$, the normalization of employment to 1 in the frictionless economy implies

$$1 = \frac{\delta^0}{\delta^0 + \delta^1} \frac{1}{\bar{a} - \underline{a}} \int_{b/\eta^1}^{\bar{a}} l_i da = \left(\frac{\delta^0}{\delta^0 + \delta^1} \right) \left(\frac{1}{\bar{a} - \underline{a}} \right) \left(\frac{1}{2\sigma} \eta^1 \left(\bar{a}^2 - \left(\frac{b}{\eta^1} \right)^2 \right) - \frac{b}{\sigma} \left(\bar{a} - \frac{b}{\eta^1} \right) \right) .$$

Setting $\underline{a} = 0$ implies that $\bar{a}$ is the positive root of a quadratic equation

$$\bar{a} = \frac{\frac{\delta^0 + \delta^1}{\delta^0} + \frac{b}{\sigma} + \sqrt{\left(\frac{\delta^0 + \delta^1}{\delta^0} + \frac{b}{\sigma} \right)^2 - \frac{b^2}{\sigma^2}}}{\eta^1 / \sigma} .$$

The dynamic transitions between the good and bad state are parametrized as $\delta^j = 0.4$, for $j = 0, 1$. This implies that created and destroyed jobs have a 70% chance to persist for one year.²³ These values are broadly consistent with estimates reported in Davis et al. (1996). The utility flow of non-market activity $b = 0.175$ yields a replacement ratio of about 40%, that is the upper end of empirically observed replacement ratios in the US. We check that the value of b implies that workers in the frictionless economy find it optimal to supply labor in the good state so that $b \leq \rho_i^1(l)$, for all a_i . Indeed, this condition is always satisfied in equilibrium for $\sigma = 0.4$ and $\eta^1 = 1$. We set η^0 equal to $b/\bar{a}$ so that firms do not hoard labor in the frictionless economy. The slope σ of the marginal revenue function yields a plausible “labor share” of 63%.²⁴

It remains to choose the values for the marginal cost of vacancy posting, c , and the scaling factor of the matching function, ξ . We set them to obtain a yearly job finding rate of 5.4, as estimated for the US by Shimer (2005), and a vacancy-unemployment ratio of 0.54, the average of estimates by Hall (2005) for the period from December 2000 to December 2002. The average vacancy and unemployment durations are therefore equal to 5.2 and 9.6 weeks, respectively. We find that these last two moments are matched when $c = 0.115$ and $\xi = 6.9$. The implied average recruitment costs are slightly below one week of average labor earnings and the unemployment rate is 6.9%.

These benchmark parameters imply positive firm turnover but no labor hoarding. To see formally why this is the case, consider the two remarks in Section 3. Remark 2 shows that when there is no entry costs, a positive fixed cost implies that the exit threshold is necessarily larger than the entry threshold and firm turnover is positive as $a^* > a^{**}$. The absence of labor hoarding when business conditions are bad ($\tilde{a} = \bar{a}$), is due to the conjunction of small frictions, sizable shocks and no firing costs $F = 0$. As we will see below, increasing firing costs lowers $\tilde{a}$ so that some firms hoard labor.

²³The Poisson distribution implies that, over a yearly time period, a created job is destroyed with probability $1 - e^{-0.4} = 0.33$.

²⁴This differs from standard matching models where the labor share is close to one. In our model, firms also incur fixed costs that amount to 14% of output in the benchmark economy. Moreover, workers bargain on the marginal surplus so that firms are able to appropriate infra-marginal profits.

<i>Fixed Parameters</i>	<i>Value</i>	<i>Target</i>	<i>Source</i>
Discount rate r	0.05	-	Standard
Elasticity matching function γ	-0.6	-	Petrongolo and Pissarides (2001)
Worker bargaining power β	0.6	-	$\beta = -\gamma$, Hosios condition.
<i>Matched parameters</i>			
Poisson hazard of shocks δ^j	0.4	Persistence of created/destroyed job to last for 1 year=70%	Davis et al. (1996)
Utility flow during unemployment b	0.175	Replacement ratio=40%	Shimer (2005)
Decreasing returns σ	0.4	Labor share=63%	Standard
Scale matching function ξ	6.9	Annual job finding rate=5.4	Shimer (2005)
Marginal vacancy cost c	0.115	$V-U$ ratio=0.54	Hall (2005)
Fixed costs f	0.4	Firm exit rate=10%	Bartelsmann et al. (2004)

Table 2: Parameter values for the benchmark economy without regulation.

At this stage, let us mention that the model is also qualitatively consistent with stylized facts on the cross-sectional distributions for wages and employment across firms with different size. The model implies a right-skewed wage distribution, a U-shaped employment-firm-size distribution, a firm-size wage premium and lower wage dispersion at large firms. The qualitative implications of the model for wages and their distribution are similar to Bertola and Garibaldi (2001) so that we do not report them more extensively. Starting from this benchmark case, we now illustrate how EPL and PMR affect firm and job turnover, unemployment and productivity.

5 Firing costs and product market regulation

In this section, we first analyze the effects of EPL and PMR on firm and job turnover. We consider a sequential increase in firing costs, bureaucratic flow costs and sunk entry costs. We show that firm turnover and job turnover per firm in regulated and unregulated economies can be quite similar. Then we analyze how firing costs affect labor productivity, unemployment and welfare. Finally, we discuss the extension of the model if we allow firms to default on the firing costs upon market exit.

Regulation and turnover. The turnover statistics for the benchmark economy are displayed in column (1) of Table 3. As mentioned in the previous section, the firm exit rate is equal to 10%. Notice that in the steady-state, the entry and exit rates are necessarily equal, so that the latter is a sufficient index of firm turnover. In the second row of Table 3, we report the size of the flows out of the unemployment pool, i.e. the job finding rate times the unemployment rate. We use this measure as a statistic of job creation since worker turnover is equal to job turnover in our model. In the third row of Table 3, we account for the changes in the mass of operating firms and report job creation per firm. This statistic is of more interest to us since available job turnover statistics are typically computed at the firm or establishment level.

We choose the parameter values of PMR and firing costs so that firm turnover and job turnover per firm in the unregulated economy (column (1)) are the same as in the regulated economy (column (4)). The effects of the different regulations on firm turnover are consistent with the predictions of the partial equilibrium analysis in Section 3. In column (2) we observe that firm turnover increases slightly to 10.5% when we raise firing costs to the average wage in the benchmark economy. Although firing costs do not change the total mass of firms that exit and enter the market, consistent

	$F = 0$	$F = 0.4$	$F = 0.4$	$F = 0.4$
	$f = 0.4$	$f = 0.4$	$f = 0.44$	$f = 0.44$
	$C = 0$	$C = 0$	$C = 0$	$C = 0.12$
<i>Equilibrium Variables</i>	(1)	(2)	(3)	(4)
Firm exit rate (in %)	10.0%	10.5%	11.5%	10.1%
Job creation	0.37	0.34	0.34	0.33
Job creation / firm	1.09	1.06	1.10	1.09

Table 3: The effect of EPL and PMR on turnover.

with the partial-equilibrium effect reported in Table 1, they concurrently lower the mass of operating firms by raising the entry and exit threshold. Hence, the turnover *rate*, which is equal to the ratio of these two masses, increases slightly with the stringency of EPL.

As predicted by the analytical results in Section 3, the effect of an increase in bureaucratic flow costs on firm turnover is much stronger. Increasing the flow costs by 10% raises firm turnover by 1 percentage point (see column (3)). Finally, we show in column (4) that, as expected, higher barriers to entry reduce firm turnover. The value of the entry costs is chosen so that the incidence of firm entry and exit in the regulated economy in column (4) is nearly the same as in the flexible economy in column (1). These numerical results illustrate how the countervailing effects of PMR and EPL may explain why cross-country data on aggregate firm turnover, as reported by the OECD (2003),²⁵ do not reveal a clear ranking among regulated and non-regulated countries.

We now show that an analogous argument can be made to explain the similar job turnover per firm across OECD countries. In the second row of Table 3, we first illustrate that firing costs reduce aggregate job flows whereas PMR has little effect. The negative impact of firing costs is not surprising given that they induce firms to hoard more labor in bad times.²⁶ PMR also has a small negative effect because it lowers the mass of operating firms and

²⁵The OECD and World Bank made a great effort to make the data comparable across countries but reporting thresholds in terms of the number of employees or sales imply that the data do not contain the universe of firms operating in each country. Thus measurement is a serious issue for firm and also job turnover statistics where it is not obvious how this affects the comparability of the job and firm turnover statistics across countries.

²⁶See Bentolila and Bertola (1990) for a theoretical analysis of the effect of EPL on job turnover in a given firm. This result has been confirmed empirically by Haltiwanger et al. (2006), Micco and Pages (2006) or Messina and Vallanti (2007), using difference-in-difference techniques on international industry or firm-level data.

so the number of posted vacancies.

These aggregate statistics hide, however, that the job turnover per firm, which corresponds more closely to the empirical job turnover statistics at the firm and establishment level, is the same in the regulated and unregulated economy (see the third row in columns (1) and (4)). This is because fixed costs exclude less efficient firms and consequently decrease the congestion in the labor market. The surviving firms face a lower equilibrium tightness θ . Hence, they post more vacancies in good times and hoard less labor in bad times since it is easier to recruit workers.²⁷ The larger firm size in good times and smaller firm size in bad times imply that fixed costs generate more job turnover at the firm level. These equilibrium effects highlight how PMR matters for the hiring-and-firing margin through firm selection. Our numerical example also illustrates that this mechanism is not negligible, as the positive effect of PMR on job turnover per firm can offset the negative effect of firing costs for reasonable parameter values.²⁸

Empirical evidence in Haltiwanger et al. (2006) for continuing firms (Table 15, column (4)) supports these results. They find that the strictness of employment protection is negatively associated with job turnover for continuing firms whereas the opposite is the case for product market regulation (although the coefficients for the latter are not significant). The size of the coefficients is of similar magnitude but opposite signs, suggesting that both types of regulation have offsetting effects on job turnover for continuing firms. The high positive correlation between EPL and PMR is thus a possible explanation for the puzzling similarity in job turnover rates at the firm level across countries with very different regulations.

Although the joint regulation of product and labor markets does not matter much for turnover in the numerical example above, the regulated economy in Table 3, column (4), has a 3 percentage point higher unemployment rate and a 2.4% lower labor productivity. These total effects, however, hide quite non-linear relationships between regulation, unemployment and productivity. We illustrate this now by focusing on the impact of firing costs.

²⁷A given firm hoards less labor but also the mass of labor-hoarding firms may be reduced when there is an equilibrium effect on the labor-hoarding threshold $\tilde{a}$ (as shown in Section 3, there is no direct effect of PMR on $\tilde{a}$). We find, for example, that fixed flow costs reduce the mass of firms which hoard a positive amount of labor as $\tilde{a}$ increases slightly.

²⁸Note that sunk entry costs in Table 3, column (4), have an additional countervailing effect by reducing firm entry and exit and thus job turnover.

Firing costs, unemployment and productivity. Figure 1 reports the effect of firing costs on the productivity thresholds, unemployment rate, labor productivity and “welfare” in the benchmark economy. The welfare measure is defined as aggregate output net of the regulation costs (see Appendix B for a formal expression).

The upper-left panel in Figure 1 plots the exogenous upper bound, $\bar{a}$, of the productivity distribution along with the endogenous labor-hoarding, exit and entry thresholds. As mentioned above, there is no labor hoarding in the benchmark economy when firing costs are zero, so that $\bar{a} = \tilde{a}$. Higher firing costs lower $\tilde{a}$ only when they exceed 0.2, i.e. roughly half of the average wage. Above this value, firing costs reduce the labor hoarding threshold $\tilde{a}$ until it equals the exit threshold a^* . Afterwards, $\tilde{a}$ and a^* coincide, as firms that would hoard labor decide to exit the market. Finally, as predicted by Remark 2, firing costs increase both the exit a^* and entry a^{**} thresholds, if only slightly for our parameter values.

The upper-right panel of Figure 1 plots the unemployment rate as a function of firing costs. Interestingly, the negative impact of EPL on employment is stronger when firing costs are low. The reason is that, as explained in the previous paragraph, firing costs have no effect on the intensive hiring-and-firing margin when they are below 0.2. The impact of firing costs on the extensive entry-and-exit margin, on the other hand, is always detrimental to employment since they reduce firm entry and increase firm exit. When firing costs also augment labor hoarding, which is the case for values of F above 0.2, their overall impact is obviously smaller. Hence, allowing for firm entry and exit increases the adverse influence of firing costs on employment. This explains why, in contrast with Bertola and Caballero (1994), our parametrization yields a positive correlation between EPL and unemployment.²⁹ In general, the total effect of firing costs on employment will depend on modeling details, especially the specification of the stochastic process, which determine the relative importance of the effect on the extensive and intensive margin. Nevertheless, the effect of EPL on the extensive margin is of interest because 30 to 40% of job flows occur through firm entry and exit empirically (see Haltiwanger et al., 2006).

The lower-left panel of Figure 1 shows that average labor productivity is an inversely U-shaped function of firing costs. As above, the reason is the countervailing effect of firing costs at the extensive and intensive margin. For low values of F , the effect of firing costs on the entry and exit threshold

²⁹Note that we find a positive effect of firing costs on unemployment although firing costs do not reduce the firms’ threat point in the Nash bargaining problem (7). In one-firm-one-worker models instead, Ljungqvist (2002) has pointed out that firing costs generally increase employment under this modeling assumption.

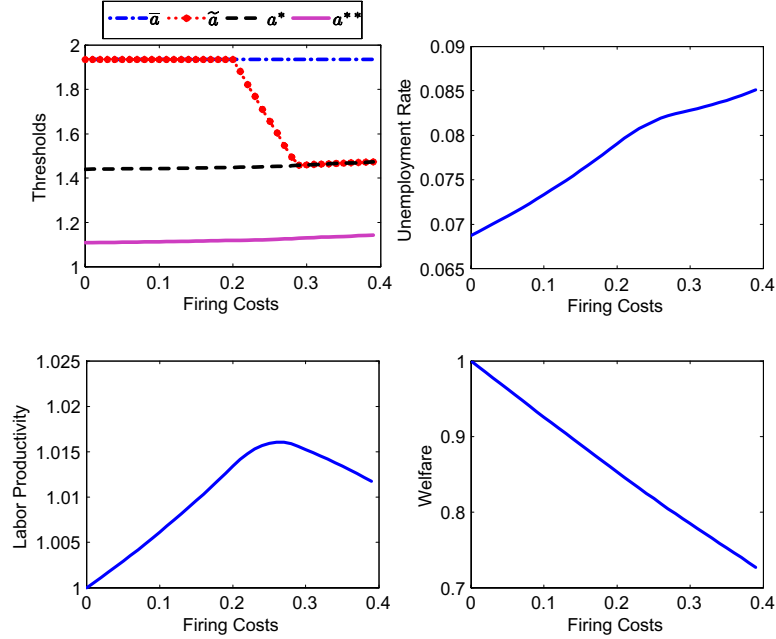


Figure 1: Effects of firing costs on the benchmark economy.

implies that firms with higher permanent productivity operate in the market. For higher values of F , this selection effect is outweighed by the usual negative impact on productivity due to labor hoarding (see the empirical evidence for the US by Autor et al., 2007, in this feature).

Finally, the lower-right panel shows that firing costs reduce welfare across steady states.³⁰ This conclusion also holds if we rebate the costs of regulation at the aggregate level: for our choice of parameters, the congestion externality is too small to generate welfare enhancing effects of regulation.

Concerning the interaction of labor and product market regulation in terms of welfare, we find that bureaucratic flow costs and sunk entry costs interact negatively by reducing both the incentives of firms to enter the market in good times and the capacity of firms to produce in bad times. However, incumbent firms benefit from higher entry costs since these costs reduce the congestion externality. Thus, if a government wanted to introduce firing costs, it could “buy” the support of these firms by compensating them with entry barriers. Finally, higher bureaucratic flow costs mechanically reduce the aggregate cost of firing regulation as less firms operate in the market.

³⁰Of course, these steady-state comparisons neglect possibly important effects in the transition periods.

Non-enforceable EPL. We now discuss the extension of our model if we allow firms to default on firing costs upon market exit. This extension of the model is of interest since for some high EPL countries like Portugal and Spain, firing costs may not be paid by the firm in case of insolvency. Either firms are granted exemptions in the event of plant closures or firms are simply not able to cover payments to workers in case of bankruptcy (see Samaniego, 2006, Section 2). Moreover, Haltiwanger et al. (2006) find that job flows are more similar across countries for small firms. This suggests that small firms can avoid the costs of regulation more easily, for example, by defaulting on payment obligations in the case of firm closure.³¹ We show in Appendix C how the equations for the firms' asset value change if firms can default on firing costs. This is non-trivial since wages and vacancy-posting behavior also change for these firms.

Not surprisingly, we find that higher firing costs increase firm turnover more if exiting firms default on firing costs.³² In this case, firing costs increase the fraction of job turnover through firm exit which is consistent with the empirical evidence that job turnover in Portugal, a country with strict EPL, occurs more through firm exit and entry than in the US (Blanchard and Portugal, 2001).³³

In terms of welfare interactions, we find that default introduces a complementarity between firing costs and low entry costs. Low entry costs then make it profitable for less efficient firms to enter the market in the good state. This may rationalize why small firms are exempted from the legislation in some countries with stringent EPL, like Germany or Italy.

6 Conclusion

We have solved a dynamic stochastic equilibrium model with equilibrium unemployment that allows to distinguish firm entry and exit from hiring and firing. We have characterized analytically how the hiring-and-firing and exit-and-entry margin depend on EPL and PMR for the specific case where returns to labor are constant. We then have extended the model to decreasing

³¹Moreover, in some countries, like Germany or Italy, firms with a size below a certain threshold (15 in Italy and 10 in Germany), are exempted from EPL.

³²This confirms the results of Samaniego (2006) who uses the neoclassical model of Hopenhayn and Rogerson (1993).

³³Blanchard and Portugal (2001) show that job turnover in Portugal is 60-70% of the turnover in the US but job destruction due to firm exit is 50% higher in Portugal than in the US. Changes of legislation in the US, due to the introduction of the "good-faith" exception in the wrongful discharge legislation, have had little impact on firm exit and entry instead (Autor et al., 2007).

returns and illustrated numerically the effect of EPL and PMR. Can these results help us understand the differences in labor market outcomes between Anglo-Saxon and continental European countries? Our model suggests that the joint regulation of labor and product markets is a potential explanation for the similarity of firm turnover and job turnover per firm across OECD countries. Our analysis also concurs with empirical findings on the negative employment effects of regulated product markets (see, for example, Bertrand and Kramarz, 2002).

The proposed framework lends itself naturally to many extensions. In ongoing research we extend the model to allow for a more realistic shock structure. The shocks in our model are rather big, much larger than most of the shocks which hit firms in reality. Accordingly, our measure of firing costs relates more closely to restrictions on collective dismissal. Introducing techniques developed by Bertola and Garibaldi (2001) would allow for a finer shock structure and thus more realistic predictions. Alternatively, future research may strive to endogenize market power by making the elasticity of the marginal revenues function an explicit function of the number of operating firms. This would certainly introduce additional channels of interaction between EPL and PMR. Finally, further analysis of the interactions between regulation in labor and product markets may also help to understand why labor market regulation has changed very little over time in most OECD countries (see Brügemann, 2007, for an analysis of the persistence of EPL).

Appendices

Appendix A: Proofs of Remarks 1 and 2.

Proof of Remark 1:

Result (i): The expression for the wage immediately follows from replacing $\rho_i^1 = a_i \eta^1$ into equation (9) and solving for the integral. We then have $\partial w_i(l, \eta^1)/\partial l = 0$. Constant returns also imply that $\partial S_i(l, \eta^1)/\partial l = 0$. Replacing these two equalities into (3), and using the fact that $S_i(\eta^0) = -F$ if $a_i \leq \tilde{a}$, yields the shadow value of labor in the good state. The employment level to which firms converge in the good state then follows from $S_i(\eta^1) = cv_i/q(\theta)$ together with the law of motion for employment $\dot{l}_i = q(\theta)v_i - \chi l_i$.

Result (ii): Workers are indifferent between being unemployed and being employed by a bad firm, so that $W_i(l, \eta^0) = W^u$. Given that $\dot{l} = 0$ in the bad state, equations (5) and (8) imply that the wage of a labor-hoarding firm in the bad state is

$$w_i(l, \eta^0) = rW^u - \delta^0 \left(\frac{\beta}{1 - \beta} \right) S_i(l, \eta^1). \quad (\text{A1})$$

Reinserting this solution into (3) and using result (i) for the case of constant returns, we find that by continuity (approaching $\tilde{a}$ from above)

$$\lim_{a_i \rightarrow \tilde{a}^+} S_i(\eta^1) = S_i(\eta^1 | a_i = \tilde{a}) = \frac{1 - \beta}{r + \chi + \delta^1} (\tilde{a}\eta^1 - \delta^1 F - rW^u) .$$

Then $\lim_{a_i \rightarrow \tilde{a}^+} S_i(\eta^0) = -F$ if and only if

$$\tilde{a} = \frac{rW^u (r + \chi + \delta^1 + \delta^0) - F ((r + \chi)^2 + (\delta^0 + \delta^1)(r + \chi))}{(r + \chi + \delta^1) \eta^0 + \delta^0 \eta^1}$$

which proves result (ii). ■

Proof of Remark 2:

Result (i): We use the exit condition $A_i(0, \eta^0 | a_i = a^*) = 0$ which by equation (10) is equivalent to $A_i(0, \eta^1 | a_i = a^*) = f/\delta^0$. Note that $\partial A_i(l, \eta^1)/\partial l = S_i(\eta^1)$ and $l_i = 0$ so that $\dot{l}_i = q(\theta)v_i$. Equation (1) then simplifies to

$$(r + \delta^1)A_i(0, \eta^1) = -\frac{c(v_i)^2}{2} - f + S_i(\eta^1)q(\theta)v_i = \frac{q(\theta)^2 S_i(\eta^1)^2}{2c} - f,$$

where the second equality uses $S_i(\eta^1) = cv_i/q(\theta)$. Plugging in $A_i(0, \eta^1 | a_i = a^*) = f/\delta^0$ and rearranging yields

$$S_i(\eta^1)^2 = \frac{r + \delta^1 + \delta^0}{\delta^0} \frac{2cf}{q(\theta)^2}.$$

Using Remark 1(i) to substitute out $S_i(\eta^1)$ and rearranging we obtain result (i).

Result (ii): We start as above but substitute $A_i(0, \eta^1 | a_i = a^{**}) = C$ into the second last equation (instead of $A_i(0, \eta^1 | a_i = a^*) = f/\delta^0$). Proceeding as before we obtain result (ii). Comparing the different expressions substituted, $C < f/\delta^0$ implies $a^{**} < a^*$ since $A_i(0, \eta^1)$ is increasing in a_i . ■

Appendix B: Equilibrium with decreasing returns.

In this appendix, we use the notation $l_i(\tau)$ and $v_i(\tau)$ which we introduced in Section 4. The amount of hoarded labor is then $l_i(0)$. Furthermore recall that there are no worker quits, $\chi = 0$, in Section 4.

Optimal labor demand.

Substituting $\rho_i^1(l) = a_i\eta^1 - \sigma l$ into equation (9) and solving for the integral, we find that

$$w_i(\tau) \equiv w_i(l_i(\tau), \eta^1) = (1 - \beta)rW^u + \beta(a_i\eta^1 - \delta^1 F) - \frac{\beta\sigma}{1 + \beta}l_i(\tau) . \quad (\text{B1})$$

Let $\omega_i(\cdot)$ denote the marginal cost of employment. By definition

$$\omega_i(\tau) \equiv w_i(l_i(\tau), \eta^1) - \frac{\partial w_i(l, \eta^1)}{\partial l} l_i(\tau) = w_i(l_i(\tau), \eta^1) - \frac{\beta\sigma}{1+\beta} l_i(\tau) ,$$

where the last equality follows from the previous equation. Note the incentive of firms to increase employment so as to reduce the marginal worker's surplus. This incentive is stronger the larger β and σ are. Reinserting this expression into (3), using (2) and differentiating with respect to the time τ spent in the good state (notice that θ does not change in the steady state), we get

$$-\dot{w}_i(\tau) - \frac{\sigma}{1+\beta} \dot{l}_i(\tau) + \frac{c\ddot{v}_i(\tau)}{q(\theta)} = (r + \delta^1) \frac{c\dot{v}_i(\tau)}{q(\theta)} . \quad (\text{B2})$$

In order to express the derivative of the wage $\dot{w}_i(\tau)$ in terms of the vacancy schedule, we notice that the asset value of employment in a firm which has been in the good state for τ periods is

$$rW_i(\tau) = w_i(\tau) + \delta^1(W^u - W_i(\tau)) + \dot{W}_i(\tau) . \quad (\text{B3})$$

As shown in BC, p. 441-442, non-enforceability of long-term contracts implies that the asset value of a worker in a firm with low productivity is equal to the outside option W^u (firms can credibly threaten workers to fire them otherwise). The Nash-bargaining solution (8) allows us to substitute the worker asset values since

$$W_i(\tau) = W^u + \frac{\beta}{1-\beta} S_i(\tau) = W^u + \frac{\beta}{1-\beta} \frac{cv_i(\tau)}{q(\theta)} , \quad (\text{B4})$$

and so

$$\dot{W}_i(\tau) = \frac{\beta}{1-\beta} \frac{c\dot{v}_i(\tau)}{q(\theta)} .$$

Inserting these two expressions into (B3) and using (2) yields

$$w_i(\tau) = rW^u + \frac{\beta c}{1-\beta} \frac{(r + \delta^1)v_i(\tau) - \dot{v}_i(\tau)}{q(\theta)} . \quad (\text{B5})$$

Differentiating (B5) with respect to τ , we finally obtain

$$\dot{w}_i(\tau) = \frac{\beta c}{1-\beta} \frac{(r + \delta^1)\dot{v}_i(\tau) - \ddot{v}_i(\tau)}{q(\theta)} .$$

Substituting $\dot{w}_i(\tau)$ into equation (B2) therefore yields

$$-\frac{\beta c}{1-\beta} \frac{(r + \delta^1)\dot{v}_i(\tau) - \ddot{v}_i(\tau)}{q(\theta)} + \frac{c\ddot{v}_i(\tau)}{q(\theta)} - (r + \delta^1) \frac{c\dot{v}_i(\tau)}{q(\theta)} - \frac{\sigma}{1+\beta} \dot{l}_i(\tau) = 0 .$$

Using the law of motion of employment, replacing $q(\theta)$ by $\xi\theta^\gamma$ and rearranging leads to

$$\ddot{v}_i(\tau) - (r + \delta^1)\dot{v}_i(\tau) - \sigma \frac{1 - \beta}{1 + \beta} \frac{\xi^2 \theta^{2\gamma}}{c} v_i = 0 .$$

The solution of this second-order differential equation subject to the boundary condition $\lim_{\tau \rightarrow \infty} v_i(\tau) = 0$ is

$$v_i(\tau) = v_i(0)e^{-\lambda\tau} \text{ with } \lambda = 1/2 \left(- (r + \delta^1) + \sqrt{(r + \delta^1)^2 + 4\sigma \frac{1 - \beta}{1 + \beta} \frac{\xi^2 \theta^{2\gamma}}{c}} \right) . \quad (\text{B6})$$

Accordingly, permanent differences between firms matter only for the absolute number of posted vacancies but not for the behavior of the vacancy policy over time (λ would depend on i if we allowed σ to differ across firms). The rate of convergence is also independent of firm entry and exit.

Wage schedules.

Let w_i^0 denote the wage of workers employed by firm i when it is in the bad state. In order to characterize explicitly w_i^0 , we have to derive the expected capital gains of a job seeker. Let $\varphi(\tau, a)$ denote the probability density that the firm contacted by a job seeker has a permanent productivity equal to a and that it has been in the good state for τ periods. Given that a and τ are independently distributed, we have

$$\varphi(\tau, a) = \frac{v(\tau, a)\delta^1 e^{-\delta^1 \tau} u(a)}{\int_{a^{**}}^{\bar{a}} \left(\int_0^\infty v(\tau, a)\delta^1 e^{-\delta^1 \tau} d\tau \right) u(a) da} = \frac{(\delta^1 + \lambda) e^{-(\delta^1 + \lambda)\tau} v(0, a) u(a)}{\int_{a^{**}}^{\bar{a}} v(0, a) u(a) da} .$$

Equation (B4) then implies that the expected gain from finding a job is

$$\begin{aligned} E[W_i(\tau)] - W^u &= \frac{\beta}{1 - \beta} \frac{c}{q(\theta)} \left(\int_{a^{**}}^{\bar{a}} \int_0^\infty v(\tau, a) \varphi(\tau, a) d\tau da \right) \\ &= \frac{\beta}{1 - \beta} \frac{c}{q(\theta)} \left(\frac{\int_{a^{**}}^{\bar{a}} \int_0^\infty (\delta^1 + \lambda) e^{-(\delta^1 + \lambda)\tau} (v(0, a))^2 u(a) da}{\int_{a^{**}}^{\bar{a}} v(0, a) u(a) da} \right) \\ &= \frac{\beta}{1 - \beta} \frac{c}{q(\theta)} \frac{\delta^1 + \lambda}{\delta^1 + 2\lambda} \left(\frac{\int_{a^{**}}^{\bar{a}} (v(0, a))^2 u(a) da}{\int_{a^{**}}^{\bar{a}} v(0, a) u(a) da} \right) . \end{aligned}$$

Notice that this expression puts a larger weight on firms with a higher initial level of vacancy posting, since a given job seeker is more likely to meet them. Equation (6) then implies that

$$rW^u = b + c\theta \frac{\beta}{1 - \beta} \frac{\delta^1 + \lambda}{\delta^1 + 2\lambda} \left(\frac{\int_{a^{**}}^{\bar{a}} (v(0, a))^2 u(a) da}{\int_{a^{**}}^{\bar{a}} v(0, a) u(a) da} \right) .$$

From equation (A1), we have

$$w_i^0 = rW^u - \delta^0 \frac{\beta}{1-\beta} \frac{cv_i(0)}{q(\theta)}, \quad (\text{B7})$$

and thus

$$w_i^0 = b + \frac{\beta}{1-\beta} c \left(\theta \frac{\delta^1 + \lambda}{\delta^1 + 2\lambda} \left(\frac{\int_{a^{**}}^{\bar{a}} (v(0, a))^2 u(a) da}{\int_{a^{**}}^{\bar{a}} v(0, a) u(a) da} \right) - \frac{\delta^0}{q(\theta)} v_i^1(0) \right). \quad (\text{B8})$$

The wage in the bad state depends positively on the total number of posted vacancies which increase the outside option; but negatively on the expected number of vacancies posted in the own firm i if good times arrive. Intuitively, workers are willing to take larger wage cuts in bad times if they are compensated in good times. To determine wages in good times we use equations (B5) and (B6) which imply

$$w_i(\tau) = rW^u + \frac{\beta c}{1-\beta} \frac{(r + \delta^1 + \lambda) v_i(0) e^{-\lambda\tau}}{q(\theta)}. \quad (\text{B9})$$

Plugging in W^u , we get

$$w_i(\tau) = b + \frac{\beta}{1-\beta} c \left(\theta \frac{\delta^1 + \lambda}{\delta^1 + 2\lambda} \left(\frac{\int_{a^{**}}^{\bar{a}} (v(0, a))^2 u(a) da}{\int_{a^{**}}^{\bar{a}} v(0, a) u(a) da} \right) + \frac{(r + \delta^1 + \lambda) e^{-\lambda\tau}}{q(\theta)} v_i(0) \right). \quad (\text{B10})$$

Note that the wage in good times depends positively on $v_i(0)$. As $\tau \rightarrow \infty$ all workers earn the same wage because firms exploit their monopsony power and hire until $w_i(\tau)$ converges to

$$w^\infty \equiv \lim_{\tau \rightarrow \infty} w_i(\tau) = b + \frac{\beta}{1-\beta} c \theta \frac{\delta^1 + \lambda}{\delta^1 + 2\lambda} \left(\frac{\int_{a^{**}}^{\bar{a}} (v(0, a))^2 u(a) da}{\int_{a^{**}}^{\bar{a}} v(0, a) u(a) da} \right).$$

To sum up: workers in firms with high permanent productivity $a_i > a^*$ earn lower wages in bad times, higher wages upon arrival of good times and the same wage as $\tau \rightarrow \infty$.

Boundary conditions for $v_i(0)$ and $l_i(0)$.

The employment and vacancy schedules are fully characterized by the initial conditions $v_i(0)$ and $l_i(0)$ since $v_i(\tau) = v_i(0)e^{-\lambda\tau}$ and

$$l_i(\tau) = l_i(0) + \frac{q(\theta)}{\lambda} (1 - e^{-\lambda\tau}) v_i(0).$$

Initial conditions of “labor hoarding” firms. We need to determine two boundary conditions. The first one ensures that workers are indifferent between employment and unemployment when the firm is in the bad state. By definition, the shadow value in the bad state is given by

$$a_i \eta^0 - \sigma l_i(0) - \omega_i(0) + \delta^0 \left(\frac{c v_i(0)}{q(\theta)} - (-F) \right) = -rF .$$

Since the wage rate of a firm in the bad state is determined by supply conditions, as explained above, we have $\omega_i(0) = w_i^0$. It follows that the previous equation simplifies to

$$v_i(0) = \frac{q(\theta)}{\delta^0 c} \left(\sigma l_i(0) - (a_i \eta^0 + (\delta^0 + r)F - w_i^0) \right) . \quad (\text{B11})$$

The second boundary condition follows from equating (B1) and (B9)

$$(1 - \beta) r W^u + \beta (a_i \eta^1 - \delta^0 F) - \frac{\beta \sigma}{1 + \beta} l_i(0) = r W^u + \frac{\beta c}{1 - \beta} \frac{r + \delta^1 + \lambda}{q(\theta)} v_i(0) .$$

Using (B7) to substitute $r W^u$, we finally obtain

$$\frac{c}{1 - \beta} \frac{r + \delta^1 + \beta \delta^0 + \lambda}{q(\theta)} v_i(0) = -\frac{\sigma}{1 + \beta} l_i(0) + (a_i \eta^1 - \delta F - w_i^0) . \quad (\text{B12})$$

Inserting, w_i^0 from (B8), the two boundary conditions (B11) and (B12) can be used to solve for $v_i(0)$ and $l_i(0)$.

Initial vacancy posting of “non-permanent” firms. Since $l_i(0) = 0$ for all firms with $a_i \leq a^*$, we only need one boundary condition to characterize their optimal labor demand schedules. The value of $v_i(0)$ can be determined as before by setting equal (B1) and (B9), so that

$$\frac{c}{1 - \beta} \frac{(r + \delta^1 + \lambda)}{q(\theta)} v_i(0) = a_i \eta^1 - \delta F - r W^u . \quad (\text{B13})$$

This completes the characterization of firm i ’s optimal policies. It remains to close the model by determining the aggregate stock of vacancies V and employment L and thus θ .

Equilibrium.

In steady state the number of firms turning good has to equal the number of firms turning bad (for each a_i). Thus the proportion of firms in the good and bad state, ϕ^0 and ϕ^1 respectively, are given by

$$\delta^1 \phi^0 = \delta^0 \phi^1 \text{ and } \phi^0 + \phi^1 = 1 .$$

It follows that

$$\phi^0 = \frac{\delta^1}{\delta^0 + \delta^1} \text{ and } \phi^1 = \frac{\delta^0}{\delta^0 + \delta^1} .$$

Given that the density of τ is exponentially distributed, aggregate vacancies V and employment L are given by

$$V = \phi^1 \delta^1 \int_{a^{**}}^{\bar{a}} \left(\int_0^\infty v_i(\tau) e^{-\delta^1 \tau} d\tau \right) dU(a) = \phi^1 \frac{\delta^1}{\delta^1 + \lambda} \int_{a^{**}}^{\bar{a}} v_i(0) u(a) da .$$

and

$$L = \phi^1 \delta^1 \int_{a^{**}}^{\bar{a}} \left(\int_0^\infty l_i(\tau) e^{-\delta^1 \tau} d\tau \right) dU(a) + \phi^0 \int_{\tilde{a}}^{\bar{a}} l_i(0) u(a) da .$$

where $\tilde{a}$ is the labor hoarding threshold. Plugging in the expression for $l_i(\tau)$, we get

$$L = \phi^1 \frac{q(\theta)}{(\delta^1 + \lambda)} \int_{a^{**}}^{\bar{a}} v_i(0) u(a) da + \int_{\tilde{a}}^{\bar{a}} l_i(0) u(a) da ,$$

where the aggregate employment level depends negatively on a^{**} and $\tilde{a}$.

Steady-state employment distribution.

We now prove that the employment distribution is ergodic for any given a . Let $T(l, a)$ denote the time elapsed in the good state such that a firm with permanent productivity a has a workforce equal to l . Given that

$$l(T, a) = l(0, a) + q(\theta) \left(\frac{1 - e^{-\lambda T}}{\lambda} \right) v(0, a) ,$$

the function $T(l, a)$ reads

$$T(l, a) = - \left(\frac{1}{\lambda} \right) \ln \left(1 - \frac{(l - l(0, a)) \lambda}{v(0, a) q(\theta)} \right) .$$

As business conditions switch to the bad state at the Poisson rate δ , the employment density $\mu(l|a)$ reads

$$\mu(l|a) = \begin{cases} \phi^0 & \text{if } l = l(0, a) \\ \phi^1 \delta^1 e^{-\delta^1 T(l, a)} & \text{if } l \in \left(l(0, a), l(0, a) + \frac{q(\theta)}{\lambda} v(0, a) \right) \\ 0 & \text{if } l \notin \left[l(0, a), l(0, a) + \frac{q(\theta)}{\lambda} v(0, a) \right] \end{cases} .$$

According to Kolmogorov's forward equation, the cross-sectional distribution is stationary when

$$-\frac{\partial \mu(l|a)}{\partial l} \dot{l}(T, a) = \delta^1 \mu(l|a) . \quad (\text{B14})$$

Differentiating the density above yields for all $l \in \left(l(0, a), l(0, a) + \frac{q(\theta)}{\lambda} v(0, a) \right)$

$$-\frac{\partial \mu(l|a)}{\partial l} = \delta^1 [v(0, a)q(\theta) - (l - l(0, a)) \lambda]^{-1} \mu(l|a) .$$

Reinserting the following equality

$$\dot{l}(T, a) = q(\theta)v(T, a) = q(\theta)v(0, a)e^{-\lambda T(l, a)} = v(0, a)q(\theta) - (l - l(0, a)) \lambda ,$$

into the stationarity condition (B14) proves that it is satisfied by the proposed steady-state distribution. In other words, when employment is distributed according to $\mu(l|a)$, inflows and outflows for any employment level balance out.

Output and Welfare.

Each firm in the good state has a “production-equivalent” flow

$$y_i^1 \equiv a_i \eta^1 l_i(\tau) - \frac{\sigma}{2} l_i(\tau)^2 - \frac{c}{2} v_i(\tau)^2 - \delta^1 F(l_i(\tau) - l_i(0)) .$$

Firms in the good state bear a steady-state mobility cost $\delta^1 F(l_i(\tau) - l_i(0))$, and costs of vacancy posting $cv_i(\tau)^2/2$ (below we add the fixed cost f which all firms have to pay). Instead each labor-hoarding firm in the bad state has a “production-equivalent” flow

$$y_i^0 \equiv a_i \eta^0 l_i(0) - \frac{\sigma}{2} l_i(0)^2 .$$

Thus, gross output is defined as

$$\begin{aligned} Y = & \phi^1 \int_{a^{**}}^{\bar{a}} \left(\int_0^\infty \delta^1 e^{-\delta^1 \tau} \left((a_i \eta^1 - \delta^1 F) l_i(\tau) - \frac{\sigma}{2} l_i(\tau)^2 - \frac{c}{2} v_i(\tau)^2 \right) d\tau \right) u(a) d(a) \\ & + \phi^0 \int_{\tilde{a}}^{\bar{a}} \left((a_i \eta^0 + \delta^0 F) l_i(0) - \frac{\sigma}{2} l_i(0)^2 \right) u(a) d(a) , \end{aligned}$$

up to a constant of integration that can be neglected if output is zero for firms that do not use labor. We compute welfare Ω adding the production-equivalent flow b for all unemployed workers and subtracting the fixed flow cost f for all firms in the market, as well as the sunk entry costs C incurred by the firms which enter the market, so that

$$\Omega = Y + bU - \phi^1 \int_{a^{**} \wedge a^*}^{a^*} (f + \delta^1 C) u(a) d(a) - \int_{a^*}^{\bar{a}} f u(a) d(a) ,$$

where $a^{**} \wedge a^* \equiv \min\{a^{**}, a^*\}$. Plugging in the expression for $l_i(\tau)$ and $v_i(\tau)$ allows us to solve for the first integral in Y explicitly

$$\begin{aligned} & \int_0^\infty \delta^1 e^{-\delta^1 \tau} \left((a_i \eta^1 - \delta^1 F) l_i(\tau) - \frac{\sigma}{2} l_i(\tau)^2 - \frac{c}{2} v_i(\tau)^2 \right) d\tau \\ &= (a_i \eta^1 - \delta^1 F) l_i(0) + \frac{q(\theta)}{\delta^1 + \lambda} (a_i \eta^1 - \delta^1 F) v_i(0) - \frac{\sigma}{2} l_i(0)^2 \\ & \quad - \frac{\sigma q(\theta)^2}{(\lambda + \delta^1)(2\lambda + \delta^1)} v_i(0)^2 - \frac{\sigma}{\lambda + \delta^1} q(\theta) l_i(0) v_i(0) - \frac{\delta^1}{2\lambda + \delta^1} \frac{c}{2} v_i(0)^2 . \end{aligned}$$

which can be integrated over $a \in [a^{**} \wedge a^*; \bar{a}]$ to compute the first term of Y .

Appendix C: Equilibrium when firing costs are non-enforceable.

We distinguish the asset values of firms that exit the market and default on the firing costs by attaching the superscript b for bankruptcy. We also apply superscripts b to their state and control variables.

Entry and Exit.

If firm i defaults in the bad state $A_i^b(\eta^0) = 0$. Thus its asset value in the good state satisfies

$$rA_i^b(l, \eta^1) = \pi_i(l, v_i^b(l, \eta^1), \eta^1) + \frac{\partial A_i^b(l, \eta^1)}{\partial l} (q(\theta) v_i^b(l, \eta^1) - \chi l) - \delta^1 A_i^b(l, \eta^1) .$$

Notice the difference with equation (1) since firing costs $-Fl$ are not deduced upon transition to the bad state. The comparison makes clear that firms which exit the market always find it optimal to concurrently default on the firing costs.

Exit rule. The option to go bankrupt implies that firms exit the market when

$$A_i(l_i^0, \eta^0) - F(l - l_i^0) < 0 .$$

Intuitively, higher firing costs make it more attractive for firms to default in the bad state, especially for firms which fire many workers. This gives rise to a time-inconsistency problem as the incentive to default increases with the employment level in good times. For some firms, it may be initially credible that they will hoard labor, which allows them to reduce their wage bill (We have seen in section 2.3 how firms which pay firing costs pass on part of that cost to workers through lower wages). As the employment level in these firms increases, however, they may find it optimal to default if hit by a bad shock. Solving this problem in which the optimal credible policies are a function of employment is clearly beyond the scope of this paper. We restrict our

attention to the case where firms are credible to hoard labor only if they never exit the market in the bad state

$$A_i(l_i^0, \eta^0) - F(\bar{l}_i - l_i^0) \geq 0 ,$$

where $\bar{l}_i = \lim_{\tau \rightarrow \infty} l_i(\tau)$ is the employment target in good times. The exit threshold a^{**} is then determined as the value of a for which this expression holds with equality.

Entry rule. The entry threshold a^{**} can be determined as before, except that if $a^{**} < a^*$ one has to use the asset value of defaulting firm so that

$$A_i^b(0, \eta^1 | a^{**}) = C \text{ if } a^{**} < a^* .$$

Labor demand of defaulting firms.

These firms do not pay the firing costs so that the shadow value of labor in the bad state is equal to zero

$$S_i^b(l, \eta^0) = 0 .$$

The optimality condition for vacancy posting, instead, is the same as before and given by (2). Whether firms default on the firing costs when business conditions turn bad is common knowledge. Following the same steps as in Section 2.3 and replacing the marginal revenues function, implies that wages in good times are

$$w_i^b(l_i^b(\tau), \eta^1) = (1 - \beta) rW^u + \beta a_i \eta^1 - \frac{\beta \sigma}{1 + \beta} l_i^b(\tau) . \quad (C1)$$

Intuitively, the wage paid by defaulting firms is higher because workers anticipate that their employer will not pay the firing costs. This increases the size of the match surplus and thus the worker's wage.

It is easy to check that the derivations of the vacancy schedule as a function of τ do not depend on the choice of the firm at the extensive margin. Hence, $v_i^b(\tau) = v_i^b(0)e^{-\lambda\tau}$ where λ is defined in equation (B6). As explained in Appendix B, this implies that the wage schedule satisfies

$$w_i^b(\tau) = rW^u + \frac{\beta c}{1 - \beta} \frac{(r + \delta^1)v_i^b(\tau) - \dot{v}_i^b(\tau)}{q(\theta)} = \frac{\beta c}{1 - \beta} \frac{(r + \delta^1 + \lambda)v_i^b(\tau)}{q(\theta)} \quad (C2)$$

Equations (C1) and (C2) evaluated at $\tau = 0$ are consistent if and only if

$$\frac{c}{1 - \beta} \frac{(r + \delta^1 + \lambda)}{q(\theta)} v_i^b(0) = a_i \eta^1 - rW^u .$$

Comparing this initial condition with (B13) shows that the option to default increases the recruitment effort of firms which exit in the bad state.

Aggregation.

Given that defaulting firms post more vacancies, we have to distinguish them from labor-hoarding firms when we derive aggregate quantities. Accordingly, the value of being unemployed is now given by

$$rW^u = b + c\theta \frac{\beta}{1-\beta} \frac{\delta + \lambda}{\delta + 2\lambda} \left(\frac{\int_{a^{**} \wedge a^*}^{a^*} (v^b(0, a))^2 u(a) da + \int_{a^{**}}^{\bar{a}} (v(0, a))^2 u(a) da}{\int_{a^{**} \wedge a^*}^{a^*} v^b(0, a) u(a) da + \int_{a^{**}}^{\bar{a}} v(0, a) u(a) da} \right) ,$$

where $a^{**} \wedge a^* \equiv \min \{a^{**}, a^*\}$. Similarly, the aggregate number of vacancies posted is

$$V = \phi^1 \frac{\delta^1}{\delta^1 + \lambda} \left(\int_{a^{**} \wedge a^*}^{a^*} v^b(0, a) u(a) da + \int_{a^*}^{\bar{a}} v(0, a) u(a) da \right) ,$$

and aggregate employment

$$L = \phi^1 \frac{q(\theta)}{(\delta^1 + \lambda)} \left(\int_{a^{**} \wedge a^*}^{a^*} v^b(0, a) u(a) da + \int_{a^*}^{\bar{a}} v(0, a) u(a) da \right) + \int_{\tilde{a}}^{\bar{a}} l_i(0) u(a) da .$$

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